Exercise session

Exercise 1. Counting melons

The goal of this exercise is to show that melons of the $O(N)^3$ model can be represented by unlabeled colored trees. The generating function of leading-order two-point graphs can be written as:

$$G_{LO}(g,\mu) = \sum_{p,q} C_{p,q} g^{p+q} \mu^q \,,$$

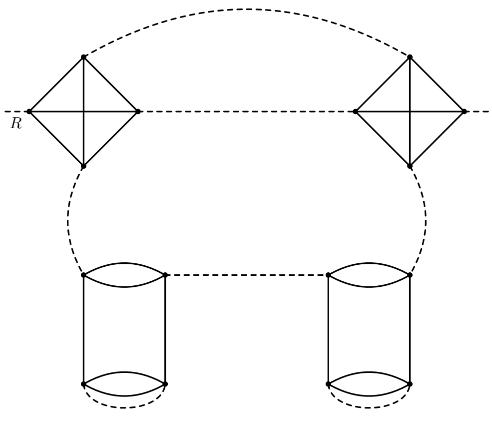
where $C_{p,q}$ counts the number of melonic two-point graphs with p melons based on tetrahedrons (type I) and q melons based on pillows (type II).

We wish to find a representation of these graphs as trees. To do so we will represent the insertion of type I and type II melons by the following vertices:

(a) Type I (b) Type II

By convention the root is labeled 0.

1. Determine the tree corresponding to the following graph.



2. What does $C_{p,q}$ count?

3. Show that $C_{p,q} = \frac{(4p+2q)!}{p!q!(3p+q+1)!}$.

Hint: We recall that the number of labeled trees with n vertices of degree $d_1, \ldots, d_n$ is $\frac{(n-2)!}{\prod_i (d_i-1)!}$.

Exercise 2. Subchromatic models

The prismatic real model is defined by the following action:

$$S = \frac{1}{2}\phi_{abc}\phi_{abc} + \frac{\lambda}{6N^\alpha}\phi_{a_1b_1c_1}\phi_{a_1b_2c_2}\phi_{a_2b_1c_2}\phi_{a_3b_3c_1}\phi_{a_3b_2c_3}\phi_{a_2b_3c_3}\,.$$

This is a model with $O(N)^3$ invariance but an interaction of order 6.

1. Give the graphical representation of the prismatic interaction as a 3-colored graph.

2. Show that for a good choice of α, this model admits a large-N expansion.

3. What are the leading order graphs?