OIST Mini Symposium
March 6–8, 2017

Viscoelasticity and Dissipative Dynamics of Rods and Membranes

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Book of Abstracts

Organizers

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Strongly irreversible gradient flows

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In the context of Damage Mechanics, *strongly irreversible* (or *unidirectional*) gradient systems have been introduced in order to describe an irreversible character of damaging phenomena (e.g., crack propagation). Indeed, damage in material evolves monotonically, and hence, particularly in view of phase-field models, phase parameters (describing the degree of damage) are supposed to be non-decreasing (or non-increasing). On the other hand, such a phase parameter is often supposed to evolve in accordance with a law of a gradient flow of some free energy. In this talk, we particularly apply ourselves to a variant of the Allen-Cahn equation,

$$u_t = \left(\Delta u - W'(u)\right)_+, \quad x \in \Omega, \quad t > 0$$

where $(\,\cdot\,)_+$ is the positive-part function, $W(\,\cdot\,)$ is a double-well potential and Ω is a smooth bounded domain of $\mathbb{R}^N$. Here $u(x,t)$ is a phase parameter describing the degree of damage in material, and hence, its evolution is constrained to be non-decreasing (due to the presence of the positive-part function) and also follows a (modified) L^2 gradient flow of a Ginzburg-Landau-Wilson free energy.

The main purpose of this talk is to exhibit partial energy-dissipation structures hidden in the strongly irreversible gradient flow. Indeed, due to the strong irreversibility, full energy-dissipation structures as in the classical Allen-Cahn equation (without $(\,\cdot\,)_+$) are not realized. On the other hand, (partial) dissipative phenomena of dynamics also emerge from the gradient flow structure as well as the parabolic nature of the equation. More precisely, we shall exhibit (partial) smoothing effect of solutions and (partial) energy-dissipation estimates, and then, we shall introduce a framework to construct a global attractor, which is different from a standard function space setting; indeed, no (compact) global attractor exists in any L^p spaces. If time permits, we shall also discuss the long-time behavior of each solution for the Cauchy-Dirichlet problem as well as Lyapunov stability for a certain class of equilibria. Finally, we shall further show some degenerate structure of entire solutions and equilibria due to the strong irreversibility.

This talk is based on a joint work with Messoud Efendiev (München).

Numerical simulations of thin viscous threads

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A thin thread made of a viscous fluid produces regular coiling patterns when impinging on a surface, as when honey is poured on a toast. In a variant of this experiment where the thread is deposited on a moving surface, more than ten different deposition patterns have been observed, some of which are quite complex; the patterns have been characterized experimentally as a function of the threads properties, of the fall height, and of the velocity of the surface. With the aim to reproduce them, we have developed a numerical tool for simulating the dynamics of a viscous thread based on a 1D model: finite rotations, viscous bending, stretching and twisting, and surface tension are all taken into account.

Interacting elastic curves on a rigid manifold

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Confinement of flexible curves endowed with finite bending stiffness to a surface is pervasive in nature. In the case of multiple curves on a surface, the equilibrium configuration of the curves is governed by short and long range interactions between them. We derive a variational framework for studying the interaction between two curves that are restricted to a two-dimensional manifold. The curves are assumed to be locally inextensible. The factors determining the equilibrium configuration of such system are the geometry of the manifold to which the curves are restricted, the bending stiffness of the curves, and the strength of their interaction potential. Using that framework, we study equilibria and stability properties of a particular system in which two elastic rods are closed and are restricted to move on a spherical surface.

Dimensional reduction of viscoelastic continua

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A general mathematical framework for constructing constitutive models for a class of viscoelastic continua will be described. Starting from three-dimensional continua, a possible dimensional reduction technique will be introduced and its relation with models for microstructured continua will be discussed. Such a reduction enables us to identify appropriate sets of kinematic descriptors and response functions apt to describe the behavior of thin shells and slender rods. Purely geometric shape energies are shown to arise as degenerate cases of the reduced models. A few remarks on the modeling of plastic behavior will also be given.

Stability and bifurcation for surfaces with constant mean curvature

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A surface with constant mean curvature (CMC surface) is an equilibrium surface of the area functional among surfaces which enclose the same volume (and satisfy given boundary conditions). A CMC surface is said to be stable if the second variation of the area is nonnegative for all volume-preserving variations. In this talk we give criteria for stability of CMC surfaces in the three-dimensional Euclidean space. We also give a sufficient condition for the existence of smooth bifurcation branches of fixed boundary CMC surfaces, and we discuss stability/instability issues for the surfaces in bifurcating branches. By applying our theory, we determine the stability/instability of some explicit examples of CMC surfaces.

Motion of a thickish elastic wire

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Caflisch and Maddocks introduced a PDE for the motion of a thickish elastic wire $\gamma(x,t)$ in the plane, and proved the existence of a solution to the PDE. The equation is derived from Hamilton's principle defined by the functional

$$F = \int_0^T E - U\,dt = \int_0^T (\|\gamma_t\|^2 + \|\gamma_{xt}\|^2) - \|\gamma_{xx}\|^2\,dt,$$

where E is kinetic energy and U is potential energy = elastic energy, and $\|*\|$ means L_2 norm. Here, the wire is arc-length parametrized ($|\gamma_x| \equiv 1$), and the term $\|\gamma_{xt}\|^2$ in E comes from the thickness of the wire.

I will give a sketch of proof for the case of higher dimensional Euclidean space, and show a plan to extend the result to the case of general Riemannian manifolds.

On the gradient flow of elastic open curves with fixed length and clamped ends

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In this talk, I will discuss regular open curves in $\mathbb{R}^n$ with clamped ends subject to a fixed length constraint and moving according to the L^2-gradient flow of the elastic energy, which is a fourth-order parabolic equation. A standard procedure in deriving the long-time existence of smooth solutions is to apply interpolation inequalities to obtain integral estimates of various order. For this equation, the main difficulty within such approach comes from the constraint of fixed total length and the clamped ends. I will focus on how to overcome the difficulty and then prove the long time existence and unique convergence to critical points.

The result is based on a joint work with Anna Dall'Acqua and Paola Pozzi.

The Kirchhoff–Plateau problem

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The Kirchhoff–Plateau problem concerns the equilibrium shapes of a system in which a flexible filament in the form of a closed loop is spanned by a liquid film, with the filament being modeled as a Kirchhoff rod and the action of the spanning surface being solely due to surface tension. In this seminar I will discuss the existence of an equilibrium shape that minimizes the total energy of the system under the physical constraint of non-interpenetration of matter, but allowing for points on the surface of the bounding loop to come into contact. In our treatment, the bounding loop retains a finite cross-sectional thickness and a nonvanishing volume, while the liquid film is represented by a set with finite two-dimensional Hausdorff measure. Moreover, the region where the liquid film touches the surface of the bounding loop is not prescribed a priori.

This is a joint work with G. G. Giusteri and E. Fried.

Using covering spaces to model soap films

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The approach to minimal surfaces based on BV functions has indisputable advantages, most notably the possibility to tackle the subject with standard Sobolev spaces like H^1 with "diffused interface" approximations and Γ-convergence following ideas started by De Giorgi. However, this is only feasible in situations where the surface can be interpreted as a transition interface between two phases. Using suitably constructed covering spaces it is possible to reinterpret soap films (i.e. a material surface with the same phase, air, on both sides) and apply the same techniques. The Plateau problem of finding a surface in 3D with a given boundary curve can be naturally treated in this context using a covering of degree two of the complement of the curve, triple junctions can also be modeled by constructing coverings with higher degree. This approach is similar to the "soap film" coverings proposed by Brakke, where surfaces are modeled using currents defined on covering spaces.

On a finite element scheme for curve shortening flow coupled to lateral diffusion

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In this talk I will propose and analyze a semi-discrete finite element scheme for a system consisting of a geometric evolution equation for a curve and a parabolic equation on that evolving curve. More precisely, curve shortening flow with a forcing term that depends on a conserved field is coupled with a diffusion equation for that field. Such a system can be considered as a prototype for more complicated problems as they may arise in applications.

The presented scheme is based on ideas of Dziuk for the curve shortening flow and Dziuk & Elliott for the parabolic equation on the moving curve. However, additional estimates, particularly with respect to the time derivative of the length element, are required. Numerical simulation results support the theoretical findings. If time permits, new numerical results for more complicated systems involving the elastic flow will be discussed.

This is joint work with Björn Stinner.

Quasistatic nonlinear viscoelasticity and gradient flows

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We consider the equation of motion for one-dimensional nonlinear viscoelasticity of strain-rate type under the assumption that the stored-energy function is λ-convex, which allows for solid phase transformations. We formulate this problem as a gradient flow, leading to existence and uniqueness of solutions. By approximating general initial data by those in which the deformation gradient takes only finitely many values, we show that, under suitable hypotheses on the stored-energy function, the deformation gradient is instantaneously bounded and bounded away from zero. Finally, we discuss the open problem of showing that every solution converges to an equilibrium state as time $t \to \infty$ and prove convergence to equilibrium under a nondegeneracy condition. We show that this condition is satisfied in particular for any real analytic cubic-like stress-strain function.

Carbon nanotubes: geometry and mechanics

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Molecular Mechanics models molecules as configurations of particles interacting via classical potentials. This perspective is here applied to carbon nanotubes, which is a carbon allotrope, namely hollow long cages of carbon atoms. The specific geometry of covalent bonding in carbon is described by the combination of an attractive-repulsive two-body interaction and a three-body bond-orientation part.

Optimal nanotube configurations are identified with local minima of the configurational energy and their fine geometry is fully characterized in terms of lower-dimensional problems. Under moderate tension, such local minimizers are proved to be periodic, which indeed validates the so-called Cauchy-Born rule in this setting.

The presentation includes results obtained in collaboration with Manuel Friedrich (Vienna), Edoardo Mainini (Genova), Hideki Murakawa (Kyushu), and Paolo Piovano (Vienna).

Method of thresholds for numerical approximation of interface network motion

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The problem of motion of three or more interfaces admitting junctions will be considered. To start with, I will present a numerical method for the precise calculation of such interfacial network moving under the steepest descent of surface energy, where one allows for arbitrary surface tensions of each interface, and for nonlocal constraints, such as preservation of enclosed volume. The method is based on the level set approach but linearizes the corresponding PDE. Then, I will focus on the problem of energy preserving motion of the same network, which includes inertial effects. Surprisingly, an approximation technique similar to the one used for the gradient flow problem can be designed for this hyperbolic motion. I will summarize results obtained in numerical analysis and point out some open problems.