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Real-time animation of synchrotron radiation

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Abstract

New mathematical method has been developed to compute radiation field from a moving charge in free space. It is not based on the retarded potential or its derivation [R.Y. Tsien, Picture of dynamic electric fields, Am. J. Phys. 40, 1972]. It solves conformal mapping of electric field lines based on the following two facts: (1) once a wave is emitted from a particle, it propagates as a spherical wave. The wavelet (a part of the wave-front) runs with speed of the light, and does not change its direction, (2) the initial direction of the wavelet is determined by the Lorentz transformation between the electron-rest-frame to the laboratory frame, which gives the light aberration effect. 2D radiation simulator has been developed based on this method, which simulates synchrotron, undulator and dipole radiation in time domain [T. Shintake, Simulation of field lines generated by a moving charge, private note 1984 March 19 at KEK, not published].

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1. Introduction

In various experimental applications of radiation, such as, the synchrotron, undulator and FEL radiations, discussions are usually made in terms of the angular and frequency spectrum of these radiations. The field properties are historically analysed by solving retarded potential for specified trajectory. Usually only the far-field radiation, whose field intensity is proportional to r^{-1}, is considered, and the Coulomb field is omitted since it decays quickly as r^{-2}. The results from this approximation have been widely used to evaluate the experimental data and its validity has been well confirmed.

However, understanding the realistic spatial distribution of radiation field and its time evolution becomes more important for studying the beam physics in electron accelerators. For example, the electron bunch-compressor for the e^+e^- Linear Collider, or the X-ray FEL, uses very short and intense bunch, and the Coherence Synchrotron Radiation (CSR) breaks the transverse emittance of the beam. To cure this effect, the understanding the radiation field and the beam dynamics is key issues of these accelerators.

2. Mathematical model

2.1. Basic equation

The Maxwell equation with field source is given by

$$\nabla \times \mathbf{H} = J + \frac{\partial D}{\partial t}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

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$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \cdot D = \rho. \tag{1}$$

Here we treat the radiation field from a single charge in free space. In the Maxwell equation, there are two driving terms, ρ and $\boldsymbol{J}$, which are related by the following continues equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0 \tag{2}$$

Therefore, when we correctly treat ρ on a moving charge, we do not need to treat $\boldsymbol{J}$ as explicit manner, which is automatically included. The magnetic field is derived by the first equation, so that it is enough to calculate only the $\boldsymbol{E}$ with one driving term ρ.

Since we have Gauss's theorem, which is always satisfied for a moving charge, the flux enclosed area d$\boldsymbol{S}$ is kept constant when we follow the "flux pipe" as shown in Fig. 1a. Thus

$$dQ = \varepsilon_0 \int E \,\mathrm{d}S \tag{3}$$

where dQ is a part of total charge Q of moving particle. If the cross-sectional area d$\boldsymbol{S}$ is known, we can determine the field strength E from Eq. (3), then the associated magnetic field from Maxwell Eq. (1a).

Along with the motion of charge, information of each event propagates outward with speed of the light as a spherical wave. Because of the causality, we can apply numbering on each wave-front as shown in Fig. 1b. The shape of the wave-front is always perfect sphere in free space, and continuously expands with speed of the light, only the origin (start point) differs for each event.

The discussion above is basically 3D problem, so that it is possible to develop 3D computer code to solve radiation field based on this method. However, to make the discussion simple, and ease code development, firstly the 2D code: Radiation 2D was developed. The field is treated in 3D manner, but the electric field distribution on 2D plane is computed and plotted. We tread only the 2D trajectory of moving charge, such as, circular, sinusoidal, or dipole oscillation. In this condition, the electric field lines and wave-fronts are treated as a 2D grid space as seen in Fig. 2, and motion of the node points (crossing point) is tracked in real-time manner. It should be noted that, this grid space is not always orthogonal. For a rest particle, the grid space becomes orthogonal, but for a moving charge the electric field line and wave-front becomes non-orthogonal. This is due to the light aberration effect, which is discussed in next section.

2.2. Initial conditions

When a wave-front is emitted from a moving charge, direction of the wavelet (a part of the wave-front) is tilted toward the velocity of motion. This is due to the light-aberration effect. When a wavelet is emitted in the direction of unit wave-vector $\mathbf{k}'$ on the electron rest frame, the observed

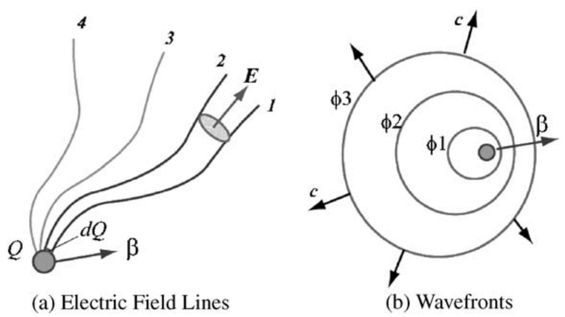

Fig. 1. Electric field lines of moving charge, and spherical wave-fronts.

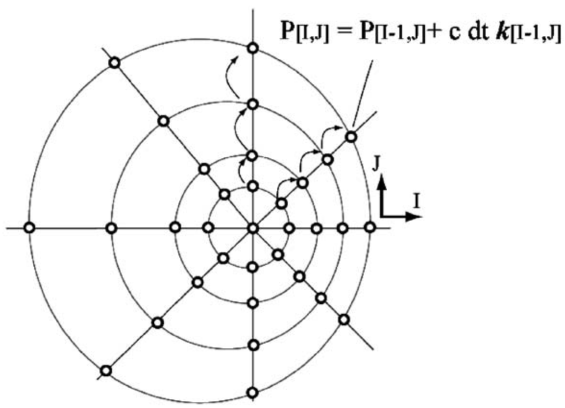

Fig. 2. 2D grid space consists from series of wave-fronts and electric field lines.

wavelet on the laboratory frame propagates along unit vector **k**

$$k = \begin{bmatrix} k_{\parallel} \\ k_{\perp} \end{bmatrix} = k \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \frac{1}{1+\beta \cdot k'_{\parallel}} \begin{bmatrix} k'_{\parallel} + \beta \\ \dfrac{k'_{\perp}}{\gamma} \end{bmatrix}. \quad (4)$$

When a particle is running at relativistic speed, the direction of the wavelet is focused in the direction of motion. This is the physical origin of the radiation power of the synchrotron or undulator radiation being focused in the cone-angle of $1/\gamma$ along the direction of particle velocity.

3. The 2D radiation simulator

The windows application has been created, which simulates 2D radiation field. It shows electric field line motion in real-time, and wave-front propagation. The code is available from our Web-site http://www-xfel.spring8.or.jp. It runs on Windows 98, 2000, XP. No Linux, nor Machintosh version is supported, at moment.

3.1. Numerical model

In the 2D radiation simulator, the positions of the node points of the electric field lines and wave-fronts are recorded in 2D matrix. One step of the computation is

(1) Move the particle in one step: $c\beta \mathrm{d}t$.
(2) Compute the direction of wavelet by Eq. (4).
(3) Move node point one step with speed of the light.
(4) Shift the address one step along electric field line.

In each time steps, new wave-front is generated from the particle, and propagates outward by the following equations:

$$\begin{aligned} \boldsymbol{P}[I,J] &= \boldsymbol{P}[I-1,J] + c\,\mathrm{d}t \cdot \boldsymbol{k}[I-1,J] \\ \boldsymbol{k} &= [I,J] = \boldsymbol{k}[I-1,J] \end{aligned} \quad (5)$$

where P is the coordinate of the grid point.

Followings are sample snapshots from the simulator.

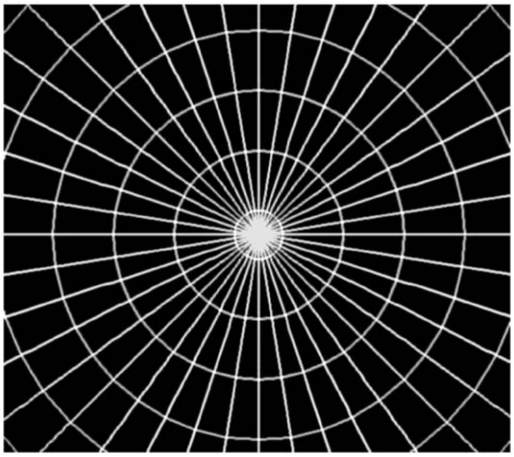

Fig. 3. The static field with the propagating spherical wave-front and field lines. Snap shot from the Radiation 2D simulator.

3.2. Static field

Fig. 3 shows the static field. Even if the particle rests, wave-fronts are always generated from the particle, and propagate outward. Since time derivation of E is zero, the magnetic field has to be zero, as a result, the pointing vector becomes zero. Therefore, there is no energy loss.

3.3. Synchrotron radiation

When a charged particle runs along a circular trajectory, it generates spiral electric field as shown in Fig. 4. The field lines are condensed in bright spiral zone, where the electric field is very high. Increasing particle velocity, the bright zone becomes narrower, which corresponds to short impulse field, which has wide frequency spectrum. This is the synchrotron radiation.

3.4. Undulator radiation

When a charged particle runs through an undulator, it is periodically deflected due to series of transverse magnetic field. In each curve, particle generates radiation in the direction of motion. Since the particle velocity is slightly lower than the speed of the light, wavelength of the accumulated periodic radiation becomes very short due to

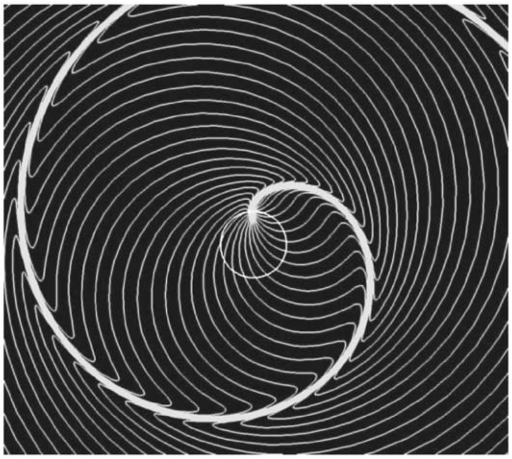

Fig. 4. Synchrotron radiation at $v = 0.9c$.

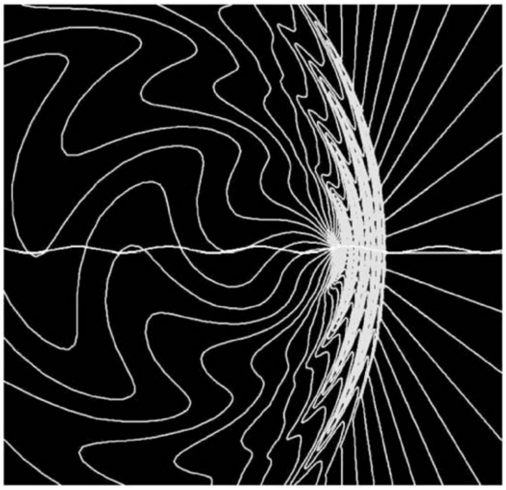

Fig. 5. Undulator radiation, $v = 0.9c$, $K = 1$.

Doppler effects. This is clearly shown in Fig. 5. It is also important to know that the undulator radiation field consists from series of spherical wave-front.

4. Discussions

This code can be extended to multi-particle problem. As seen in the snapshot of the undulator radiation, at the location where the radiation power is high, we have much data point (node point), which provide enough spatial resolution. This is a kind of auto-zooming function. This will be suitable to particle tracking of short bunch and high frequency field problem, like CSR.