SPECIAL RELATIVITY HOMEWORK – MIDTERM EXAM

These exercises are set in 2+1d spacetime. The linear nature of spinor space is quite non-trivial from the spacetime point of view. Our goal here will be to find the spacetime meaning of spinor addition.

Exercise 1. *Let's define the vector product in 2+1d spacetime in the obvious way, as* $(a \times b)^\mu = \eta^{\mu\nu}\epsilon_{\nu\rho\sigma}a^\rho b^\sigma$. *Find the vector products of our favorite three vectors (given here in the standard* (t, x, y) *basis):*

$$u^\mu = (1, 1, 0); \quad v^\mu = (1, -1, 0) \ ; \quad y^\mu = (0, 0, 1) \ . \tag{1}$$

Exercise 2. *Let* $A^\alpha{}_\beta, B^\alpha{}_\beta$ *be the spinor-matrix representation of two spacetime vectors* a^μ, b^μ. *What is the spacetime-geometric meaning of the matrix product* $(AB)^\alpha{}_\beta$*? Specifically, what's the meaning of its trace and of its traceless part?*

Exercise 3. *Let* ψ^α *and* χ^α *be the spinor square roots of two null vectors* ℓ^μ *and* n^μ. *Describe, using spacetime langauge, the vector represented by the spinor matrix* $\psi^{(\alpha}\chi^{\beta)}$.

Exercise 4. *Now, what is the spacetime-geometric meaning of the sum of two spinors* $\psi^\alpha + \chi^\alpha$*?*