SPECIAL RELATIVITY MIDTERM EXAM ≡ WEEK 5 HOMEWORK

Exercise 1. *Let's repeat our analysis of the projective lightcone, but in 2+1 spacetime dimensions instead of 3+1.*

1. *In 2+1d, the projective lightcone is a circle. Let's represent it by the circular section* $(t, x, y) = (1, \cos\phi, \sin\phi)$. *How does* ϕ *transform under a boost in the tx plane?*

2. *Consider the section of the lightcone* $x_\mu x^\mu = 0$ *by the plane* $t = x + 1$. *Parameterize this section as* $(t, x, y) = (t(\zeta), x(\zeta), \zeta)$, *i.e. find the embedding functions* $t(y)$ *and* $x(y)$.

3. *Identify the Lorentz transformations that correspond to* $\zeta \to \alpha\zeta$, $\zeta \to \zeta + \beta$ *and* $\zeta \to -1/\zeta$.

4. *Write a general element of the Lorentz group* $SO(2, 1)$ *as a transformation on* ζ. *Check that the dimension of the group comes out correctly.*

Exercise 2 (We're in 3+1d again)**.** *A particle of mass* m *and charge* q *is moving in a spacetime-independent electromagnetic field* $F_{\mu\nu}$. *Find the most general trajectory* $x^\mu(\tau)$, *where* τ *is proper time. Feel free to work in a maximally convenient Lorentz frame.*

Exercise 3. *A particle of mass* m *is moving in a spin-0 force field, obeying the equation of motion:*

$$m\frac{du^\mu}{d\tau} = (\delta^\mu_\nu + u^\mu u_\nu)F^\nu \ , \tag{1}$$

where τ *is proper time, and* u^μ *is the 4-velocity. Consider a* lightlike *spacetime-independent* F^μ. *As in the previous question, find the most general trajectory* $x^\mu(\tau)$, *working in a convenient Lorentz frame.*