GENERAL RELATIVITY HOMEWORK – WEEK 8

Exercise 1. *Find the correct values of the numerical coefficients* α, β *in the definition of the Weyl tensor from the lecture:*

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \beta(R_{\mu\rho}g_{\nu\sigma} - R_{\nu\rho}g_{\mu\sigma} - R_{\mu\sigma}g_{\nu\rho} + R_{\nu\sigma}g_{\mu\rho}) - \alpha R(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \ . \tag{1}$$

In the following exercises, we will derive in steps the Newtonian limit of the Einstein equation.

Exercise 2. *Rewrite the Einstein equation* $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$ *in the form:*

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \gamma T g_{\mu\nu}\right) \ , \tag{2}$$

where γ *is a numerical coefficient. What is the value of* γ*?*

Exercise 3. *Assume that the metric is given by* $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$*, where* $h_{\mu\nu}$ *is a small perturbation over the flat Minkowski metric* $\eta_{\mu\nu}$*. Express the Riemann tensor in terms of* $h_{\mu\nu}$ *and its derivatives, neglecting all but the linear terms. Verify the index symmetries of* $R_{\mu\nu\rho\sigma}$*.*

Exercise 4. *Now, assume further that we are in a non-relativistic limit, i.e. that time derivatives* ∂_t *are negligible with respect to spatial derivatives* ∂_i*. Express the Ricci component* R_{tt} *in terms of the Newtonian potential* $\phi = -h_{tt}/2$ *and its derivatives.*

Exercise 5. *Now, assume a further property of the non-relativistic limit: the energy density* $\rho = T^{tt}$ *(which is basically mass density) is much larger than all other components of* $T^{\mu\nu}$*. Derive the Poisson equation for the Newtonian potential as the* tt *component of eq.* (2)*. What happens if we change the spacetime dimension from 4d to 3d? How about 2d?*