

GENERAL RELATIVITY HOMEWORK – WEEK 6

Exercise 1. *The Lie derivative of vectors and covectors along a vector field u^μ is given by:*

$$\mathcal{L}_u v^\mu = u^\nu \partial_\nu v^\mu - v^\nu \partial_\nu u^\mu ; \quad \mathcal{L}_u c_\mu = u^\nu \partial_\nu c_\mu + c_\nu \partial_\mu u^\nu . \quad (1)$$

Show that these satisfy the tensor transformation law.

Exercise 2. *The Riemann tensor (for a metric-compatible, torsion-free connection) has the index symmetries:*

$$R_{\mu\nu\rho\sigma} = R_{[\mu\nu][\rho\sigma]} = R_{\rho\sigma\mu\nu} ; \quad R_{[\mu\nu\rho\sigma]} = 0 . \quad (2)$$

How many independent components does this tensor have on a manifold with dimensions $n = 1$? $n = 2$? $n = 3$? $n = 4$?