SPECIAL RELATIVITY HOMEWORK – WEEK 6

Exercise 1. *As in the lecture, consider an electric potential inside a region of length L:*

$$A^t = \begin{cases} V & 0 < x < L \\ 0 & elsewhere \end{cases} \tag{1}$$

A particle with mass m and charge q enters the region from the left, with velocity $\mathbf{v}$ (remember that spacetime has also y and z axes). Assume $qV > 2m$. What are the conditions on $\mathbf{v}$ that enable forward-in-time transmission? What are the conditions for backward-in-time transmission? In both cases, find the amount of time (positive or negative) spent by the particle inside the potential.

Exercise 2. *Consider our fancy worldline action for a free spin-0 particle:*

$$S[x^\mu(\lambda), e(\lambda)] = \int_{\lambda_i}^{\lambda_f} \left(\frac{1}{2e}\dot{x}_\mu \dot{x}^\mu - \frac{1}{2}m^2 e + qA_\mu(x)\dot{x}^\lambda \right) d\lambda \ , \tag{2}$$

where $x^\mu(\lambda)$ is the particle's position, and $e(\lambda)$ is the worldline gravitational field.

1. *Use e's equation of motion to eliminate it from the action, obtaining an action $S[x^\mu(\lambda)]$.*
2. *Alternatively, let's be naive, and just set $e = 1$, i.e. let us consider the non-gravitating worldline theory. Write the resulting action and its equations of motion. These equations are just fine, as long as we impose an extra constraint on the initial data $\dot{x}^\mu(\lambda_i)$. Find this constraint for a massive particle with mass m, and for a massless particle with initial 4-momentum $p_\mu(\lambda_i)$.*

Exercise 3. *Consider a gas of n particles per unit volume. Each particle has the same mass m and velocity of magnitude v. The directions of the particles' velocities are uniformly distributed. Show that the stress-energy tensor has the form:*

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \ , \tag{3}$$

and find the values of ε and p. What is the physical meaning of p? For massless particles, how are ε and p related?