SPECIAL RELATIVITY HOMEWORK – WEEK 4

For this exercise set, we live in 3d Minkowski spacetime $\mathbb{R}^{1,2}$, with coordinates (t, x, y). The Lorentz group has 3 generators:

$$J_{tx} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad J_{ty} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad ; \quad J_{xy} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \ . \tag{1}$$

Exercise 1. *Compute the Lorentz matrices $e^{\theta J_{ty}}$, $e^{\theta J_{xy}}$, and $e^{\theta(J_{ty}+J_{xy})}$, where θ is a scalar parameter.*

Points on the lightcone $x_\mu x^\mu = 0$ can be coordinatized as:

$$(t, x, y) = \rho(1, \cos\phi, \sin\phi) \ . \tag{2}$$

Notice that ϕ is a coordinate on the <u>projective</u> lightcone, which is a circle.

Exercise 2. *Rewrite eq. (2) in terms of a different lightcone coordinate $\xi \equiv \tan\frac{\phi}{2}$. You may find it convenient to redefine the scale factor ρ as well.*

Exercise 3. *Work out how the projective lightcone coordinate ξ transforms under the following spacetime symmetries:*

1. $e^{\theta J_{tx}}$
2. $e^{\theta(J_{ty}+J_{xy})}$
3. *The spatial reflection* $(t, x, y) \to (t, -x, y)$
4. $e^{\theta(J_{ty}-J_{xy})}$ *(hint: try assembling it from the previous answers)*