GENERAL RELATIVITY HOMEWORK – WEEK 4

Exercise 1. *A particle at rest with mass M decays into two lighter particles, with masses $M/2$ and $M/3$. Using 4-momentum conservation, find the velocities of the outgoing particles.*

Exercise 2. *The angular momentum of a moving particle is given by the spatial tensor $J^{ij} = x^i p^j - x^j p^i$. Let us explore its spacetime generalization $J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu$.*

1. *How many components does $J^{\mu\nu}$ have in addition to those in J^{ij}?*
2. *Write a formula for these new components in terms of t, $\mathbf{x}$, E, and $\mathbf{p}$.*
3. *The conservation of these new components implies a relation between E, $\mathbf{p}$, and the particle's velocity $\mathbf{v}$. What is it?*
4. *What is the symmetry associated with this "new" conservation law?*

Exercise 3. *In the lecture, we saw that the stress-energy tensor for a "dust" of particles moving with velocity v^i takes the form:*

$$T^{\mu\nu} = \rho \begin{pmatrix} 1 & v^j \\ v^i & v^i v^j \end{pmatrix} , \qquad (1)$$

where ρ is the energy density.

Now, consider a "gas" of particles that move with velocities of the same magnitude $|\mathbf{v}| \equiv v$, but in random directions. As before, let ρ be the energy density of the gas.

1. *What is the form of $T^{\mu\nu}$ in this case?*
2. *Find a relation between ρ, v, and the gas' pressure P.*
3. *For what value of v does the trace T^μ_μ vanish?*