GENERAL RELATIVITY HOMEWORK – WEEK 4

Exercise 1. *Consider a particle moving along the worldline* $x^\mu(\theta) = (\rho \sinh\theta, \rho\cosh\theta, 0, 0)$ *at fixed "radius"* ρ. *Find the 4-velocity* u^μ, *the 4-acceleration* α^μ *and its magnitude* $\alpha_\mu\alpha^\mu$.

Exercise 2. *Consider a gas with* n *particles per unit volume. The particles are moving with velocities of the same magnitude* $v = |\mathbf{v}|$, *but in random directions (isotropically distributed). Find the gas' stress-energy tensor* $T^{\mu\nu}$. *Show that for a gas of photons,* $T^{\mu\nu}$ *is traceless:* $T^\mu_\mu = 0$.

Exercise 3. *Consider again the action of electromagnetism, now written in 4-tensor notation:*

$$\begin{aligned} S = \sum_{particles} \int_\gamma (-m d\tau + q A_\mu dx^\mu) - \frac{\varepsilon_0}{4} \int F_{\mu\nu}F^{\mu\nu}\, d^4x \ , \\ \textit{where} \quad d\tau = \sqrt{-dx_\mu dx^\mu} \quad \textit{and} \quad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} \ . \end{aligned} \tag{1}$$

Derive the Lorentz force law $m\frac{du_\mu}{d\tau} = qF_{\mu\nu}u^\nu$ *and the Maxwell equation* $\partial_\nu F^{\mu\nu} = j^\mu/\epsilon_0$ *as the Euler-Lagrange equations for* $x^\mu(\lambda)$ *and* $A_\mu(x^\nu)$, *respectively.*