

SPECIAL RELATIVITY HOMEWORK – WEEK 4

Exercise 1. *In class, we identified the subgroups of Lorentz that preserve a spacelike, timelike or lightlike vector.*

1. *Find the subgroups that preserve a spacelike, timelike or lightlike direction.*
2. *Find the subgroups that preserve a 2d plane of signature $(+,+)$, $(-,+)$ or $(0,+)$.*

Exercise 2. *In the lecture, we discussed conformal transformations of the 2d plane. Here, we'll discuss conformal transformations of 3+1d spacetime itself (though the lessons are dimension-independent).*

1. *Consider the inversion transformation $x^\mu \rightarrow \frac{x^\mu}{x_\nu x^\nu}$. Find the resulting transformation on infinitesimal lengths $dx_\mu dx^\mu$, and show that an inversion is a conformal operation.*
2. *Find the effect on x^μ of the chain (inversion \rightarrow translation by a vector $a^\mu \rightarrow$ inversion). This is known as a special conformal transformation.*
3. *Consider a particle at rest at $\mathbf{x} = (0, 0, 0)$. Find the shape of the particle's worldline after a special conformal transformation along the spacelike vector $a^\mu = (0, a, 0, 0)$. Don't stop at an equation – the answer can be described in simple words. For inspiration, consider the limit of small t .*