GENERAL RELATIVITY HOMEWORK – WEEK 3

Exercise 1. *Using the formulas we developed for determinants and inverse matrices, find the derivative of a determinant $|A|$ with respect to the elements of the matrix $A^i{}_a$:*

$$\frac{\partial |A|}{\partial A^i{}_a} \;=\; ? \tag{1}$$

Exercise 2. *Consider a matrix $A^i{}_a$ on a 4d space, with determinant $|A|$. As we saw, the inverse matrix can be written as:*

$$(A^{-1})^a{}_i = \frac{\epsilon^{abcd}\epsilon_{ijkl}A^j{}_b A^k{}_c A^l{}_d}{3!|A|} \;. \tag{2}$$

Find a similar formula for the antisymmetrized product $(A^{-1})^{[a}{}_{[i}(A^{-1})^{b]}{}_{j]}$, i.e. rewrite this expression in terms of the original matrix $A^i{}_a$ and its determinant. Hint: if you wish to avoid a brute-force calculation, consider thinking of $A^i{}_a$ as a basis transformation.