**SPECIAL RELATIVITY HOMEWORK – WEEK 2**

In the lecture, we counted degrees of freedom for isometries – coordinate transformations $x_i \to \tilde{x}_i(x)$ that preserve a metric $g_{ij}$. Taylor-expanding around $x_i = 0$, we found that the $n$'th derivative $\partial_{i_1} \dots \partial_{i_n} \tilde{g}_{jk}$ of the new metric is governed by the $(n+1)$'st derivative $\partial_{i_1} \dots \partial_{i_{n+1}} \tilde{x}_j$ of the new coordinates. In the following questions, denote the dimension of our space by $D$.

**Exercise 1.** *Let's review our d.o.f. counting procedure, and sneak in a little GR lesson.*

1. *How many degrees of freedom are there in* $\partial_{i_1} \dots \partial_{i_{n+1}} \tilde{x}_j$*? How many are in* $\partial_{i_1} \dots \partial_{i_n} \tilde{g}_{jk}$*?*
2. *How many degrees of freedom are in a rank-4 tensor* $R_{ijkl}$ *with the following combination of index symmetries:*

$$R_{ijkl} = R_{[ij][kl]} \ ; \quad R_{ijkl} = R_{klij} \ ; \quad R_{[ijkl]} = 0 \ . \tag{1}$$

3. *Compare the answer to Part 2 with that of Part 1 for* $n = 2$.

**Exercise 2.** *Now, consider conformal transformations: instead of preserving the entire metric* $g_{ij}$*, we only want to preserve it up to a (possibly* $x$*-dependent) scalar prefactor,* $g_{ij} \cong \rho(x) g_{ij}$.

1. *How many constraints appear now at each order* $n$*?*
2. *How many degrees of freedom do conformal transformations have at* $n = 0$*?* $n = 1$*?* $n = 2$*? What is the new degree of freedom at* $n = 0$*?*
3. *In the special case* $D = 2$*, how many degrees of freedom do conformal transformations have at each* $n$*?*