**SPECIAL RELATIVITY HOMEWORK – WEEK 2**

**Exercise 1.** *For matrices* $A_{ij}$ *over* $\mathbb{R}^n$*, the determinant* $\det A$ *can be defined via:*

$$\epsilon_{j_1\ldots j_n} A_{i_1 j_1} \ldots A_{i_n j_n} = (\det A)\epsilon_{i_1\ldots i_n} \ , \tag{1}$$

*where* $\epsilon_{i_1\ldots i_n}$ *is the Levi-Civita tensor. Meditate on this until you see it! Then, derive a closed-form formula for the determinant, i.e.* $\det A = \ldots$

**Exercise 2.** *Descending now into* $\mathbb{R}^2$*:*

1. *Rewrite the product* $\epsilon_{ij}\epsilon_{kl}$ *in terms of the identity matrix* $\delta_{ij}$*.*
2. *Rewrite the determinant* $\det A$ *of a matrix* $A_{ij}$ *in terms of matrix products and traces.*

**Exercise 3.** *In* $\mathbb{R}^{1,1}$*, using the lightlike coordinates* $u = t + x$ *and* $v = t - x$*, write formulas of the form* $f(u,v) = 0$ *for the most general worldline of:*

1. *A particle at rest.*
2. *A particle at constant velocity.*
3. *A particle at constant proper acceleration* $\sqrt{\alpha_\mu \alpha^\mu}$*.*

**Exercise 4.** *Consider the conformal transformations:*

$$u \to \frac{au + b}{cu + d} \ ; \quad v \to \frac{\alpha v + \beta}{\gamma v + \delta} \ . \tag{2}$$

*Show that under such transformations, a particle at constant proper acceleration remains at constant proper acceleration.*

**Exercise 5.** *Now, consider the special case:*

$$u \to \frac{u}{au + 1} \ ; \quad v \to \frac{v}{av + 1} \ . \tag{3}$$

*For a particle at rest at* $x = x_0$ *before the transformation, what will be its worldline after the transformation?*