

SPECIAL RELATIVITY HOMEWORK – WEEK 2

Consider, as in the lecture, the 2-sphere of the sky, coordinatized by the angles (θ, ϕ) . Let us now replace them with a complex coordinate:

$$\zeta = \cot \frac{\theta}{2} e^{i\phi}. \quad (1)$$

Exercise 1. Consider a small square in the complex ζ plane, with center at $\zeta = \zeta_0$, and vertices at $\zeta = \zeta_0(1 \pm \epsilon \pm i\epsilon)$, where ϵ is a small real number. What is the corresponding shape on the (θ, ϕ) sphere?

Exercise 2. Identify which elements of the Lorentz group induce the following transformations on the sky:

- $\zeta \rightarrow \alpha\zeta$, where α is a complex constant. Hint: consider modulus and phase separately.
- $\zeta \rightarrow \zeta + \beta$, where β is a complex constant. Hint: it may be easier to consider small β .
- $\zeta \rightarrow 1/\zeta$.
- $\zeta \rightarrow \bar{\zeta}$.