GENERAL RELATIVITY HOMEWORK – WEEK 1

Exercise 1. *Consider the motion* $\mathbf{r}(t) = (x(t), y(t), z(t))$ *of a non-relativistic object of mass* m *in the Earth's gravitational field, governed by the action:*

$$S = m \int dt \left(\frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - gz \right) \ , \tag{1}$$

assuming a constant free-fall acceleration g.

Find the object's trajectory, as a function of its initial position $\mathbf{r_1}$ *at time* t_1 *and its final position* $\mathbf{r_2}$ *at time* t_2. *Find the value of the action* $S(t_2, \mathbf{r_2}; t_1, \mathbf{r_1})$ *for this trajectory. Verify that the derivatives* $\partial S/\partial \mathbf{r_2}$ *and* $-\partial S/\partial t_2$ *coincide with the momentum* $\mathbf{p} = m\dot{\mathbf{r}}$ *and energy* $E = m\left(\frac{1}{2}\dot{\mathbf{r}}^2 + gz\right)$ *at time* t_2.

Exercise 2. *Consider some charge density* $\rho(\mathbf{r})$ *and* $\mathbf{j}(\mathbf{r})$. *Assume that these are stationary, i.e.* t*-independent (note that this requires* $\mathbf{j}$ *to have vanishing divergence* $\boldsymbol{\nabla} \cdot \mathbf{j} = 0$*). Under these assumptions, write integral formulas for the following fields as functions of position* $\mathbf{r}$*:*

1. *The electric (also called scalar) potential* $\phi(\mathbf{r})$.
2. *The magnetic (also called vector) potential* $\mathbf{A}(\mathbf{r})$.
3. *The electric field* $\mathbf{E}(\mathbf{r})$.
4. *The magnetic field* $\mathbf{B}(\mathbf{r})$.