

# Representation Theory of Hecke Algebras and Categorification

Okinawa Institute of Science and Technology

June 5–10, 2023

## Description

The last decade has seen a flurry of research activity in categorification, largely driven by work of Khovanov, Lauda, and Rouquier, categorifying quantum groups by quiver Hecke algebras. In type A, these are graded lifts of cyclotomic Hecke algebras, and this fact has supplied the latter algebras with tools from graded representation theory. This starting point has on the one hand spurred on a plethora of further research in categorification, and, on the other hand, has reinvigorated the study of representations of cyclotomic Hecke algebras.

This workshop will bring together experts from both sides of this coin – those whose focus is on categorification, and those who are world-renowned experts in symmetric groups, their Hecke algebras, as well as various related families of diagrammatic algebras. The workshop format will include both invited talks and contributed talks, as well as two afternoon 'lightning sessions' for participants to give 15-minute presentations on their recent or ongoing work. This session allows for a quick introduction of a broad array of mathematical ideas to the workshop audience, as well as giving some junior attendees an opportunity to present their work.

## Organizing Committee

Susumu Ariki  
Osaka University, Japan

Alexander Kleshchev  
University of Oregon, USA

Robert Muth  
Duquesne University, USA

Liron Speyer  
Okinawa Institute of Science and Technology, Japan

# Timetable

IT: Invited Talk, CT: Contributed Talk, LT: Lightning Talk. All talks will take place in Lab 4, Room E48.

## Monday June 5

8:00	Taxis from Seaside House		
9:10	Taxis from Peridot		
9:30–10:00	Welcome		
10:00–11:00	IT	<b>Chris Bowman</b> University of York, UK	Meta-Kazhdan–Lusztig combinatorics
11:15–12:00	CT	<b>Chun-Ju Lai</b> Academia Sinica, Taiwan	Quantum wreath product and Schur–Weyl duality
12:00–2:00	Lunch		
2:00–3:00	IT	<b>Kai Meng Tan</b> NUS, Singapore	Adjustment matrices of Hecke algebras and $q$ -Schur algebras
3:00–3:30	Coffee		
3:30–4:15	CT	<b>Qi Wang</b> Tsinghua University, China	Representation type of cyclotomic quiver Hecke algebras of type $A_\ell^{(1)}$
4:15–5:00	CT	<b>Sinéad Lyle</b> University of East Anglia, UK	Some decomposition number for Rouquier / RoCK blocks of Ariki–Koike algebra
6:30–8:00	Dinner on campus		

## Tuesday June 6

8:00	Taxis from Seaside House		
9:10	Taxis from Peridot		
9:30–10:30	IT	<b>Masaki Kashiwara</b> RIMS, Japan and KIAS, Korea	Monoidal categories associated with quiver Hecke algebras and quantum affine algebras
10:30–11:00	Coffee		
11:00–12:00	IT	<b>Jon Brundan</b> University of Oregon, USA	The nil-Brauer category
12:00–2:00	Lunch		
2:00–3:00	IT	<b>Karin Erdmann</b> University of Oxford, UK	Finite-dimensional Hecke algebras and beyond
3:00–3:30	Coffee		
3:30–4:15	CT	<b>Alistair Savage</b> University of Ottawa, Canada	Diagrammatics for real supergroups
4:15–4:30	LT	<b>Alison Parker</b> University of Leeds, UK	Some representation theory of Kadar–Martin–Yu algebras
4:30–4:45	LT	<b>Nick Davidson</b> College of Charleston, USA	Superalgebra Deformations of Webs
4:45–5:00	LT	<b>Tianyuan Xu</b> Haverford College, USA	2-roots for simply laced Weyl groups
6:00	Taxis to Onna		
8:00	Taxis back to accommodations		

IT: Invited Talk, CT: Contributed Talk, LT: Lightning Talk. All talks will take place in Lab 4, Room E48.

## Wednesday June 7

8:00	<b>Taxis from Seaside House</b>	
9:10	<b>Taxis from Peridot</b>	
9:30–10:30	IT	<b>Inna Entova-Aizenbud</b> Ben Gurion University, Israel Representation stability for $GL_n(F_q)$
10:30–11:00	<b>Coffee</b>	
11:00–12:00	IT	<b>Andrew Mathas</b> University of Sydney, Australia Cyclotomic KLR algebras
12:00–2:00	<b>Lunch</b>	
2:00–2:45	CT	<b>Carl Mautner</b> UC Riverside, USA Symmetric groups, Schur algebras and symmetric products of the plane
2:45–3:15	<b>Coffee</b>	
3:15–3:30	LT	<b>Mahir Bilen Can</b> Tulane University, USA Almost Toric Schubert Varieties
3:35–3:50	LT	<b>Jun Murakami</b> Waseda University, Japan On Temperley–Lieb algebra at odd root of unity
3:55–4:10	LT	<b>Berta Hudak</b> OIST, Japan Homomorphisms into Specht modules labelled by hooks when $e = 2$
4:20–4:35	LT	<b>Gonzalo Jiménez</b> University of Chile, Chile Path morphisms in Reduced expressions graphs
4:45–5:00	LT	<b>Julianne Rainbolt</b> Saint Louis University, USA Using Degrees of Irreducible Characters to Construct a Supercharacter Theory
6:30–8:00	<b>Dinner on campus</b>	

## Thursday June 8

8:00	<b>Taxis from Seaside House</b>	
9:10	<b>Taxis from Peridot</b>	
9:30–10:30	IT	<b>Se-Jin Oh</b> Sungkyunkwan University, Korea Non-commutative polynomials and Categorification
10:30–11:00	<b>Coffee</b>	
11:00–12:00	IT	<b>Michela Varagnolo</b> CY Cergy Paris University, France Critical convolution algebras and quantum loop groups
12:00–1:00	<b>Lunch</b>	
1:00–6:30	<b>Excursion</b>	
6:30–8:00	<b>Banquet at Kafuu Hotel</b>	

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## Friday June 9

8:00	<b>Taxis from Seaside House</b>		
9:10	<b>Taxis from Peridot</b>		
9:30–10:30	IT	<b>Peng Shan</b> Tsinghua University, China	Modularity for W-algebras and affine Springer fibers
10:30–11:00	<b>Coffee</b>		
11:00–12:00	IT	<b>Syu Kato</b> Kyoto University, Japan	A geometric realization of Catalan functions
12:00–2:00	<b>Lunch</b>		
2:00–2:45	CT	<b>Iva Halacheva</b> Northeastern University, USA	Bethe algebras for $gl(n)$ , Gelfand-Tsetlin patterns, and cacti
2:45–3:15	<b>Coffee</b>		
3:15–3:30	LT	<b>Mee Seong Im</b> United States Naval Academy, USA	Universal construction, one-dimensional cobordisms with defects, and pseudocharacters
3:35–3:50	LT	<b>Lorenzo Putignano</b> University of Florence, Italy	The multiplicity freeness of the 3-weight blocks of the Iwahori–Hecke algebra of type B
3:55–4:10	LT	<b>Alexandre Minets</b> University of Edinburgh, UK	Semicuspidal categories and microlocalization
4:20–4:35	LT	<b>Eoghan McDowell</b> OIST, Japan	Determination of characters of the symmetric and alternating groups
4:45–5:00	LT	<b>Alice Dell’Arciprete</b> University of East Anglia, UK	Full runner removal theorem for Ariki–Koike algebras
6:00	<b>Taxis to Onna</b>		
8:00	<b>Taxis back to accommodations</b>		

## Saturday June 10

8:00	<b>Taxis from Seaside House</b>		
9:10	<b>Taxis from Peridot</b>		
9:30–10:30	IT	<b>Joe Chuang</b> City University, London, UK	On James’s conjecture for symmetric groups
10:30–11:00	<b>Coffee</b>		
11:00–11:45	CT	<b>Jon Kujawa</b> University of Oklahoma, USA	Positivity and web bases for Specht modules
11:45–2:00	<b>Lunch</b>		
2:00–3:00	IT	<b>Ben Elias</b> University of Oregon, USA	Reduced expressions for double cosets in Coxeter groups
3:00	<b>END</b>		

## Monday June 5

### Meta-Kazhdan–Lusztig combinatorics

**Chris Bowman**

IT

University of York, UK

We discuss the algebraic structure of Khovanov arc algebras by way of the anti-spherical Hecke categories of maximal parabolics of finite symmetric groups. There is a wealth of extra, richer combinatorial information which can be encoded into the Dyck tableaux underlying these ( $p$ -)Kazhdan–Lusztig polynomials. Instead of looking only at the sets of Dyck tableaux (which enumerate the  $p$ -Kazhdan–Lusztig polynomials) we look at the relationships for passing between these Dyck tableaux. In fact, this “meta-Kazhdan–Lusztig combinatorics” is sufficiently rich as to completely determine the complete Ext-quiver and relations of these categories.

### Quantum wreath product and Schur–Weyl duality

**Chun-Ju Lai**

CT

Academia Sinica, Taiwan

In this talk we introduce a new notion called the quantum wreath product that produces an algebra from a given algebra and a choice of parameters. Important examples include many variants of the Hecke algebras, such as the (1) Ariki–Koike algebras, (2) affine Hecke algebras of type A/C and their degenerate version, (3) Wan–Wang’s wreath Hecke algebras, (4) Rosso–Savage’s (affine) Frobenius Hecke algebras, (5) Kleshchev–Muth’s affinization algebras, and (6) Hu algebra, which quantizes the wreath product  $S_m \wr S_2$  between the symmetric groups. Our uniform approach to their structure/representation theory encompasses many known results which were proved in a case by case manner. Our theory is motivated by (and has application to) the Ginzburg–Guay–Opdam–Rouquier conjecture on quasi-hereditary covers of Hecke algebras for complex reflection groups. This is a joint work with Dan Nakano and Ziqing Xiang

### Adjustment matrices of Hecke algebras and $q$ -Schur algebras

**Kai Meng Tan**

IT

NUS, Singapore

I will give a survey of the rules for computing adjustment matrices of Hecke algebras and  $q$ -Schur algebras.

## Representation type of cyclotomic quiver Hecke algebras of type $A_\ell^{(1)}$

**Qi Wang**

CT

Tsinghua University, China

One of the fundamental problems in representation theory is determining the representation type of algebras. In this talk, we will introduce the representation type of cyclotomic quiver Hecke algebras, also known as cyclotomic Khovanov–Lauda–Rouquier algebras, in affine type A. Our main result relies on novel constructions of the dominant maximal weights of integrable highest weight modules for quantum groups in affine type A. This work is a collaboration with Susumu Ariki and Linliang Song.

## Some decomposition number for Rouquier / RoCK blocks of Ariki–Koike algebra

**Sinéad Lyle**

CT

University of East Anglia, UK

The Rouquier or RoCK blocks are important and much-studied blocks of the Hecke algebras of type A. We shall discuss an analogue for the Ariki–Koike algebras. The definition we give is combinatorial. We shall see how we can use Scopes' equivalences to equate some Rouquier blocks, although unlike in the type A case there does not seem to be an obvious explicit way of enumerating the blocks up to Scopes' equivalence. We shall also demonstrate a closed formula for the decomposition numbers of the Rouquier blocks in the case where the multipartitions have a common e-core and sketch how this is found.

## Tuesday June 6

### Monoidal categories associated with quiver Hecke algebras and quantum affine algebras

**Masaki Kashiwara**

IT

RIMS, Japan and KIAS, Korea

The module category over quiver Hecke algebras and the one over quantum affine algebras has a structure of monoidal categories, which have similar properties. Indeed, generalized Schur–Weyl duality functors connects their monoidal category structures. I will explain the results on these structures obtained with Myungho Kim, Se-jin Oh and Euiyong Park.

### The nil-Brauer category

**Jon Brundan**

IT

University of Oregon, USA

I will introduce the nil-Brauer category and explain how it categorifies the split  $i$ -quantum group of rank one. This is based on joint work with Ben Webster and Weiqiang Wang.

### Finite-dimensional Hecke algebras and beyond

**Karin Erdmann**

IT

University of Oxford, UK

Let  $H$  be some finite-dimensional Hecke algebra over a field. We discuss the category of finite-dimensional  $H$ -modules and its stable category, from the perspective of the representation theory of finite-dimensional algebras, with main focus on associated quasi-hereditary algebras.

### Diagrammatics for real supergroups

**Alistair Savage**

CT

University of Ottawa, Canada

We introduce diagrammatic monoidal supercategories controlling the representation theory of real forms of the general linear, orthosymplectic, periplectic, and isomeric supergroups. As a consequence, we obtain first fundamental theorems for these real supergroups and equivalences between monoidal supercategories of tensor supermodules over the real forms of a complex supergroup. This is joint work with Saima Samchuck-Schnarch.

## Some representation theory of Kadar–Martin–Yu algebras

**Alison Parker**

LT

University of Leeds, UK

Kadar–Martin–Yu introduced a new chain of subalgebras of the Brauer algebra. These algebras start with Temperley–Lieb and end with the Brauer algebra and build in representation theoretic intensity. This gives a new tool to tackle the long standing problem of understanding the representation theory of the Brauer algebra. We present an introduction to these new algebras and some results about their representation theory. This is joint work with my PhD student N. M. Alraddadi.

## Superalgebra Deformations of Webs

**Nick Davidson**

LT

College of Charleston, USA

Webs are a diagrammatic category used to describe certain categories of representations of  $\mathfrak{gl}_n$ . In this talk, I will describe *superalgebra deformations* of webs, which, roughly speaking, correspond to decorating these web diagrams with elements of a superalgebra  $A$ . By specializing  $A$ , these constructions recover several existing constructions of web categories which have appeared in the literature, and also give a diagrammatic presentation for the so-called “Schurifications” of Kleschev–Muth. This work is joint with Jon Kujawa, Rob Muth, and Jieru Zhu.

## 2-roots for simply laced Weyl groups

**Tianyuan Xu**

LT

Haverford College, USA

We introduce and study “2-roots”, which are symmetrized tensor products of orthogonal roots of Kac–Moody algebras. We concentrate on the case where  $W$  is the Weyl group of a simply laced Y-shaped Dynkin diagram with three branches of arbitrary finite lengths  $a$ ,  $b$  and  $c$ ; special cases of this include types  $D_n$ ,  $E_n$  (for arbitrary  $n \geq 6$ ), and affine  $E_6$ ,  $E_7$  and  $E_8$ .

We construct a natural codimension-1 submodule  $M$  of the symmetric square of the reflection representation of  $W$ , as well as a canonical basis  $\mathcal{B}$  of  $M$  that consists of 2-roots. The module  $M$  may be viewed as a specialization of a certain Kazhdan–Lusztig cell module of the Hecke algebra of  $W$ . We prove that with respect to  $\mathcal{B}$ , every element of  $W$  is represented by a column sign-coherent matrix in the sense of cluster algebras. We also prove that if  $W$  is not of affine type, then the module  $M$  is completely reducible in characteristic zero and each of its nontrivial direct summands is spanned by a  $W$ -orbit of 2-roots. (This is joint work with Richard Green.)



## Wednesday June 7

### Representation stability for $GL_n(F_q)$

**Inna Entova-Aizenbud**

IT

Ben Gurion University, Israel

I will present recent results on the Deligne categories for the family of groups  $GL_n(F_q)$ ,  $n > 0$ , based on a joint project with T. Heidersdorf. This family of symmetric monoidal categories interpolates the tensor categories of complex representations of  $GL_n(F_q)$  and have been previously constructed by F. Knop. I will describe some properties of these categories, as well as the relation to the category of algebraic representations of the infinite group  $GL_\infty(F_q)$ .

### Cyclotomic KLR algebras

**Andrew Mathas**

IT

University of Sydney, Australia

I will report on recent progress with Tubbenhauer in obtaining (cellular) bases for the cyclotomic KLR algebras, mostly of affine type. Most of the advances here have come from using Webster's diagrammatic KLRW algebras. In affine type C, my work with Evseev gives a more algebraic approach that yields much more information. I will finish by discussing the graded decomposition numbers of these algebras and some of the unexpected features that have been revealed in joint work with Chung and Speyer.

### Symmetric groups, Schur algebras and symmetric products of the plane

**Carl Mautner**

CT

UC Riverside, USA

In joint work with Tom Braden we introduce a new algebra, a close cousin to the Schur algebra. Like the Schur algebra, it can be defined in terms of the symmetric group and has a nice diagrammatic description. Geometrically, this algebra appears in our study of perverse sheaves on  $S^n(\mathbb{C}^2)$ , the  $n$ -fold symmetric product of the plane. In this talk I will aim to describe our motivation for this work and some interesting properties of the algebra.

## Almost Toric Schubert Varieties

**Mahir Bilen Can**

LT

Tulane University, USA

The complexity of the action of a reductive group on a variety  $X$  can be defined as the transcendence degree of the field of Borel subgroup invariant rational functions on  $X$ . In particular, if the action of a torus on a normal variety  $X$  has complexity 0, then  $X$  is a toric variety. In this talk, we will first introduce a notion of a nearly toric variety. Then we will show that if a singular Schubert variety in  $G/B$  is a nearly toric variety (for the maximal torus  $T$ ) of  $G$ , then  $X$  is a spherical variety with respect to an appropriate Levi subgroup of  $G$ . If time permits, then we will discuss how Iwahori–Hecke algebra of  $G$  can be used to determine all singular nearly toric Schubert varieties. This talk is based on my joint work with Pinaki Saha from IIT Bombay, India, and Nestor Diaz from Tulane University, USA.

## On Temperley–Lieb algebra at odd root of unity

**Jun Murakami**

LT

Waseda University, Japan

We extend the Temperley–Lieb algebra at an odd root of unity so that it relates to the monoidal category generated by the fundamental representation of the small quantum group of  $sl_2$  at a root of unity of odd order.

## Homomorphisms into Specht modules labelled by hooks when $e = 2$

**Berta Hudak**

LT

OIST, Japan

Let  $H_n$  denote the Iwahori–Hecke algebra of type  $A$ . For every partition  $\lambda$  of  $n$ , there exists a corresponding Specht module  $S^\lambda$ . These modules arise as the cell modules for  $H_n$  and therefore are the simple  $H_n$ -modules in the semisimple case. In his paper, using the Brundan–Kleshchev isomorphisms, Loubert completely determined the homomorphisms between Specht modules labelled by arbitrary shapes and those labelled by hooks in the case when the quantum characteristic is not equal to 2.

## Path morphisms in Reduced expressions graphs

**Gonzalo Jiménez**

LT

University of Chile, Chile

In this presentation, we will talk about path morphisms in the reduced expressions graph of elements in the symmetric group  $S_n$ . Our goal is to know when two different paths define the same morphism. We show recent results in that line and the current work in progress.

## Using Degrees of Irreducible Characters to Construct a Supercharacter Theory

**Julianne Rainbolt**

LT

Saint Louis University, USA

The talk will begin with an introduction to Supercharacter Theory as defined by P. Diaconis and I.M. Isaacs (Trans. Amer. Math. Soc., 2008). A possible Supercharacter Theory based on the degrees of the irreducible characters of a finite group will then be discussed. A proof that this Supercharacter Theory applies for certain families of finite groups but does not apply for other families will be exhibited.

**Thursday June 8**

**Non-commutative polynomials and Categorification**

***Se-Jin Oh***

IT

Sungkyunkwan University, Korea

In this talk, I will discuss “non-commutative” polynomials that can be understood as a “kind” of characters of simple modules over quiver Hecke algebras of every finite type. This talk is based on the works with almost all of my coworkers.

**Critical convolution algebras and quantum loop groups**

***Michela Varagnolo***

IT

CY Cergy Paris University, France

I will introduce a new family of algebras attached to quivers with potentials, using critical K-theory. They generalize the convolution algebras attached to quivers defined by Nakajima and are in some sense doubles of K-theoretical Hall algebras recently introduced by Padurariu. As an application I will give (for Dynkin types) a geometrical construction of Kirillov-Reshetikhin and prefundamental representations of the quantum loop group (or a shifted version). This is a joint work with Eric Vasserot.

**Friday June 9**

## **Modularity for W-algebras and affine Springer fibers**

**Peng Shan**

IT

Tsinghua University, China

We will explain a bijection between admissible representations of affine Kac-Moody algebras and fixed points in affine Springer fibers. We will also explain how to match the modular group action on the characters with the one defined by Cherednik in terms of double affine Hecke algebras, and extensions of these relations to representations of W-algebras. This is based on joint work with Dan Xie and Wenbin Yan.

## **A geometric realization of Catalan functions**

**Syu Kato**

IT

Kyoto University, Japan

Catalan functions form a distinguished family of symmetric functions specified by Dyck paths. They are introduced by Chen–Haiman and studied by Blasiak–Morse–Pun–Summers. In this talk, we realize Catalan polynomials as the graded characters of the space of global sections of line bundles on certain smooth projective varieties indexed by Dyck paths. This enables us to prove a vanishing conjecture originally posed by Broer (1994) in a vague way, and later refined and generalized by Shimozono–Weyman (2000) and Chen–Haiman (2010).

This talk is based on arXiv:2301.00862.

## **Bethe algebras for $\mathfrak{gl}(n)$ , Gelfand–Tsetlin patterns, and cacti**

**Iva Halacheva**

CT

Northeastern University, USA

For a reductive Lie algebra  $\mathfrak{g}$ , the Bethe algebras are a family of maximal commutative subalgebras of the Yangian of  $\mathfrak{g}$  parametrized by regular elements of the maximal torus. When  $\mathfrak{g}=\mathfrak{gl}(n)$ , this family has been shown to extend to the Deligne–Mumford compactification of  $M(0,n+2)$ , the moduli space of smooth genus 0 curves with  $n+2$  marked points. If we consider parameters in the real locus of this compactification, the corresponding Bethe subalgebras act with simple spectrum on a given tame representation of the Yangian, leading to a covering space of the real locus with fiber this spectrum. I will discuss the resulting monodromy action of a cactus group, focusing on the fibers over the most degenerate curves, i.e. caterpillar curves, where the eigenlines for the Bethe subalgebra correspond to Gelfand–Tsetlin patterns. This is joint work with Anfisa Gurenkova and Lenya Rybnikov.

## Universal construction, one-dimensional cobordisms with defects, and pseudocharacters

**Mee Seong Im**

LT

United States Naval Academy, USA

I will explain the universal construction in topological theory and topological quantum field theory, with a focus on one-dimensional cobordisms with defects, and their relationship to pseudocharacters, an essential tool in modern number theory, and characters. If I have time, I will also discuss their connections to pseudo-holonomies and holonomies. This is joint with Mikhail Khovanov and Victor Ostrik.

## The multiplicity freeness of the 3-weight blocks of the Iwahori–Hecke algebra of type B

**Lorenzo Putignano**

LT

University of Florence, Italy

In the representation theory of finite groups the notion of weight of a block quantifies how “complicated” the block is: smaller the weight is, easier its structure is. In the case of the symmetric group  $S_n$  (and of the Iwahori–Hecke algebra of type A), it has been shown that blocks of weight  $w \leq 3$  are multiplicity free algebras, i.e. all decomposition numbers are at most 1 (M.Richards for  $w \leq 2$ , M.Fayers for  $w = 3$ ). In general, this is no more true if  $w \geq 4$ . In the case of the Iwahori–Hecke algebra of type B, M.Fayers generalized the idea of weight defining an analogous “difficulty quantifier” of blocks (still called weight) and showed the multiplicity freeness of blocks of weight at most 2. In this talk we will see that this property is shared by the 3-weight blocks too.

## Semcuspidal categories and microlocalization

**Alexandre Minets**

LT

University of Edinburgh, UK

Let  $Q$  be an affine quiver, and let  $R(\alpha)$  be the corresponding KLR algebras, where  $\alpha$  is a dimension vector. The categories  $R(\alpha)\text{-mod}$  admit a stratification by semicuspidal categories  $C(\theta)\text{-mod}$ , parameterized by positive roots. The ones corresponding to imaginary roots are poorly understood in positive characteristic. I will explain how to access the algebras  $C(\theta)$  geometrically, with methods inspired by Donaldson–Thomas theory. The talk is based on some ongoing projects with B. Davison, R. Maksimau.

## Determination of characters of the symmetric and alternating groups

**Eoghan McDowell**

LT

OIST, Japan

How much information about an ordinary irreducible character determines the character? I will discuss this question for the symmetric and alternating groups, with a focus on two interpretations of 'information': the values on  $p'$ -classes, and the zero values. Interest in the zeros of a character date back to Burnside, while our question in the case of values on  $p'$ -classes also tells us about the modular representation theory of the group: characters agreeing on  $p'$ -classes corresponds to equal rows in the decomposition matrix. I will highlight an apparent connection between these two determination problems that is the subject on ongoing work.

## Full runner removal theorem for Ariki-Koike algebras

**Alice Dell'Arciprete**

LT

University of East Anglia, UK

The determination of the decomposition numbers, i.e., the composition multiplicities of the simple modules  $D$  in the Specht modules  $S$  is one of the most important outstanding problems in representation theory of the symmetric groups and related algebras. For the Iwahori-Hecke algebras of the symmetric group Fayers proved a theorem which relates decomposition numbers for different values of  $e$ , by adding 'full' runners to the abacus displays for the labelling partitions. I will describe the background of Fayers' theorem, and then talk about a 'runner removal' theorem for Ariki-Koike algebras that generalises Fayers' one.

**Saturday June 10**

**On James's conjecture for symmetric groups**

*Joe Chuang*

IT

City University, London, UK

We describe some primitive idempotents in certain diagrammatic algebras of Webster and explain how this translates via Koszul duality to the truth of James's conjecture for parts of blocks of symmetric groups. This is joint work with Hyohe Miyachi and Kai Meng Tan.

**Positivity and web bases for Specht modules**

*Jon Kujawa*

CT

University of Oklahoma, USA

We show that the transition matrix from the standard basis to the web basis for a Specht module of the Hecke algebra is unitriangular and satisfies a strong positivity property whenever the Specht module is labeled by a partition with at most two parts. This generalizes results of Russell-Tymoczko and Rhoades. This work is joint with Sam Heard.



## Reduced expressions for double cosets in Coxeter groups

Ben Elias

IT

University of Oregon, USA

There is a rich and well-developed theory of reduced expressions for elements of Coxeter groups. For example, one has Matsumoto's theorem, which states that any two reduced expressions for a given element are related by a sequence of braid relations. Reduced expressions also play a role in many representation-theoretic and geometric constructions. For any element  $w$  in a Weyl group  $W$ , the Bruhat decomposition yields a corresponding  $B$ -orbit in the flag variety  $G/B$ . The closure of this orbit is the Schubert variety  $X_w$ . To any expression (i.e. word in the simple reflections), one can construct a Bott–Samelson variety which maps to the flag variety. This is a “resolution of singularities” of  $X_w$  if and only if the word is a reduced expression for  $w$ .

Meanwhile, consider two parabolic subgroups  $W_J$  and  $W_K$  inside a Weyl group  $W$ . To a double coset  $[w]$  inside  $W_J W/W_K$ , one can associate a  $P_J$  orbit inside  $G/P_K$ , where  $P_J$  and  $P_K$  are the corresponding parabolic subgroups of  $G$ . The closure of this orbit is a variety  $X_{[w]}$ . In order to study these varieties and construct resolutions of singularities, Geordie Williamson in his thesis introduced a theory of expressions and reduced expressions for double cosets.

In recent work with Hankyung Ko, we develop the theory of reduced expressions for double cosets. We provide several equivalent definitions for a reduced expression, easier to work with than the original definition of Williamson. We prove a relationship between the Bruhat order on double cosets and the appropriate analogue of subexpressions. Most importantly, we introduce the double coset braid relations, and prove the double coset Matsumoto theorem: that any two reduced expressions for  $[w]$  are related by the double coset braid relations. For example, in type  $A_2$ , the ordinary braid relation  $sts = tst$  actually follows from a sequence of four (smaller and more elementary) double coset braid relations.

We also briefly describe some applications of this work to the study of Demazure operators and singular Soergel bimodules (work in progress, joint with Ko, Libedinsky, and Patimo).

# Participants

Noriyuki Abe, University of Tokyo, Japan  
Jonas Antor, University of Oxford, UK  
Susumu Arika, Osaka University, Japan  
Elijah Bodish, MIT, USA  
Chris Bowman, University of York, UK  
Jon Brundan, University of Oregon, USA  
Mahir Can, Tulane University, USA  
Aaron Chan, Nagoya University, Japan  
Joe Chuang, City, University of London, UK  
Chris Chung, OIST, Japan  
Tiago Cruz, MPI Bonn, Germany  
Alfred Dabson, City University, London, UK  
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