

Homework 1: Manifolds and Lie Groups

Due 20th June 2022

Question 1 (10 points)

Consider the circle S^1 , embedded in the complex plane.

- a) Show that S^1 together with complex multiplication is a Lie group.
- b) Consider the map $f : S^1 \rightarrow S^1$ that sends $z \mapsto z^2$. What is happening to the circle? Is this a homeomorphism?

Question 2 (5 points)

Let M and N be smooth manifolds of dimension m and n , respectively. Show that the product $M \times N$ is a smooth manifold of dimension $m + n$.

Question 3 (5 points)

Explain to me the following, in your own words. Also, don't waste your time writing pages and pages of explanation, the fun is in trying to be as succinct as possible. A few sentences per point is fine.

1. The difference between a vector space and a module.
2. The similarities and differences between a tangent vectors and a vector field.
3. Integration of a covector field.
4. de Rham's Theorem.
5. The exponential map $exp : \mathfrak{g} \rightarrow G$.