Fibre Bundles and Spin Structures

Homework 3: Connections and Curvature

Due July 25th

Question 1 (5 points) Let ∇^1 and ∇^2 be linear connections on E. Show that a convex combination of ∇^i is again a connection. Specifically: if ρ_1 and ρ_2 are two real-valued functions on M that sum to 1, show that $\nabla := \rho_1\nabla^1 + \rho_2\nabla^2$ is a connection on E.

Question 2 (5 points)
The curvature F_∇ of the connection ∇ is a map $F_\nabla : \mathfrak{X}(M)\times\mathfrak{X}(M)\times\Gamma(E) \to \Gamma(E)$ given by

$$F_\nabla(v, w, s) := \nabla_v\nabla_w s - \nabla_w\nabla_v s - \nabla_{[v,w]}s.$$

Show that F_∇ is $C^\infty(M)$-linear in the first two arguments, that is, show that

1. $F_\nabla(fv, w, s) = fF_\nabla(v, w, s)$, and
2. $F_\nabla(v, fw, s) = fF_\nabla(v, w, s)$

hold for any f in $C^\infty(M)$.

Question 3 (10 points)
In your own words, explain the similarities and differences between the Lie derivative and the Covariant derivative. Give as much detail as you can.