

# Homework 2: Bundles

Due 11th July 2022

## Question 1 (5 points)

1. Is  $S^1 \times S^1$  parallelizable?
2. What about  $S^n$ , with  $n$  even?
3. Is  $S^2$  a Lie group?

## Question 2 (10 points)

Let  $P$  and  $Q$  be two principal  $G$ -bundles over the same space  $M$ . Show that any morphism from  $P$  to  $Q$  is necessarily an isomorphism.

## Question 3 (5 points)

In your own words, explain the following concepts. Try and use as much natural language as you can – i.e. as little math notation as possible.

1. The construction of the tangent bundle
2. The fibres of a principal  $G$ -bundle
3. The frame bundle
4. Reduction of the structure group
5. Associated bundles