

**Nina Holden (Courant Inst.)**

Title: Conformal welding of Liouville quantum gravity surfaces: recent results and applications

Abstract: Liouville quantum gravity (LQG) surfaces form a natural family of random fractal surfaces. A powerful tool in the study of LQG surfaces is conformal welding, where multiple LQG surfaces are combined into a single LQG surface. The interfaces between the original surfaces are typically described by variants of the random fractal curves known as Schramm-Loewner evolutions (SLE). We will present a few recent conformal welding results for LQG surfaces and their applications, which range from SLE and LQG to planar maps and random permutations. Based on joint works with Ang and Sun, with Lehmkuehler, and with Borga, Sun and Yu.

**Yilin Wang**

Title: Holography of the Loewner energy

The Loewner energy is a conformally invariant quantity that measures the roundness of Jordan curves on the Riemann sphere. It is the large deviation rate function of the family of random curves SLE (arising as interfaces in critical 2D lattice models) and can be expressed using zeta-regularized determinants of Laplacians. Motivated by AdS/CFT correspondence and the fact that the conformal group of the Riemann sphere coincides with the isometry group of the hyperbolic 3-space  $H^3$ , we look for quantities defined geometrically in  $H^3$  which equal the Loewner energy of a curve in the conformal boundary. We show that the Loewner energy equals the renormalized volume (closely related to the Einstein-Hilbert action) of a submanifold of  $H^3$  constructed using the Epstein surfaces. This is a work in progress with Martin Bridgeman (Boston College), Ken Bromberg (Utah), and Franco Vargas-Pallete (Yale).

**Eveliina Peltola (Aalto Univ.)**

Title: On large deviations of SLEs, real rational functions, and zeta-regularized determinants of Laplacians

Abstract: When studying large deviations (LDP) of Schramm-Loewner evolution (SLE) curves, a "Loewner energy", and "Loewner potential", that describe the rate function for the LDP, were recently introduced. While these objects were originally derived from SLE theory, they turned out to have several intrinsic, and perhaps surprising, connections to various fields. I will discuss some of these connections and interpretations towards Brownian loops, semiclassical limits of certain correlation functions in conformal field theory, and rational functions with real critical points (Shapiro-Shapiro conjecture in real enumerative geometry).

(Based on joint work with Yilin Wang - IHES, France.)

**Makoto KATORI (Chuo University)**

Title: Stochastic log-gases, multiple SLEs, and Gaussian free fields

Abstract: Multiple extensions of Schramm-Loewner evolution (SLE) have been usually studied by the method using commutation relations of SLEs and the absolute continuity of measures. This method is,

however, available only in the case such that the curves generated by commuting SLEs are separated from each other. Recently we have found that this restriction can be removed if we impose the coupling between the multiple SLEs and the appropriately modified Gaussian free fields (GFFs), where we have followed the work by Dubédat, Sheffield, Miller, and others on the coupling of a single SLE and GFF. By Itô's stochastic calculus, we prove that proper couplings between multiple SLEs and GFFs are established if and only if the driving processes of multiple SLEs are of a specified type of interacting particle systems. They are identified with the log-gases extensively studied as dynamical extensions of the eigenvalue distributions in random matrix ensembles. We will show that this new connection between SLE and random matrix theory (RMT) enables us to enjoy variety of coupled systems due to the variety of ensembles in RMT. The law of large numbers are then clarified as the Wigner semicircle law and its variations in the RMT level and as the Loewner chains driven by the processes solving the Burgers-type PDEs in the SLE level. The present talk is based on the joint work with Shinji Koshida (Aalto University).

### **Noriko Yui (Queen's Univ.)**

Title: Modularity of Calabi-Yau motives of rank  $\leq 4$  Speaker:

Abstract: We will focus on Calabi-Yau motives of rank  $\leq 4$ , and discuss their modularity/automorphy in the framework of the Langlands Philosophy.

In the last thirty years or so, we have witnessed tremendous advances on the modularity questions for Calabi-Yau varieties of dimension  $\leq 3$ , namely, elliptic curves, K3 surfaces and Calabi-Yau threefolds. Most of these results rest on the modularity of the two-dimensional Galois representations associated to them. The next simplest cases are Calabi-Yau threefolds of Hodge type  $(1,1,1,1)$ . However, establishing the (Siegel) modularity of such Calabi-Yau threefolds has been a challenge.

Instead, we will consider Calabi-Yau motives of rank  $\leq 4$  with various weight. I will present modularity results on some examples of such Calabi-Yau motives.

### **Laurent Baulieu (LPTHE)**

Title: Beltrami parametrization, Gravitons, BRST BMS4 Symmetry and its Cocycles from Horizontality Conditions

Abstract : The BRST structure of the extended Bondi-Metzner-Sachs symmetry group of asymptotically flat manifolds is investigated using the recently introduced framework of the Beltrami field parametrization of four-dimensional metrics. The latter identifies geometrically the two physical degrees of freedom of the graviton as fundamental fields as excitations of the Beltrami differential of the asymptotic celestial spheres along the null directions of asymptotically flat spaces. The graded BRST BMS4 nilpotent differential operator relies on four horizontality conditions giving a Lagrangian re- formulation of the asymptotic BMS4 symmetry. A series of cocycles is found which indicate the possibility of anomalies for three-dimensional Lagrangian theories to be built in the null boundaries of asymptotically flat spaces from the principle of BRST BMS4 invariance.

**Timothy Budd (Radboud Univ.)**

Title: Tree bijections and the geometry of random hyperbolic surfaces

Abstract: A natural family of random surfaces, which has received considerable attention recently in the context of JT gravity, is obtained from the Weil-Petersson measure on the moduli space of hyperbolic metrics on a genus- $g$  surface with geodesic boundaries. In this talk I will describe how methods from random maps, i.e. random discrete surfaces, can be adapted to the hyperbolic world and give insight into the probabilistic aspects of the geometry of these hyperbolic surfaces when the number of boundaries becomes large. This talk is based on joint works with Nicolas Curien and with Thomas Meeusen and Bart Zonneveld.

**Kazuhiro Sakai (Meiji Gakuin)**

Title: Matrix model approach to JT gravity

Abstract: Jackiw-Teitelboim (JT) gravity is a simple model of 2d dilaton gravity known since the 1980s. It has attracted attention in recent years, due to its connection with quantum chaos and also as a tool to study black hole information paradox. In 2019, Saad, Shenker and Stanford found out a matrix model description of JT gravity. In this talk, I will explain how JT gravity is understood in the context of the traditional theory of 2d quantum gravities and one-matrix models. I then formulate a systematic method of calculating the partition function of general topological gravity, which includes JT gravity as a special case, on a surface with multiple boundaries and handles, with/without branes. The talk is based on a series of collaborations with K. Okuyama.

**Motoko Kato (Ryukyu Univ.)**

Title: Thompson's groups and ring groups of homeomorphisms of the circle

Abstract: In this talk, we introduce ring groups of homeomorphisms of the circle, which is originally considered by Kim, Koberda and Lodha. A ring group is a group generated by finitely many homeomorphisms of the circle whose supports form a "ring" of open intervals. We also assume that the subgroups generated by two of the generators are isomorphic to Thompson's group  $F$  when their supports have nonempty intersection. We show that Higman-Thompson groups  $T_n$ , which are generalizations of Thompson's group  $T$ , admit the structure of ring groups. We explicitly construct a finite generating set of  $T_n$  as a ring group. With such a generating set, we may attribute the discussion on  $T_n$  to the discussion on  $F$ , not to  $F_n$ . This is useful since we may avoid technical difficulties in  $n$ -adic cases. As an application, we show that every  $T_n$  has a fixed point property for semi-simple actions on complete  $CAT(0)$  space of finite covering dimension.

**Séverin Charbonnier (Geneve Univ.)**

Title: Ciliated maps, minimal conformal models coupled to gravity and topological gravity

Abstract: In 1990, supposing that two approaches to 2d gravity are equivalent, Witten conjectured that the intersection numbers of Chern classes on the moduli space of Riemann surfaces (related to topological

gravity) are computed by the Korteweg--de Vries hierarchy (related to the so-called "large N limit" of combinatorial maps).

I will expose a model of maps - Ciliated maps - that generalise the graphs employed by Kontsevich to prove Witten's conjecture. We proved that the correlation functions built from this model can be computed by a universal procedure called Topological Recursion, in which all the information to compute the correlation functions is encoded in a Spectral Curve.

I will show how, by tuning the various parameters of the model in two ways, one can relate ciliated maps to  $(2(r-1)+1,2)$  minimal models coupled to gravity (Kontsevich--Witten's story corresponds to the case  $r=2$ ):

- on the one hand, it is possible get information on the large N limit of ciliated maps by getting some critical exponents of large maps, consistent with the KPZ exponents obtained for  $(2(r-1)+1,2)$  minimal Liouville gravity;

- on the other hand, the partition function of ciliated maps is a solution to the  $r$ -Korteweg--de Vries hierarchy and as such, is related to Witten's class intersection numbers and topological gravity.

Last, I will sketch how ciliated maps can be used in free probability to show that both moments and free cumulants of a specific hermitian matrix model satisfy Topological Recursion.

### **Wenliang Li (Sun Yat Sen Univ.)**

Title: How to get something from nothing: null state, bootstrap and Dyson-Schwinger

Abstract: I will begin with a brief overview of the 2D minimal model conformal field theories, the first successful examples of the conformal bootstrap program. The null state/vector condition for the minimal models can be viewed as a quantization condition for these strongly coupled systems. I aim to explain that the null state condition can be applied in a broader context, i.e., without assuming conformal symmetry. I will discuss the application of the null state condition in the quantum mechanical bootstrap, using both the Hamiltonian and Lagrangian formalisms. In the Lagrangian case, the underdetermined Dyson-Schwinger equations are solved by imposing the null state condition. In the end, I will discuss the application to random matrix models and loop equations.

### **Hidehiko Shimada (Yukawa Institute Kyoto)**

Title: Spectral winding and quantum anomaly

Abstract: In the study of quantum anomalies, an important role is played by the winding number of the determinant of a family of Dirac operators.

In this talk, I will present a work in progress which aims to consider a refined version of this concept, which I call the "spectral winding". The spectral winding is characterised by "collective" winding numbers of a spectrum of complex eigenvalues of a family of Dirac operators. In particular, several eigenvalues can cooperate to produce a single winding number of the determinant of the Dirac operator. Thus these eigenvalues can have "fractional" winding numbers.

After introducing the concept of the spectral winding and explaining its relation with the quantum anomaly, I will present simple examples of quantum field theories exhibiting the spectral winding.

I will also discuss 2D CFTs describing localised degrees of freedom around vortex-like configurations of four-dimensional quantum field theories associated with fractional winding numbers.

**Bertrand Duplantier (IPhT, Paris-Saclay University)**

Title: Hamiltonian Paths, Liouville Quantum Gravity and KPZ

Abstract: We consider various configuration exponents of Hamiltonian paths drawn on *cubic*, *bicubic*, or *bicolored* random planar maps. Estimates from exact finite size results are compared with predictions based on the **Knizhnik-Polyakov-Zamolodchikov** (KPZ) relations in Conformal Field Theory (CFT) and Liouville Quantum Gravity (LQG), as applied to regular critical exponents on the hexagonal lattice, which in principle correspond to Schramm-Loewner Evolution with SLE parameter 8. Surprisingly, in the *bicubic* case, a naive use of KPZ does not reproduce all the measured exponents, but a certain Ansatz may possibly account for the observed discrepancies. We further describe Hamiltonian cycles on various families of *bicolored* planar maps, which in the scaling limit fall into two universality classes and CFTs, with respective central charges  $c = -1$  or  $c = -2$ . The first group comprises  $p$ -regular maps of fixed vertex valency  $p$  greater or equal to 3, and the second, maps of mixed vertex valencies, and a so-called rigid case. For each class, a universal configuration exponent, as well as a novel critical exponent associated with long-distance contacts along a Hamiltonian cycle, are obtained via LQG and KPZ from corresponding SLE exponents on regular (hexagonal or square) lattices. This time, the KPZ predictions are numerically confirmed by exact enumeration results for  $p$ -regular maps with  $p=3,4,\dots,7$ , and for maps with mixed valencies (2,3), (2,4) and (3,4).

Based on joint works with Ph. Di Francesco, O. Golinelli and E. Guitter.