

ESR studies of low P-doped Si: T_1 measurement at high fields and DNP at low temperatures

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FIR FU is carrying out R&D of the far-infrared region with world class “**Gyrotrons**” which have been originally developed in Fukui.

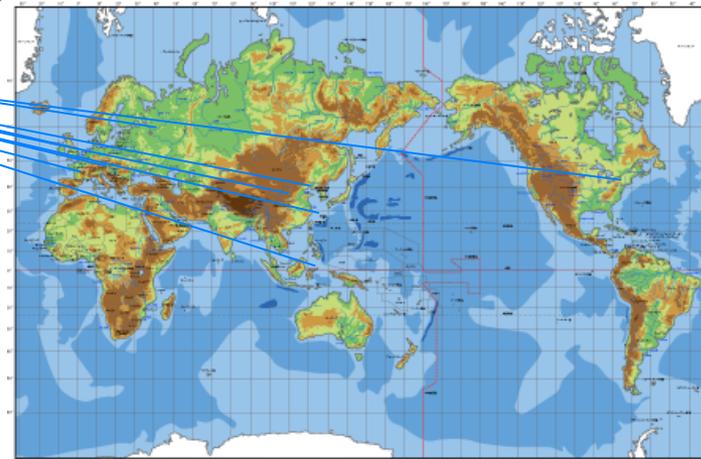
FIR FU

<http://fir.u-fukui.ac.jp/>



World wide collaborations

- Univ. Warwick (UK) (DNP-NMR)
 - ESRF (France) (XDMR)
 - Princeton Univ. (USA) (Plasma)
- etc...



Gyrotron development at Fukui (FIR FU)

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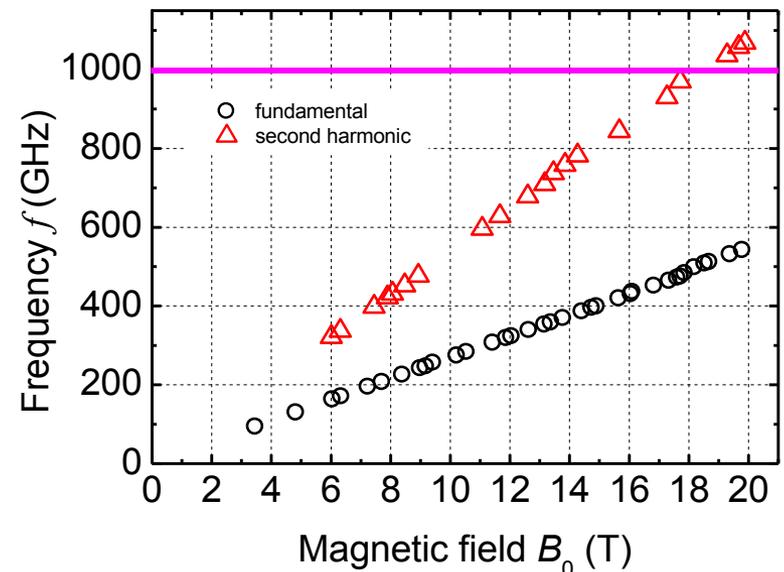
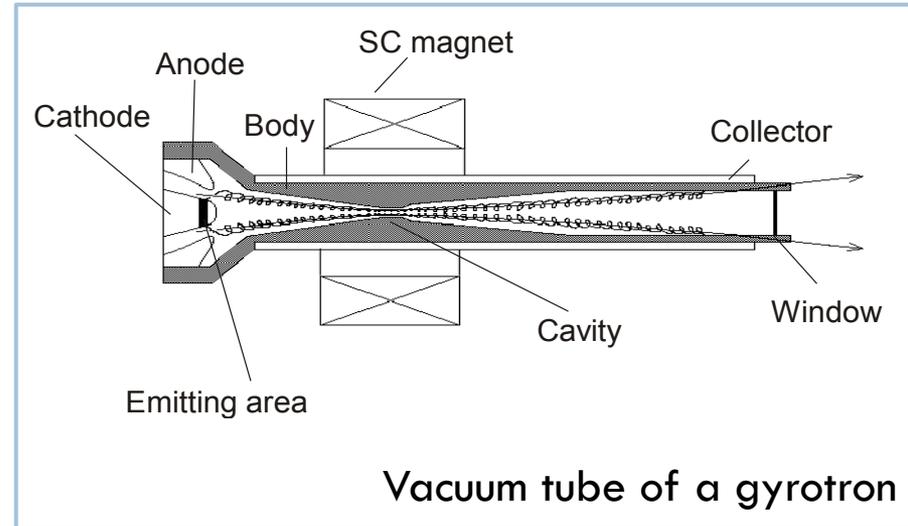
Radiation sources of SMMW

- Backward Wave Oscillator
- Molecular Laser
- Free Electron Laser
- **GYROTRON**

in FIR FU

- High frequency up to 1 THz
- Tunable frequency (38~889GHz)
- High power radiation with kW class

(World first ESR measurement by using a gyrotron as radiation source)



Outline

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1. Introduction:
 - ▣ P-doped Si (Si:P)
 - ▣ Kane's model of quantum computer
2. cw-ESR set-up and experimental results
3. Numerical solution of Bloch equations under inhomogeneously-broaden line
4. Discussion of field dependence of electron $T_1(T, H)$
5. Dynamic polarization of ^{31}P nuclear spins
6. ESR/NMR double resonance experiment
7. Summary

Refer to M. Song *et al.*, J. Phys.: Condens. Matter **22** (2010) 206001

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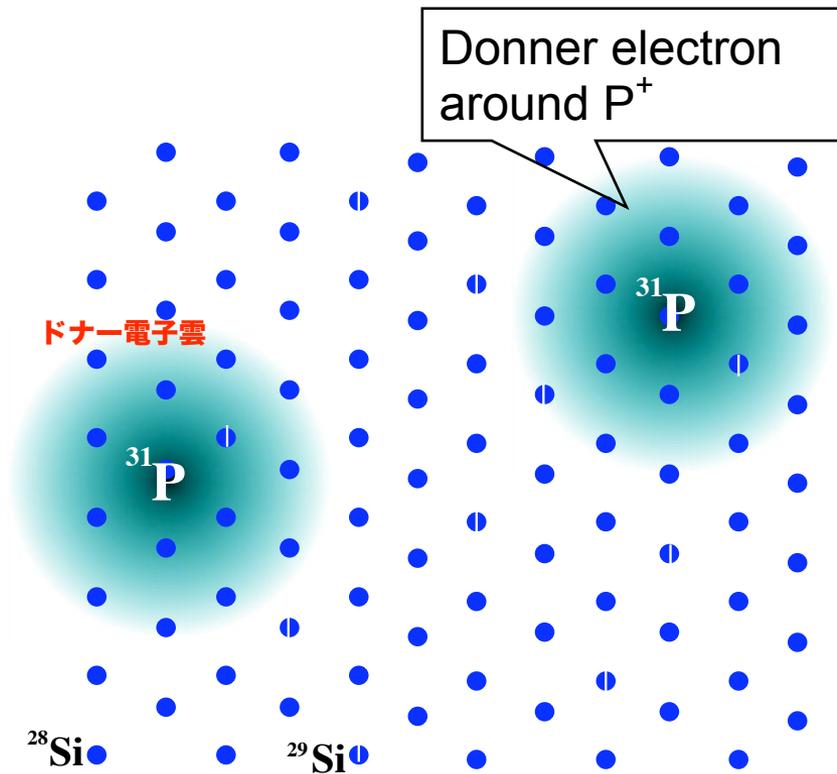
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P-doped Si (Si:P) : typical n-type semiconductor

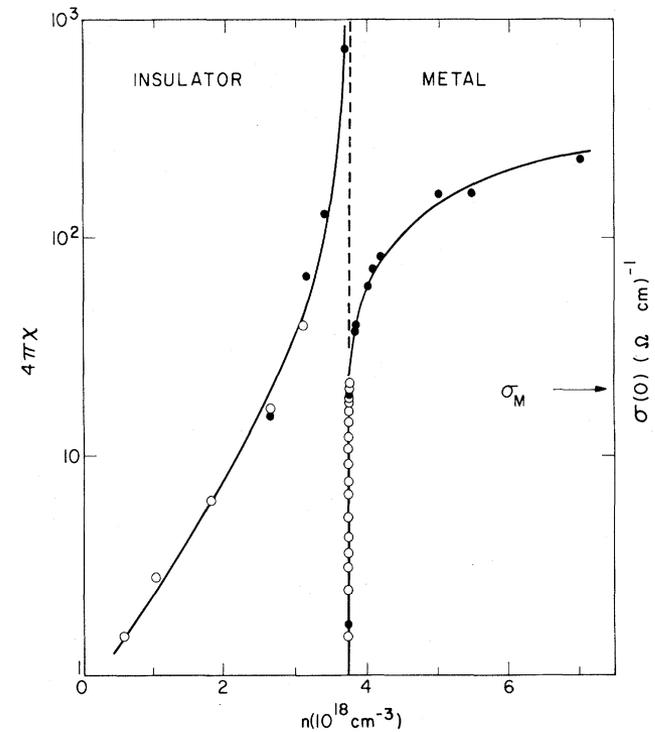
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Critical concentration $n_c = 3.7 \times 10^{18} \text{ cm}^{-3}$
 $\rightarrow n_c^{-1/3} \sim 60 \text{ \AA}$

$n > n_c$: metallic
 $n < n_c$: insulator



A schematic draw of phosphorus (P) doped silicon (Si)



Metal-insulator transition in a doped semiconductor

T. F. Rosenbaum *et al.*: PRB **27** (1983) 7509

4-level scheme in Si:P (isolated donor)

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Two spin system coupled via hyperfine interaction A ($S=1/2, I=1/2$)

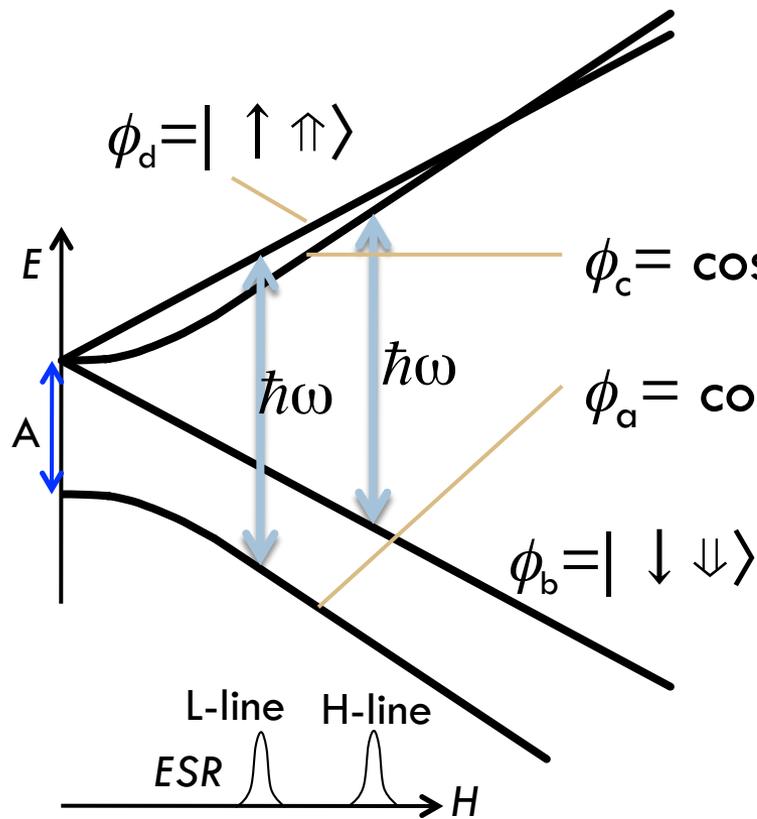
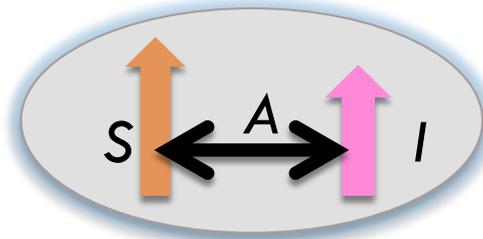
$$\mathcal{H} = -\gamma_S \hbar S^z H_0 + \mathbf{S} \cdot \tilde{\mathbf{A}} \cdot \mathbf{I} - \gamma_I \hbar I^z H_0$$

$(H_0 \parallel z, \gamma_S > 0, \gamma_I > 0)$

A = Hyperfine coupling energy
 $(A/h = 117.5 \text{ MHz for P in Si:P})$

$\gamma_I/2\pi = 17.2 \text{ MHz/T for } ^{31}\text{P}$

Wavefunction $|m_S, m_I\rangle$ ($m_S, m_I = \pm 1/2$)



$$\phi_c = \cos\theta |\uparrow\downarrow\rangle + \sin\theta |\downarrow\uparrow\rangle \approx |\uparrow\downarrow\rangle$$

$$\phi_a = \cos\theta |\downarrow\uparrow\rangle + \sin\theta |\uparrow\downarrow\rangle \approx |\downarrow\uparrow\rangle$$

$$\tan\theta = A/2g\mu_B H$$

4-levels scheme in Si:P and Overhauser effect

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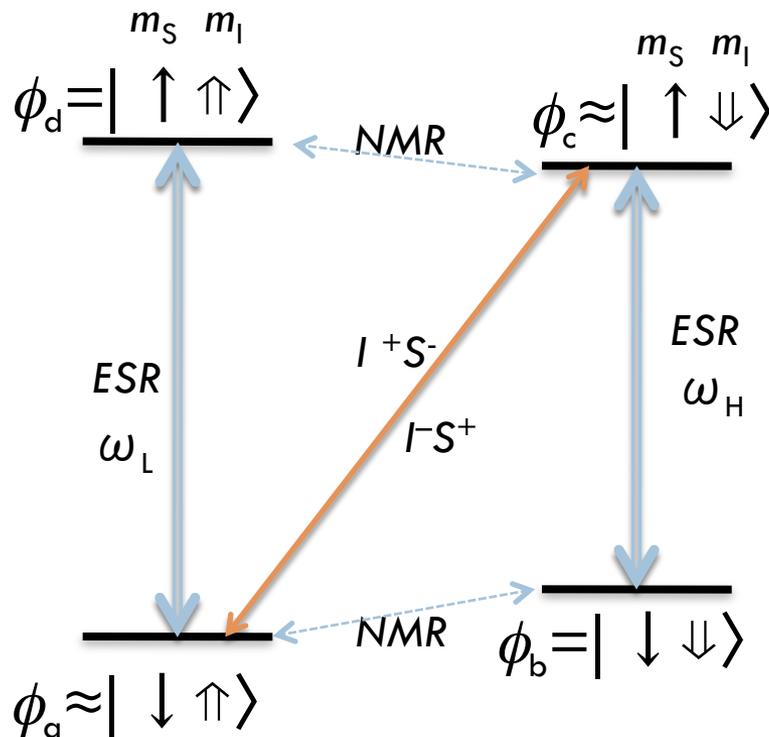
Two spin system coupled via hyperfine interaction A ($S=1/2, I=1/2$)

$$\mathcal{H} = -\gamma_S \hbar S^z H_0 + \mathbf{S} \cdot \tilde{\mathbf{A}} \cdot \mathbf{I} - \gamma_I \hbar I^z H_0$$

($H_0 \parallel z, \gamma_S > 0, \gamma_I > 0$)

$$\left(\begin{array}{l} A = \text{Hyperfine coupling energy} \\ (A/h = 117.5 \text{ MHz for P in Si:P}) \\ \gamma_I/2\pi = 17.2 \text{ MHz/T for } ^{31}\text{P} \end{array} \right)$$

Wavefunction $|m_S, m_I\rangle$ ($m_S, m_I = \pm 1/2$)



For ^{31}P in Si:P, hyperfine coupling constant A is a scalar so that

$$\mathbf{S} \cdot \tilde{\mathbf{A}} \cdot \mathbf{I} = A \mathbf{S} \cdot \mathbf{I} = A S^z I^z + \frac{1}{2} (S^+ I^- + S^- I^+)$$

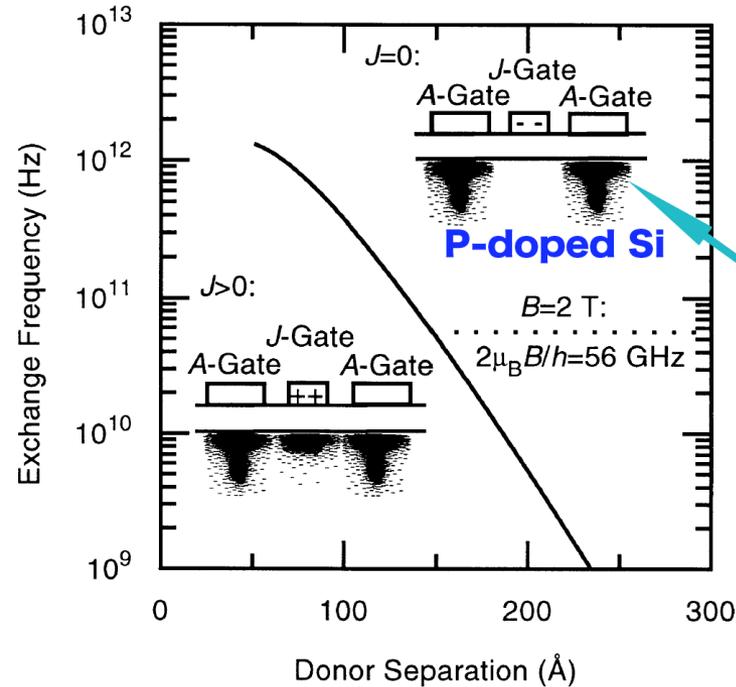
Thus, $S^+ I^+$ and $S^- I^-$ terms remain strictly forbidden.

Dynamic nuclear polarization (DNP) occurs when populations of sublevels are changed from thermal equilibrium by ESR transitions.
= **Overhauser effect**

Kane's model of quantum computer with P-doped silicon (Si:P)

(B. E. Kane : Nature **393** (1998) 133)

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量子ゲート

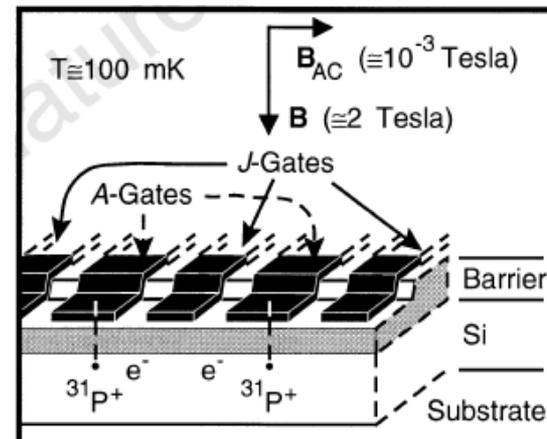
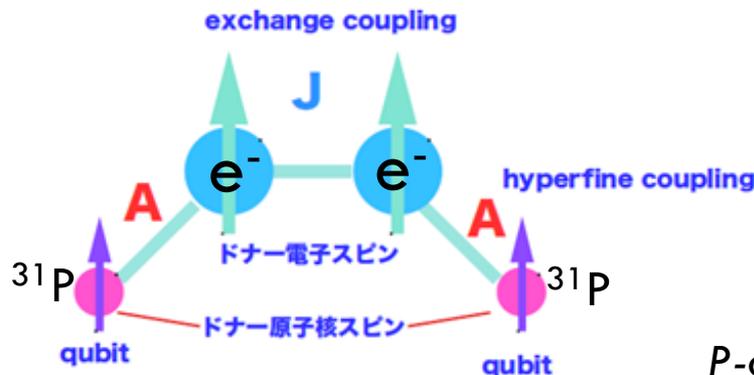
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P-doped Si: ^{31}P ($I = 1/2$), Qubit

A-Gate selects a qubit by Stark effect.

J-Gate turns on and off the electron-mediated coupling between nuclear spins.

→ Entanglement can be controlled by J-Gate voltage V_J



P-doped Si is a candidate to realize a quantum computation with ^{31}P nuclear spin.

For realization of Kane's model quantum computer

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- Electric field control of hyperfine field
- **Spin dynamics** at $B > 2$ T and $T \sim 0.1$ K (required in order to quench the electron spin freedom)

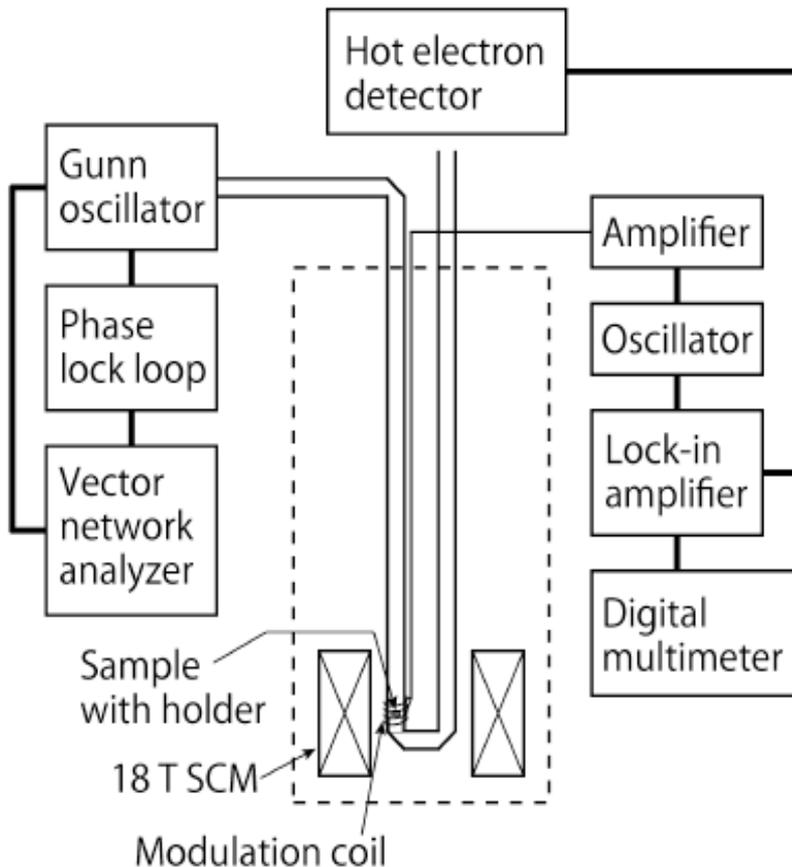
- **Spin dynamics !!**
 - **DNP** can be used to initialize qubits
 - Study in a wide range of B
 - Pulsed ESR studies have been reported at a few values of B .
→ We need more data in a wide range of B .
 - No reports of NMR for insulator Si:P

cw-ESR at high magnetic fields $B \sim 3$ T

- 〈advantage〉 cw-ESR can be performed easily in a wide range of frequency.
- 〈problem〉 Direct measurement of T_1 is difficult.
 - ▣ T_{1e} can be measured with a help of numerical calculation
 - ▣ Nuclear spin dynamics is also observed via ESR
- Sample: Si:P prepared at KAIST
 - thin film $\sim 3 \times 3$ mm²
 - P concentration: $n = 6.5 \times 10^{16}$ cm⁻³ $\ll n_c$
 - Natural Si (4 % ²⁹Si)

High frequency ESR system with field modulation

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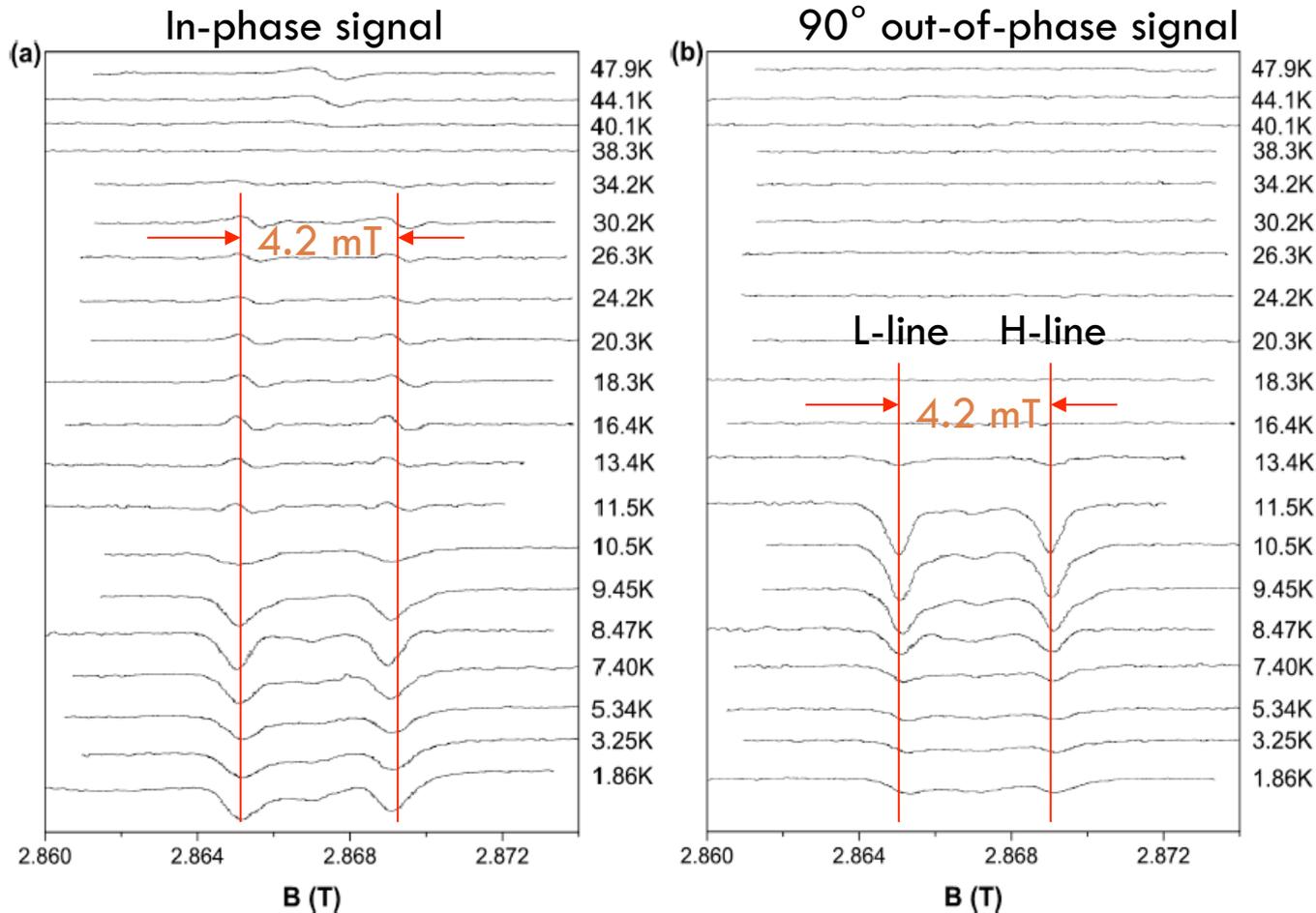
mm-wave cw-ESR of
FIR Center, Univ. Fukui

- Gunn oscillator was phase-locked to Millimeter-Wave Vector Network Analyzer. (20 - 600 GHz)
- Hot electron detector (In-Sb) cooled to 4.2 K.
- Field modulation method: **In-phase and out-of-phase signals** are obtained as the lock-in output with respect to ac-field modulation.
- $B = 2.9 \text{ T}$, $\omega = 80 \text{ GHz}$, $\omega_m = 330 \text{ Hz}$ (590 Hz, 15 kHz)

cw-ESR study on P-doped Si

Song et al.: J. Phys.: Cond. Matter **22**.(2010) 206001

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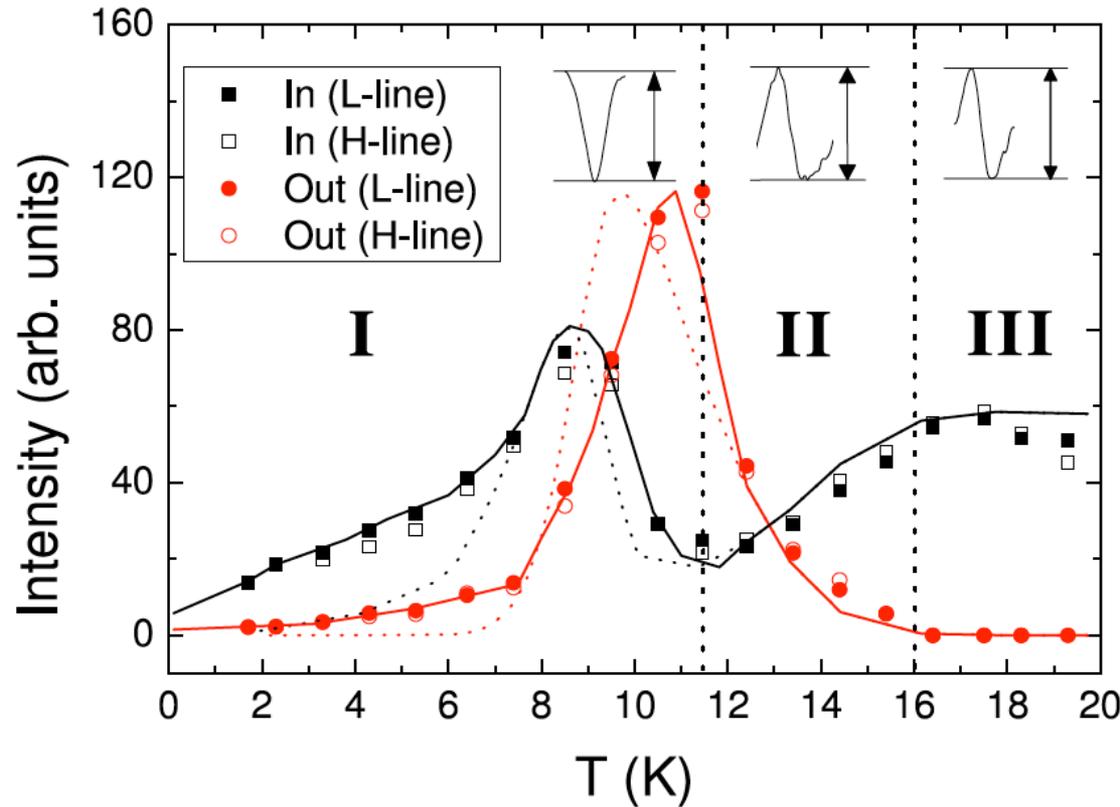


$\omega = 80$ GHz,
 $\omega_m = 330$ Hz

Two ESR lines split by hyperfine coupling
L-line (lower field)
H-line (higher field)

$$\mathcal{H} = g\mu_B B S_z + A I \cdot S - \gamma_N \hbar B I_z \rightarrow \omega_{res} = \frac{g\mu_B}{\hbar} \left(B \pm \frac{1}{2} \frac{A}{g\mu_B} \right), \text{ where } \frac{A}{g\mu_B} = 4.2 \text{ mT}$$

T-dependence of normalized ESR signal intensity



- Intensity is measured according to the change of the line shape
- Out-of-phase signal appears! Maximum ~12 K
- In-phase signal showed complicated dependence on T: Maxima ~8 K, ~17 K
- Decreasing intensity below 8 K with decreasing temperature.

$B_m \sim 0.1$ mT
 $\omega_m \sim 2 \times 10^3$ rad/s

B_1 unknown

Normalized Intensity = $I(\text{obs}) / B_S(g\mu_B SH/k_B T)$,
 where $M_0 \sim B_S(g\mu_B SH/k_B T)$ is the Brillouin function.

Bloch Equations in a rotating frame with ω'

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Bloch eqs. $\left(\begin{array}{l} \frac{dM_x}{dt} = -\gamma b_z M_y - \frac{M_x}{T_2} \\ \frac{dM_y}{dt} = \gamma b_z M_x - \gamma B_1 M_z - \frac{M_y}{T_2} \\ \frac{dM_z}{dt} = \gamma B_1 M_y - \frac{M_z - M_0}{T_1} \end{array} \right. \quad \begin{array}{l} b_z = \frac{(\omega - \omega')}{\gamma} + B_m \cos \omega_m t \\ (\delta\omega = \omega - \omega') \end{array}$

- M_0 : equilibrium amplitude of \mathbf{M}
- T_1, T_2 : relaxation constants $\rightarrow T_1$: not known (parameter)
 T_2 : known at low field

$$T_2 = T_1 \text{ for } T_1 < 10^{-4} \text{ s, } T_2 = 1 \times 10^{-4} \text{ s for } T_1 > 10^{-4} \text{ s}$$

- Solve \rightarrow time evolution of \mathbf{M} (initial condition $\mathbf{M} = (0,0,1)$)
by GNU Scientific Library (GSL) on UNIX
Embedded Runge-Kutta-Fehlberg method

Passage conditions

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Passage conditions

$\varepsilon_A \gg 1$: Adiabatic passage

$$\varepsilon_A = \frac{\gamma B_1^2}{B_m \omega_m}$$

← $\varepsilon_A = 4.7$: (quasi-)adiabatic passage even at the resonance point

$\varepsilon_R \gg 1$: Rapid passage

$$\varepsilon_R = \frac{B_m}{B_1} \omega_m T_1$$

$\varepsilon_F \gg 1$: Fast passage

$$\varepsilon_F = \omega_m T_1 < \varepsilon_R$$

$\varepsilon_F = 1$

$\varepsilon_R = 1$

Region I

$\varepsilon_R, \varepsilon_F \gg 1$
rapid and fast

Region II

$\varepsilon_R \gg 1$ & $\varepsilon_F \ll 1$
rapid and non-fast

Region III

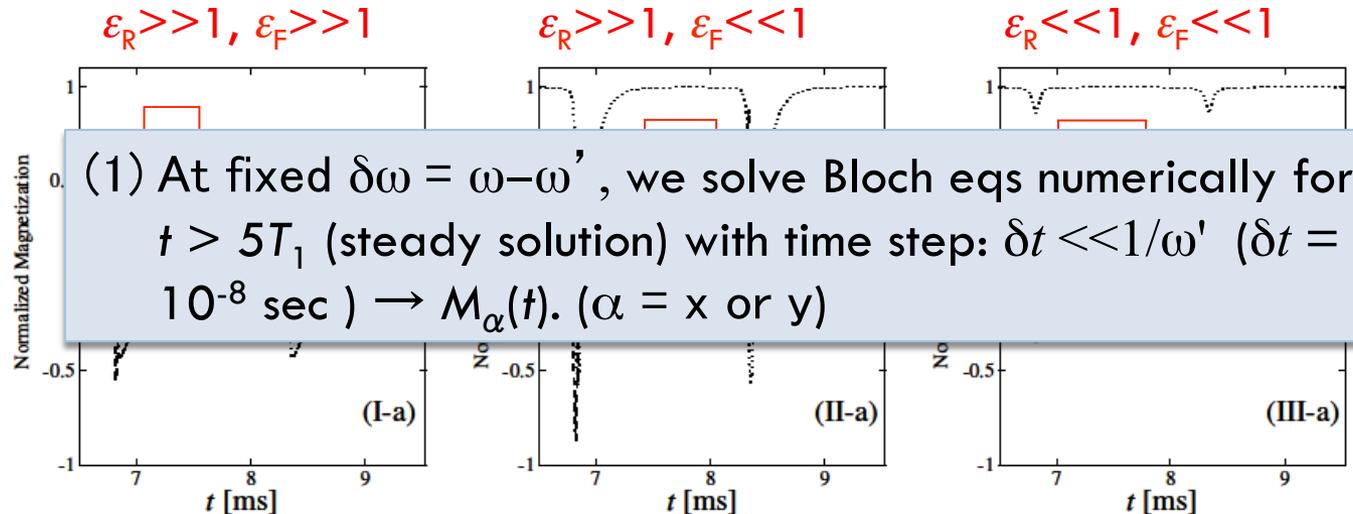
$\varepsilon_R, \varepsilon_F \ll 1$
slow (non-rapid)
and non-fast



Numerical solutions of Bloch eqs & lock-in detected ESR signals

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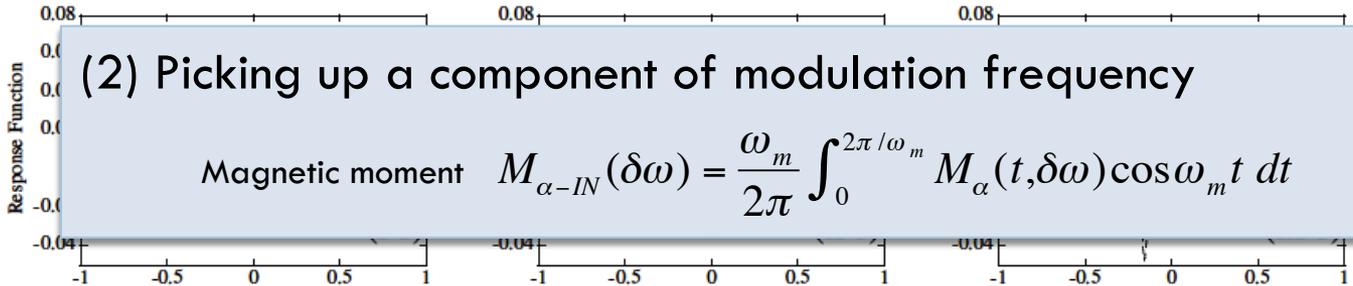
$M_x(t), M_y(t)$



(1) At fixed $\delta\omega = \omega - \omega'$, we solve Bloch eqs numerically for $t > 5T_1$ (steady solution) with time step: $\delta t \ll 1/\omega'$ ($\delta t = 10^{-8}$ sec) $\rightarrow M_\alpha(t)$. ($\alpha = x$ or y)

$M_{\alpha-IN}(\delta\omega),$

$M_{\alpha-OUT}(\delta\omega)$



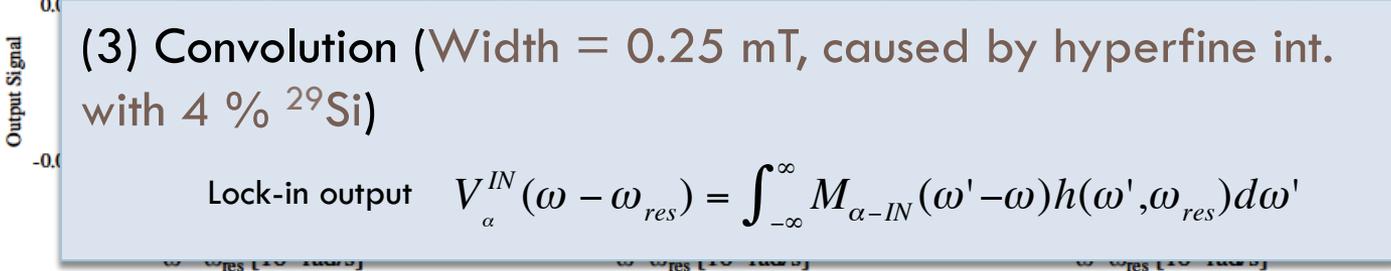
(2) Picking up a component of modulation frequency

Magnetic moment
$$M_{\alpha-IN}(\delta\omega) = \frac{\omega_m}{2\pi} \int_0^{2\pi/\omega_m} M_\alpha(t, \delta\omega) \cos \omega_m t dt$$

Signal shape change according to passage conditions

$V_{\alpha-IN}(\omega - \omega_r),$

$V_{\alpha-OUT}(\omega - \omega_r)$



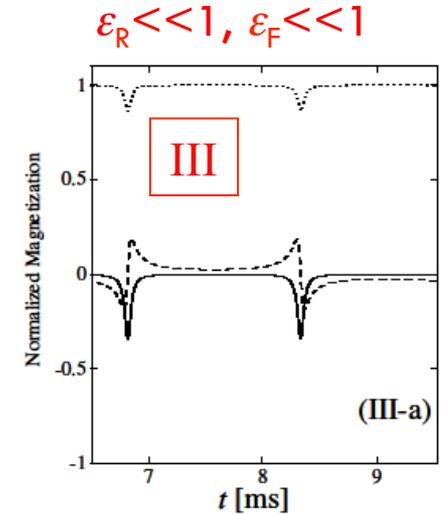
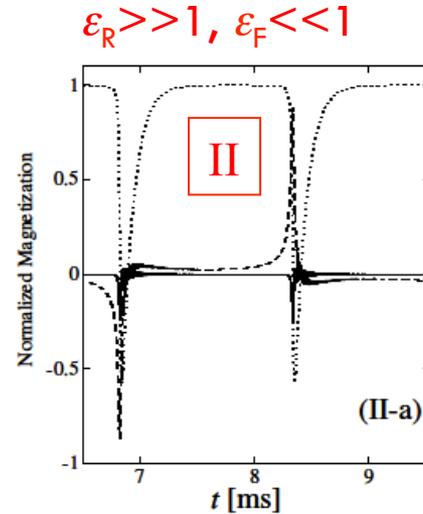
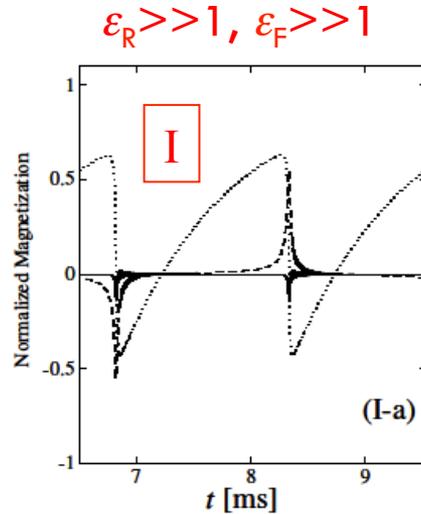
(3) Convolution (Width = 0.25 mT, caused by hyperfine int. with 4 % ^{29}Si)

Lock-in output
$$V_\alpha^{IN}(\omega - \omega_{res}) = \int_{-\infty}^{\infty} M_{\alpha-IN}(\omega' - \omega) h(\omega', \omega_{res}) d\omega'$$

Numerical solutions of Bloch eqs & lock-in detected ESR signals

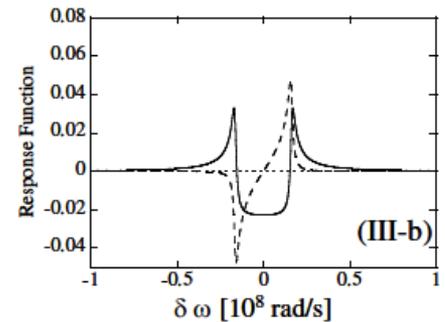
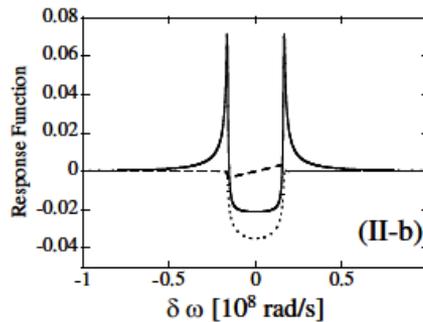
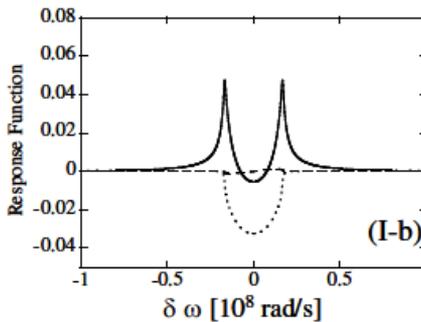
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$$M_x(t), M_y(t)$$



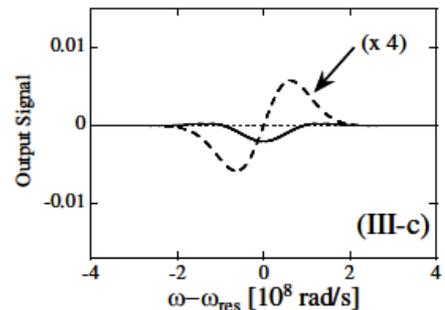
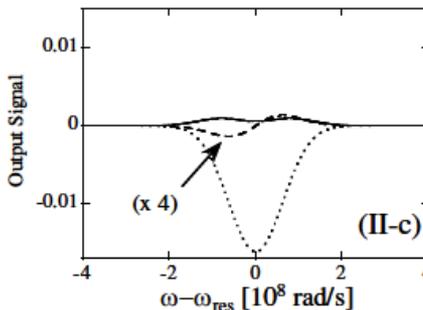
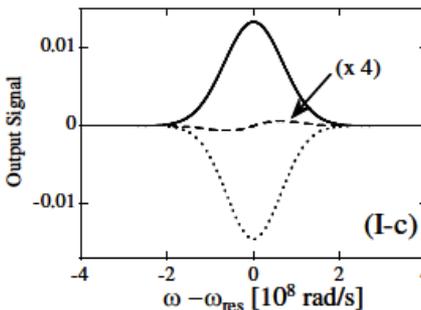
$$M_{\alpha-IN}(\delta\omega),$$

$$M_{\alpha-OUT}(\delta\omega)$$



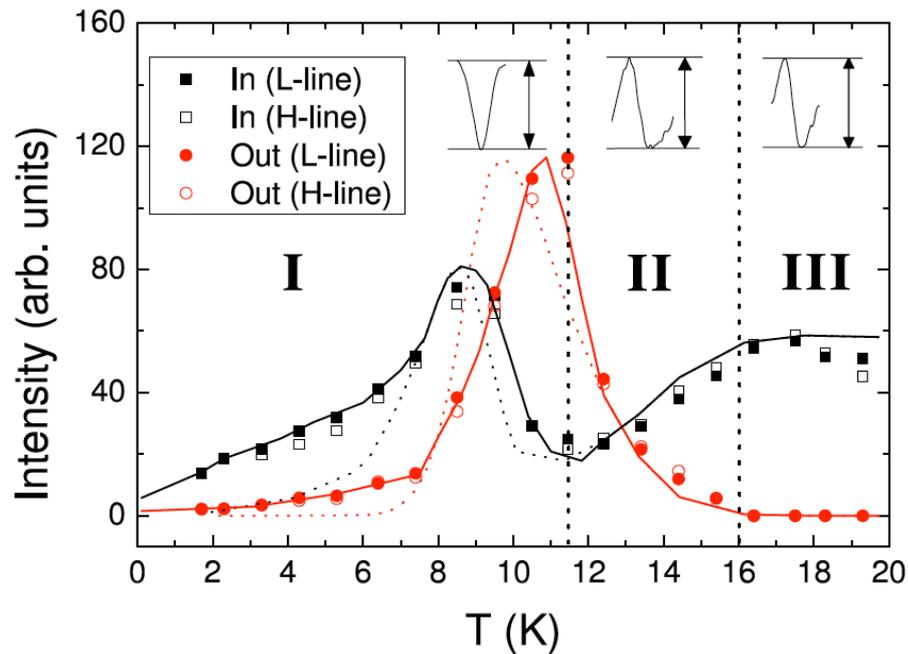
$$V_{\alpha-IN}(\omega - \omega_R),$$

$$V_{\alpha-OUT}(\omega - \omega_R)$$



Normalized ESR signal intensity as a function of T_1 by Bloch eqs.

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Solid line: Numerical calculation

$$\text{Normalized Intensity} = I(\text{obs}) / B_S (g\mu_B S H / k_B T)$$

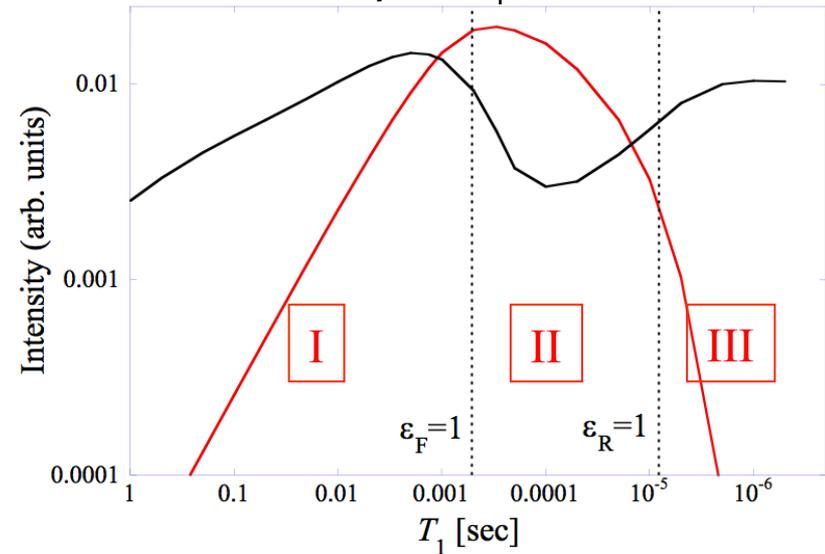
Black: In-phase,

$M_y \gg M_x$ for $\epsilon_R \gg 1$ and

Red: Out-of-phase

$M_x \gg M_y$ for $\epsilon_R \ll 1$.

Calculation: intensity vs. T_1



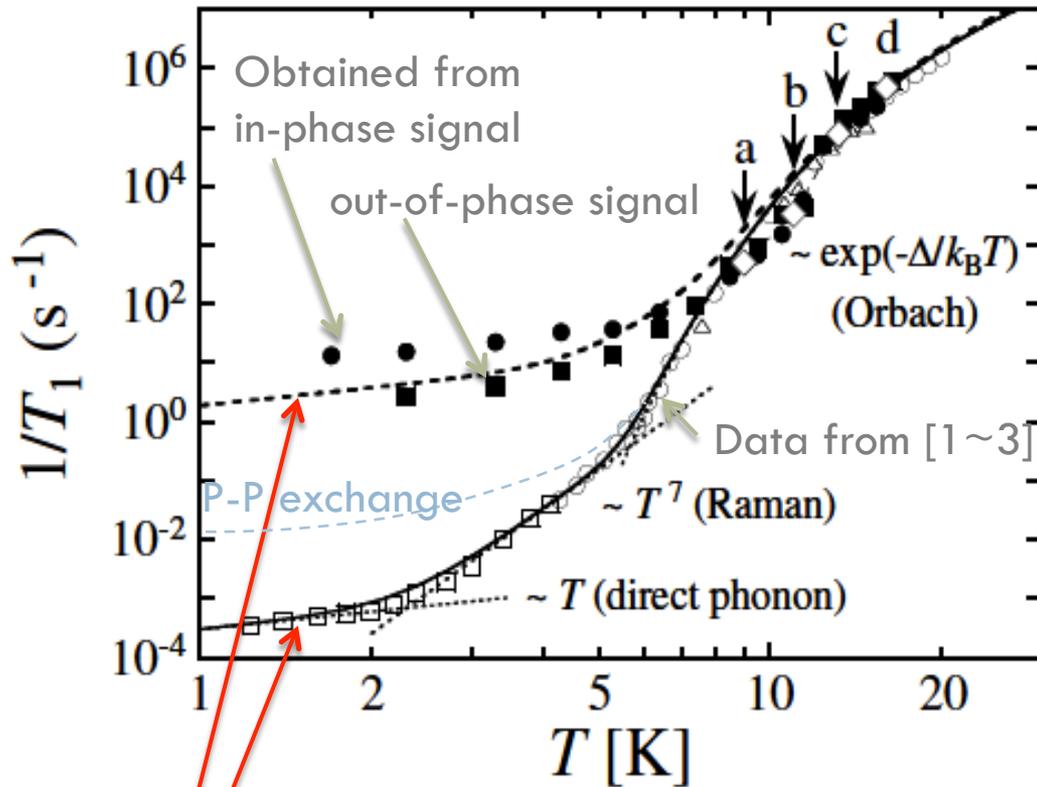
(4) Take peak-peak intensity

Parameter Fitting:

- 1) B_1 is chosen by fitting $T_1(T)$ at $\epsilon_R = 1$.
- 2) The mixing ratio between M_y and M_x is chosen by fitting the peak height in $\epsilon_F < 1$ and a constant value in $\epsilon_R > 1$ for the black line $:(4 M_y + M_x)$.

Temperature dependence of $T_1(T)$ at 2.9 T

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T_{1e} is represented fairly well by 3 relaxation processes which have been discussed at $B \sim 0.3$ T



T_{1e} process changes at high field for $T < 6$ K.

- $B < 0.8$ T: P-P exchange for $n > 10^{16} \text{ cm}^{-3}$ [1]
- $B > 1$ T: Direct/Raman process overcomes P-P exchange process for $n = 6.5 \times 10^{16} \text{ cm}^{-3}$

$$T_1^{-1} = aB^4T + bB^2T^7 + c \exp(\Delta/k_B T)$$

Solid line: $T_1^{-1}(T)$ for isolated P electrons $n < 10^{16} \text{ cm}^{-3}$ at $B < 0.3$ T [1~3]

Dashed line: $T_1^{-1}(T)$ for $B = 2.9$ T with the same values of a, b, c and $\Delta/k_B = 122.5$ K.

[1] Feher & Gere: PR **114** (1959) 1245

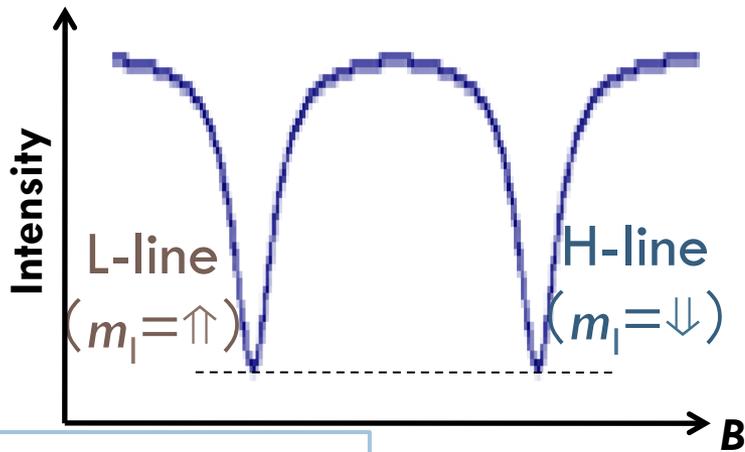
[2] Castner: PR **130** (1963) 58

[3] Tyryshkin et al.: PRB **68** (2003) 193207

ESR spectrum of Si:P with DNP

Schematic drawing of field sweep cw-ESR spectrum

☞ Each of two lines corresponds a specific direction of nuclear spin.



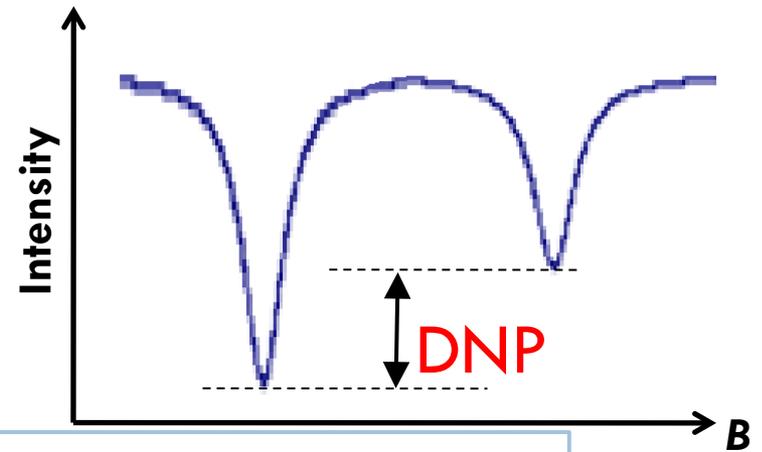
Thermal equilibrium

$$P_b/P_d = P_a/P_c$$

intensity of absorption
 $I(L) \sim P_b - P_d, I(H) \sim P_a - P_c$

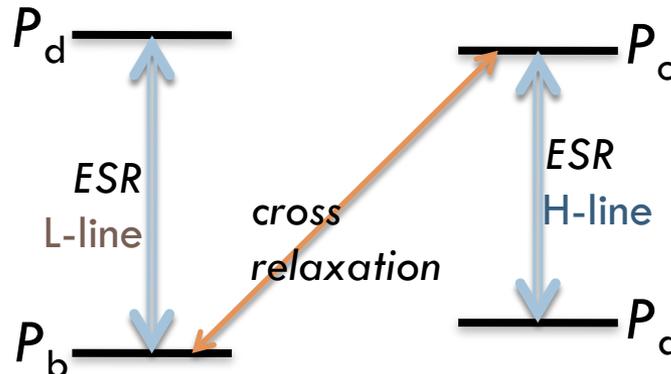
$$\underline{I(L) = I(H)}$$

P_i : poulations $\sum P_i = 1$



After saturating H-line by ESR

Asymmetric spectrum suggests DNP.

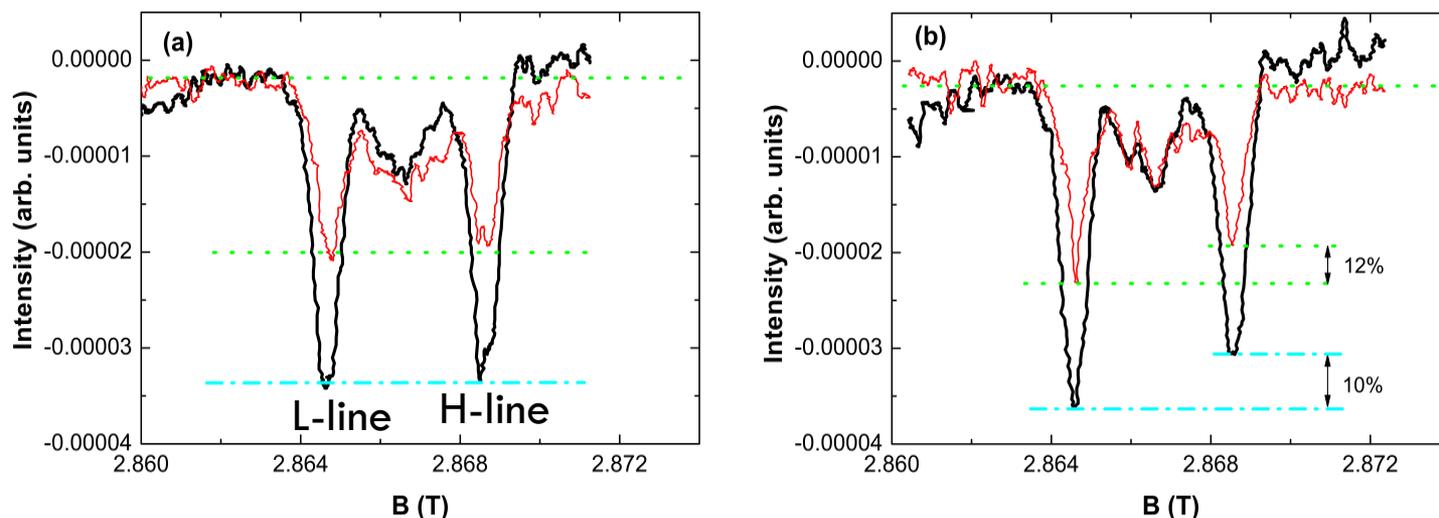


$$P_b/P_d \gg P_a/P_c$$

$$P_b \gg P_a$$

$$\underline{I(L) > I(H)}$$

Possibility of dynamic nuclear polarization (DNP) of ^{31}P



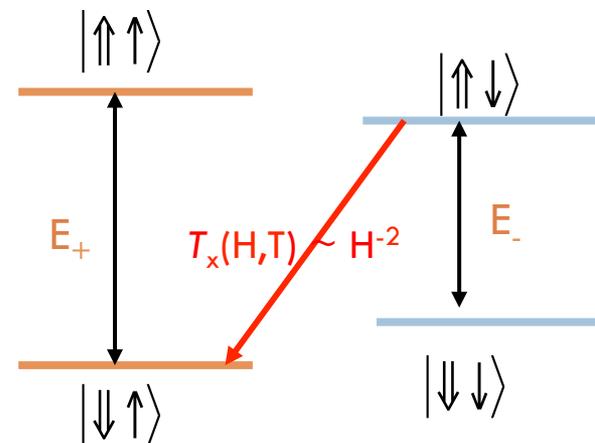
(a) Right after cooling down from 70 K to 6.9 K. (b) After radiation of microwave for 20 minutes at H-line. Black : in-phase signal, red : 90° out-of-phase signal.

Asymmetric ESR spectrum suggesting DNP effect.

$$\frac{I(L) - I(H)}{I(L) + I(H)} = 10 \% \quad \Rightarrow \quad \frac{\langle I_z \rangle}{\langle I_{\text{equi}} \rangle} = 6 \times 10^2$$

at 6.9 K, 2.9 T

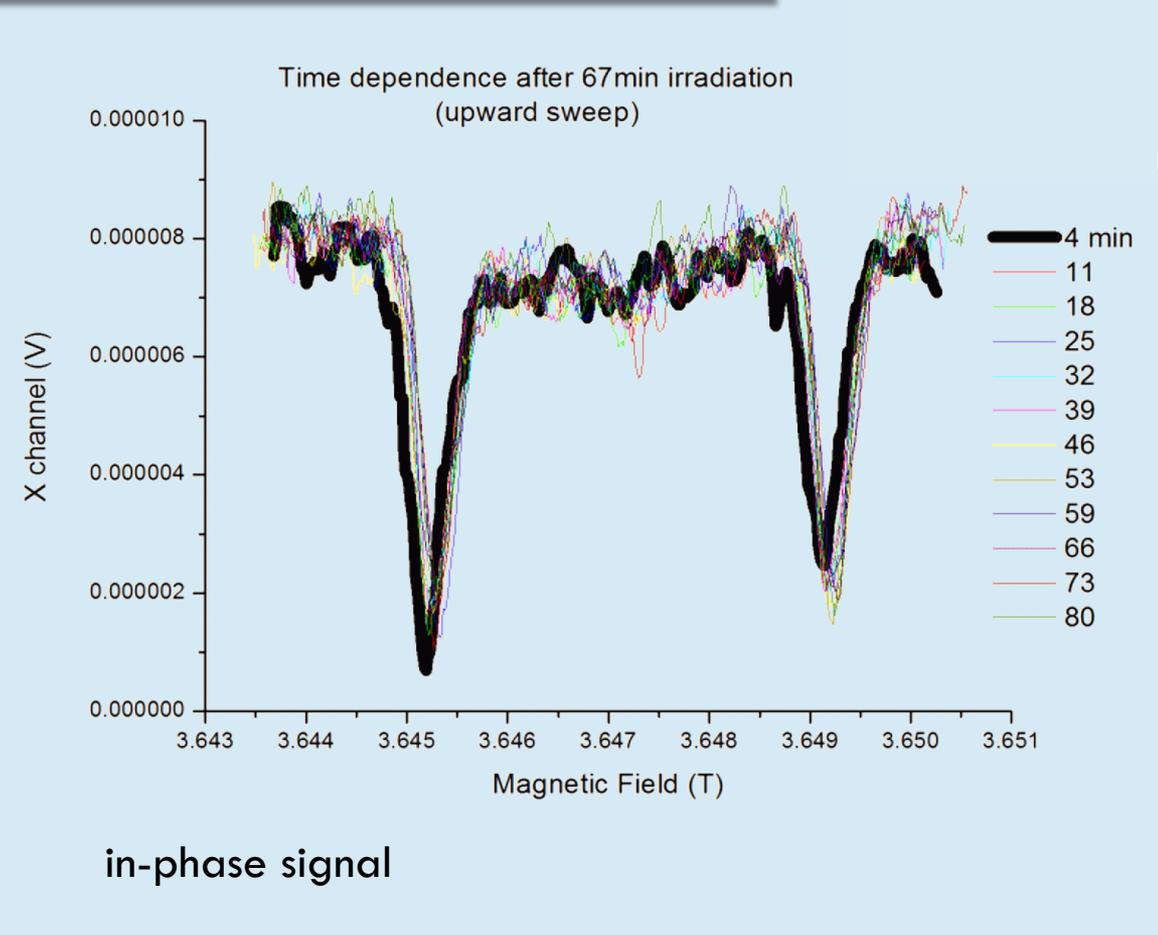
DNP effect



DNP and relaxation to thermal equilibrium

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Time evolution of nuclear polarization



$T=1.44$ K

$B=3.64$ T (102.2 GHz)

Preparation: Irradiation at H-line for 67 min. with full power microwave and field modulation.

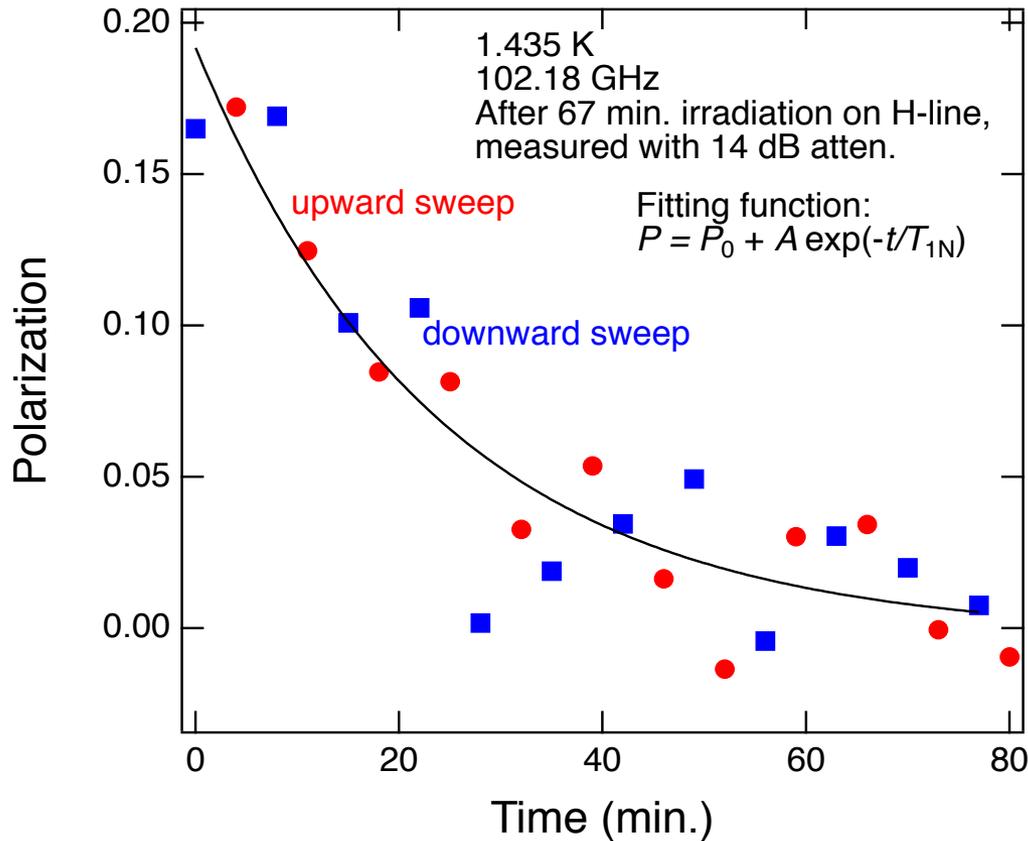
Measurement:
Sweep both direction consecutively with attenuated microwave.

Microwave: 102.2 GHz, $B_1 \sim 2 \times 10^{-6}$ T

Modulation frequency : 541 Hz, Modulation field ~ 0.03 mT

DNP and relaxation to thermal equilibrium (2)

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T_{1N} measured by ESR

$T_{1N} = 24 \pm 7$ min.
at $T=1.44$ K and $B=3.65$ T

DNP effect

Maximum nuclear spin polarization
= 19 %

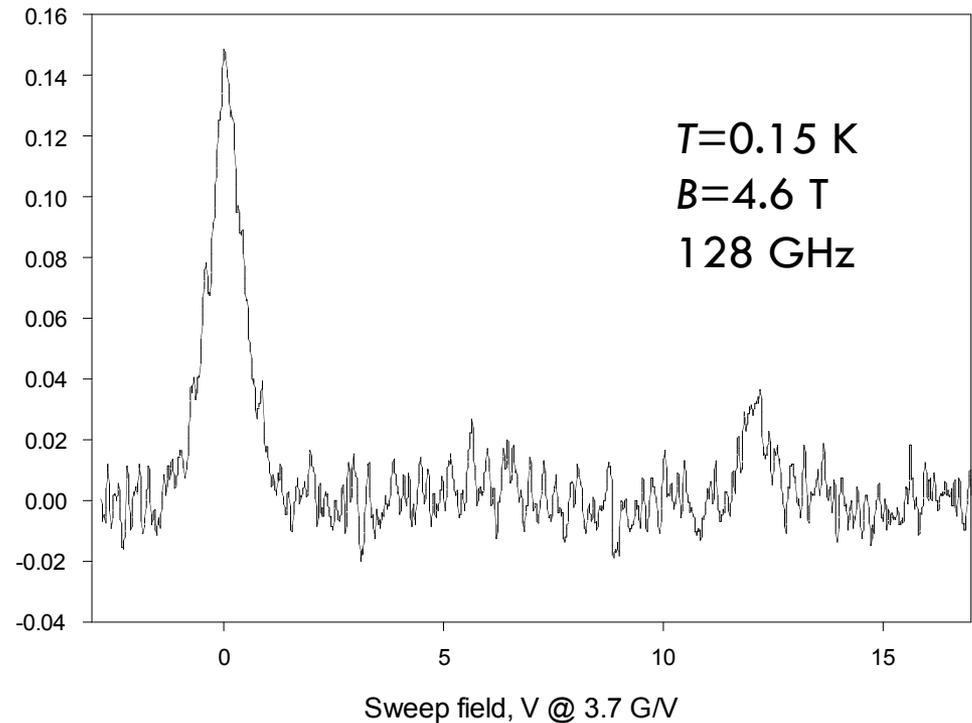
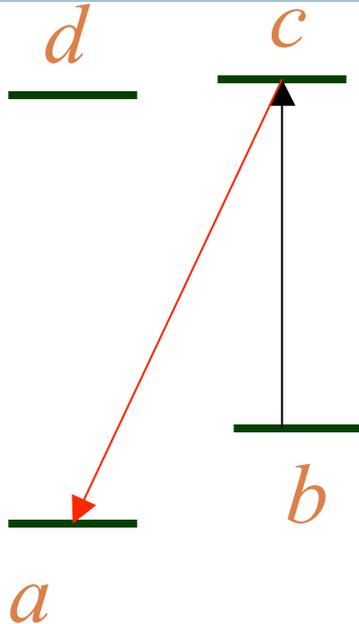
$$\frac{\langle I_z \rangle}{\langle I_{\text{equi}} \rangle} = 2 \times 10^2$$

(depends on the sweep direction)

Thermal equilibrium polarization at
1.44 K, 3.65 T

$\tanh (g \mu_B H / 2kT) = \underline{94 \%}$ for
electron spin

Imperfect saturation of electron spins?

Dynamic nuclear polarization after 300 s pumping of b - c transition

α - c transition probability

$$W_{ac} = \left(\frac{A}{2g\mu_B B} \right)^2 W_{bc} \approx 1.6 \cdot 10^{-7} W_{bc}$$

if $T_{1e} = 1$ s, then $T_{\alpha-c \text{ pumping}} \sim 10^7$ s

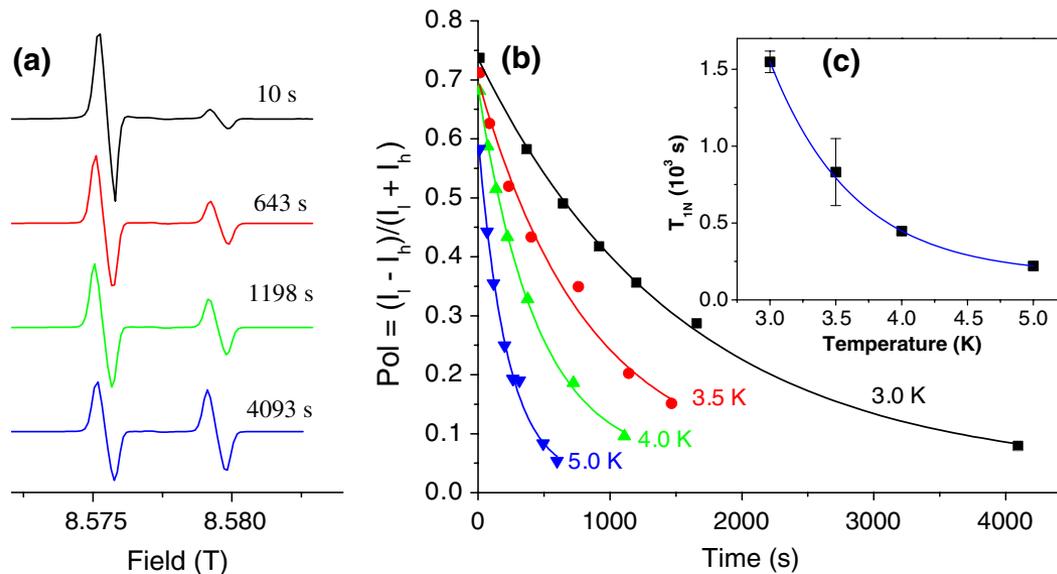


much faster in real system

J. van Tol *et al.*: Appli. Magn. Reso. **36** (2009) 259, High-Field Phenomena of Qubits

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- Sample: $3 \times 3 \times 1 \text{ mm}^3$ piece of crystalline silicon from Wacker Siltronic with $[P] = 1 \times 10^{15} \text{ cm}^{-3}$
- 240 GHz ESR = 99.99% electron spin polarization is reached at 2.1 K

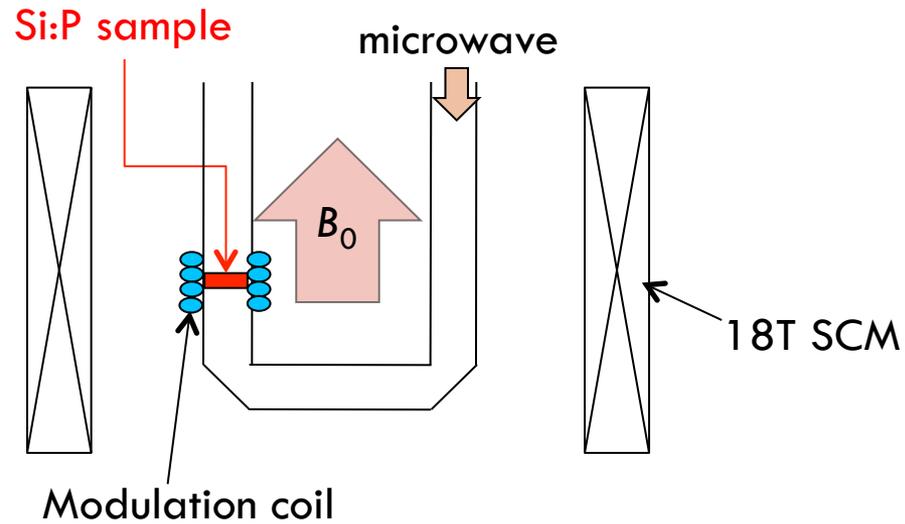


mm-wave source:
Gunn diode, 120GHz 40
mW max., 240 GHz 9 mW
max. with doubler

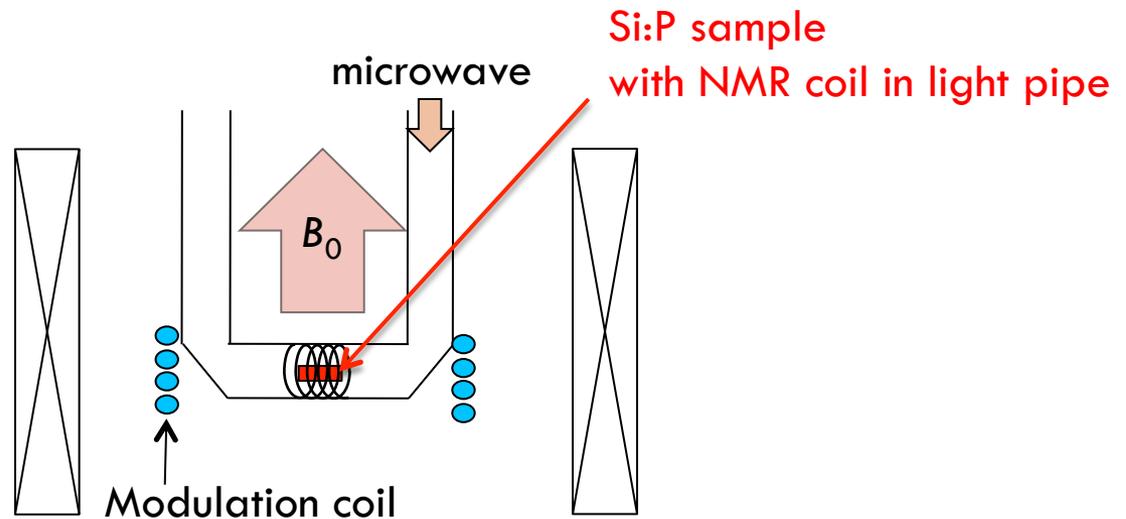
- EPR spectra at 3K measured at very low power after saturating the high-field hyperfine component for **5 min** at high power.
- ~75 % polarization below 5 K
- T_{1N} is measured as a decay of polarization. $T_{1N} \sim 3.5$ min at 5 K.

Change of sample position for adding NMR coil

cw-ESR

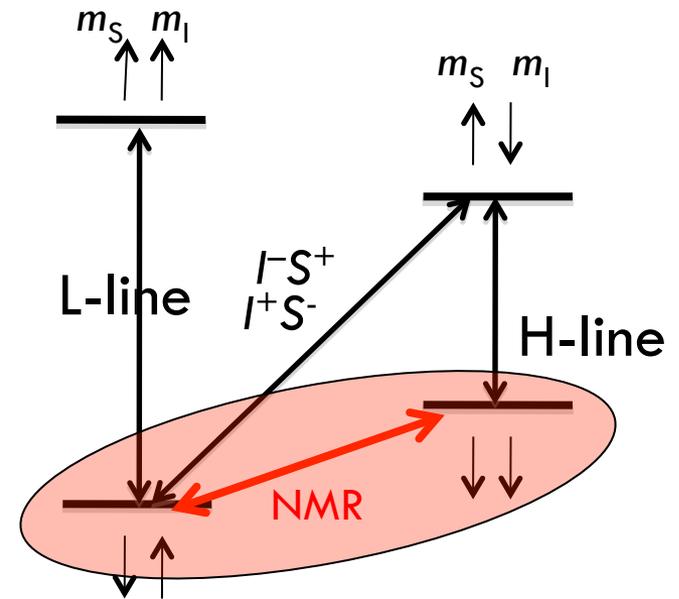
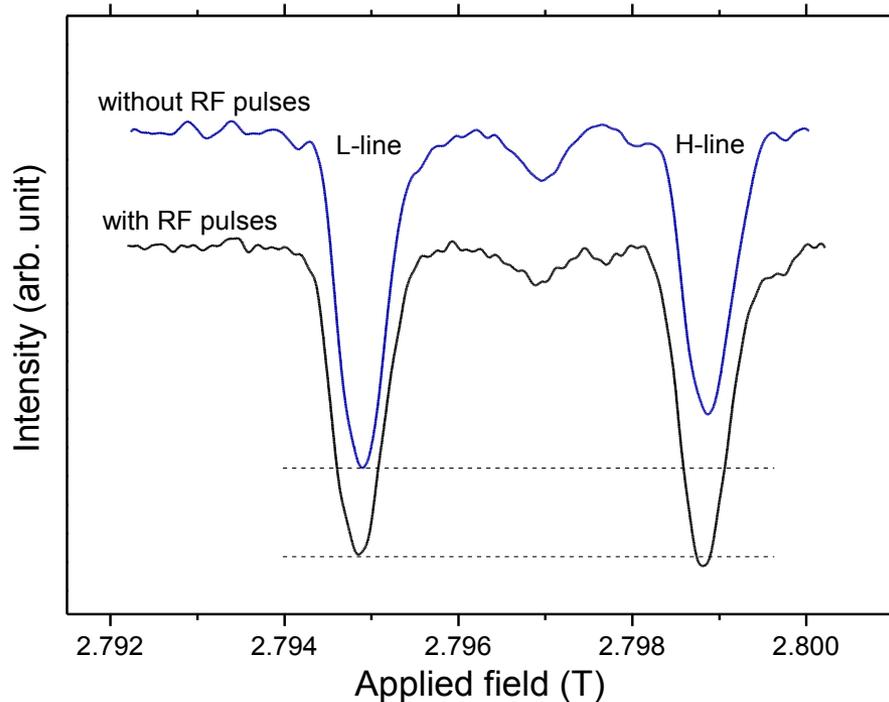


Double resonance



Double resonance experiments

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ESR spectra recorded after irradiating H-line for 20 minutes at 1.5 K.

$$f = 78.45 \text{ GHz}, f_{\text{RF}} = 107.1 \text{ MHz}$$

Nuclear spin polarization was changed by applying RF.

Summary

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- T_{1e} obtained from cw-ESR spectrum by numerical calculations of time evolution of Bloch equations
 - ▣ Shape change according to passage conditions
 - ▣ Reasonable T - and B -dependence
$$T_1^{-1} = aB^4T + bB^2T^7 + c \exp(\Delta/k_B T)$$
- DNP effect found at $B=2.9$ T and 3.6 T and $T < 7$ K
 - ▣ T_{1N} is measured
 - ▣ What restrict nuclear polarization?
- ESR/NMR double resonance measurements
 - ▣ nuclear spin polarization changed by RF
 - ▣ ENDOR

Future plan and prospect

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- Future plan
 - ▣ Using cavity to increase B_1
 - ▣ DNP system with ^3He - ^4He dilution temperature
- ^{31}P -DNP-NMR could be directly observed if nuclear polarization enhances $\sim 10^3$
 - ▣ $n \sim 10^{19} \text{ cm}^{-3}$ (metallic) $\rightarrow n \sim 10^{16} \text{ cm}^{-3}$ (insulator)

END

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Thank you!