Spin superfluidity in antiferromagnets

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Spin waves

Holstein-Primakoff transformation

\[
\hat{S}_+ = \sqrt{2S\hbar} a_0 \sqrt{1 - \frac{\hbar a_0^{\dagger} a_0}{2S}} \\
\hat{S}_- = \sqrt{2S\hbar} \sqrt{1 - \frac{\hbar a_0^{\dagger} a_0}{2S}} a_0^{\dagger} \\
\hat{S}_z = S - \hbar a_0^{\dagger} a_0 = S - \hbar \hat{N}_0
\]
The main landmark of BEC is the changing of frequency at relaxation. The magnon density decay changes the chemical potential!

The first observation:

Ideal gas

Quantum gas

BEC, superfluidity

Paramagnetic, Fermi liquid

Magnetically ordered

Coherent precession

\[ \omega = \gamma H_{\text{loc}} \]

\[ i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi \]

\[ \omega = \omega_0 \]

\[ S_x + iS_y = S \sin \beta e^{i\omega t + i\alpha} \]
Spin superfluidity and BEC of magnons was found in a 5 different states of superfluid 3He. In one of this states the induction signal can live more then one hours. Its corresponds to a 99.999% of magnons to be condensed! For an atomic BEC the 30% condensation was only achieved!

\[ S_x + iS_y = S \sin \beta e^{i\omega t + i\alpha} \]
Spin waves in superfluid $^3$He are ready to be condensed!

$$N_M = \int d^3r |\Psi|^2 = \int d^3r \frac{S - S_z}{\hbar}$$

$$\Psi = \sqrt{2S/\hbar} \sin \frac{\beta}{2} e^{i\omega t + i\alpha},$$

$$\omega(k) = \omega_L - \omega + \frac{c^2 k^2}{\omega_L} \equiv \omega_L - \omega + \frac{k^2}{2m_M}$$

$$m_M \sim \frac{\omega_L}{v_F^2} \sim m_3 \frac{\omega_L}{E_F} \quad (\hbar = 1; \quad c \sim v_F)$$

$$n_M \sim S \sim \chi H/\gamma \sim \omega_L N_F \sim n_3 \omega_L / E_F$$

where $N_F \sim m_3 p_F$ is the density of states and $n_3 \sim p_F^3$ is the density of atoms.

Temperature of BEC

$$T_{BEC} \sim \left(\frac{nc^3}{\omega_L}\right)^{1/2} \sim E_F$$

$$T_F = 0.1K, \quad T_{\text{exp}} = 0.0003K$$

G. Volovik
Gross-Pitaevskii equation

\[ \frac{\delta F}{\delta \Psi^*} = 0 \]

\[ F = \int d^3r \left( \frac{\left| \nabla \Psi \right|^2}{2m_M} + (\omega_L(z) - \omega)\left| \Psi \right|^2 + F_D \right) \]

Spin Supercurrent


Dipole-dipole spin-orbit energy

Gradient energy

Spectroscopic energy

True energy at the rotating frame

Chemical potential

\[ \mu = \omega_L(z) - \omega \]

\[ F = A + (B+\mu)\left| \Psi \right|^2 + C\left| \Psi \right|^4 \]

\[ \begin{align*}
\text{C < 0} & \quad \text{B+}\mu > 0 \\
\text{C > 0} & \quad \begin{cases}
\text{B+}\mu < 0 & \mu = 0 \\
\text{B+}\mu > 0 & \mu = 0
\end{cases}
\end{align*} \]
Josephson effect

\[ \xi = \frac{1.088 c_{||}}{\sqrt{\omega (\omega - \gamma H)}} \]

**Diagram and Figures**

**Diagram:**
- Arrows indicating magnetic fields and RF (Radio Frequency) signals.
- josephson effect diagram with denoted field lines and labels.

**Figure Legends:**
Superfluid $^3$He-A in squeezed aerogel

$$F = \int d^3r \left( \frac{\left| \nabla \Psi \right|^2}{2m_M} + (\omega_L(z) - \omega)|\Psi|^2 + F_D \right)$$

$$F_D = \frac{\chi \Omega_L^2}{4} \left[ -2 \frac{|\Psi|^2}{S} + \frac{|\Psi|^4}{S^2} + \left( -2 + 4 \frac{|\Psi|^2}{S} - \frac{7}{4} \frac{|\Psi|^4}{S^2} \right) \sin^2 \beta_L \right]$$

BEC in antiferromagnet with Pulling

Coupled Nuclear electron precession


Suhl – Nacamura interaction

\[ F = \int d^3 r \left( \frac{\left| \nabla \Psi \right|^2}{2m_M} + (\omega_L(z) - \omega)|\Psi|^2 + F_{SN} \right) \]
CW NMR experiments

Broadening was about 2 MHz and RF field was only about 300 kHz

$\text{CsMnF}_3$

$\text{$^3$He-A in aerogel}$
**CW NMR experiments**

Broadening was about 2 MHz and RF field was only about 300 kHz

$CsMnF_3$

$^3$He-A in aerogel
FID amplitude vs. RF pulse length at different magnetic fields

Signal intensity vs. magnetic field
FID amplitude vs. RF pulse length at different magnetic fields

Signal intensity vs. magnetic field
Non-linear Stationary Spin Waves in Flared out texture

NMR of Rotated superfluid 3He-B

\[
\omega^2 = (\gamma H)^2 + n_\perp^2 \Omega_D^2
\]

\[
(\omega - \omega(z)) M^+ = \frac{\Omega_D^2}{2\omega(z)} \left( \frac{2}{5} \sin^2 \beta_L(r) - \frac{24}{65} \xi_d^2 \nabla^2 \right) M^+
\]
Grenoble experiments with Non-linear Stationary Spin Waves

\[ \omega = \gamma H + \nabla H \]

Quantum billiard

Computer simulation
Grenoble 2004

Follow Voislav Golo algorithm

Calculations of a spatial deflection of spin and orbit on basis of Poisson brackets and Takagi relaxation
Magnons condensation in Q-ball in the orbital texture trap

Yu.M. Bunkov “Persistent Signal; Coherent NMR state Trapped by Orbital Texture”


\[ F_D = \chi \Omega_L^2 \left[ \frac{4 \sin^2 (\beta_L/2)}{5S} |\Psi|^2 - \frac{\sin^4 (\beta_L/2)}{S^2} |\Psi|^4 \right] \]

\[ Q \sim |\Psi|^2 r_Q^2 \dot{\Psi}_Q, \]

\[ U(r, z) \propto \kappa^2 r^2 z^2 / (r^2 + z^2) \]

\[ (\omega - \omega_L(z)) \Psi = \frac{\delta F}{\delta \Psi^*}, \]

\[ F = \int d^3r \left( \frac{|
abla \Psi|^2}{2m_M} + F_D \right), \]

\[ \omega_{xy}^2 r^2 + \omega_z^2 z^2 \]
Non-ground-state Bose-Einstein condensates of magnons in superfluid 3He-B

3D axisymmetric trap.

\[ \omega_{xy}^2 r^2 + \omega_z^2 z^2 \]
Non-ground-state Bose-Einstein condensates of magnons in superfluid 3He-B

\[ f - f_0 = a_r(\Omega)(2n_r + 1) + a_\gamma(2n_\gamma + 1/2), \]

\[ P = 29 \text{ bar}, \ T = 0.25 T_c, \ f_L = 0.865 \text{ MHz} \]

\[ \Delta H/H = 1.65 \times 10^{-3}, \ \omega_z/2\pi = 27.7 \text{ Hz} \]

\[ \Omega = 0.8 \text{ rad/s}, \ \omega_r/2\pi = 243 \text{ Hz} \]

\[ \Delta H/H = 0.83 \times 10^{-3}, \ \omega_z/2\pi = 22.3 \text{ Hz} \]

\[ \Omega = 0.6 \text{ rad/s}, \ \omega_r/2\pi = 168 \text{ Hz} \]
$P = 0.5 \text{ bar, } T = 0.14 T_c, \; f_L = 0.827 \text{ MHz}$

$\Omega = 0.9 \text{ rad/s (vortices)}$

ground level, $\tau = 5.36 \text{ s}$

(2,0) level, $\tau = 0.74 \text{ s}$

rf pumping off

rf pumping
The Spin Superfluidity, the magnetic counterpart of mass superfluidity and electric superconductivity was discovered and widely studied in Moscow, Grenoble, Lancaster, Ithaca, Helsinki, Kosice, Kyoto, Tokio, Los-Alamos and by many theoreticians.

There was discovered and observed:

1. The coherent transport of magnetization in superfluid 3He-B.
2. Its based only on an antiferromagnetic properties of 3He and can be found in other magnetically ordered materials.
3. The formation of domain with coherent precession of magnetization.
4. This excited state correspond to a Bose-Einstein Condensation of spin waves.
5. Josephson phenomena, critical current and phase slippage in a channel.
5. Different modes of HPD oscillations.
6. Horizontal and vertical Spin vortex.
7. HPD techniques was applied for studies of counterflow and mass vortices in 3He, new types of vortex - spin-mass vortex was observed
8. 6 different states with coherent precession in A and B phases of 3He
9. Magon BEC was found recently in JYG. (Demokritov) Nature 443, 430 (2006).
The end