Electrons on Helium?

(A very Brief History)

- Surface-state electrons
- Rydberg states
- Electron mobility
- Many electron effects
- Wigner crystal
- Restricted geometries
- Single electronics
- Electron manipulation
- Qubits?

Many research groups

Mike Lea
Two-dimensional electrons on helium

Generation:
Thermionic emission
Glow discharge
Photoemission

Holding field $E_z$

Weak Image Charge
$Q = \frac{(\varepsilon - 1)e}{(\varepsilon + 1)} = 0.028e\ (^4\text{He})$

$\varepsilon = \frac{V}{d}$

Saturation
$n_s = \frac{\varepsilon e_0 V}{ed}$

Liquid $^4\text{He}$ or $^3\text{He}$

Gas atoms

Ripplons
Quasiparticles in $^3\text{He}$

$10^5 < n < 10^9\ \text{cm}^{-2}$

In-plane: Interelectron distance is $\sim 1\ \mu\text{m}$

Vertical motion quantised: Rydberg states
Bohr radius 7.6 nm ($^4\text{He}$)

Helium $z<0$
Vacuum $z>0$

$\Delta E \approx 6K$
$m=2$
$m=1$

$\ell_B \approx 76\ \text{Å}$

History
Experiment: Crandall and Williams (1971)
Two-dimensional electrons on helium

Bulk Helium
$^3\text{He}, \; ^4\text{He}$

Channels

Films

Vast parameter range: $n,\; T,\; B,\; E_z$

- Diameter: $26\; \text{mm}, \; d = 2.6\; \text{mm}$
- Kimitoshi Kono ($^3\text{He}$), Denis Konstantinov, Valeriy Shikin + others

- Width: $20\; \mu\text{m}, \; d = 1.5\; \mu\text{m}$
- "Bulk helium"
- Control electrodes
- Microfabrication
- Dave Rees, Hiroki Ikegami, Steve Lyon + many others

- Van der Waals helium films
- $d \sim 20 – 100\; \text{nm}$
- High electron density $n < 10^{15}\; \text{m}^{-2}$
- Roughness
- Structured corrugations
- Paul Leiderer, Hideki Yayama, Alexandr Smorodin
Rydberg states

Image charge $U_0(z) = \frac{-0.007e^2}{4\pi\varepsilon_0 z}$ for $z > 0$

Grimes: $U_0(z) = \frac{-0.007e^2}{4\pi\varepsilon_0 (z + b)}$ for $z > 0$

$= V_0$ for $z \leq 0$

Stark tuning $U(z) = U_0(z) + eE_z z$

$E_m = \frac{-R_e}{m^2}$

$f_{12} = 119.3$ GHz

($^4$He)

Experiment for $E_z = 0$:
125.9 GHz at 1.2 K

Transitions from $m = 1 \rightarrow M$

$M = 2$

$\alpha'$

$E_z$ (kV/m)
Resonant microwave absorption

Ground state to first excited Rydberg state on $^4\text{He}$ – linewidth $\gamma$

Stark shift

$E_z$ (kV/m)

$\gamma(T)$ (MHz)

$\gamma = AT + BN_{\text{gas}}$

Theory: Ando (1976)

Brown, Grimes, Zipfel, 1976
Ground state to first excited Rydberg state on $^4$He – linewidth $\gamma$

Stark shift

$\gamma(T)$ (MHz)

$\gamma = AT + BN_{\text{gas}}$

Ripplon Gas atom Scattering

Theory: Ando (1976)

Brown, Grimes, Zipfel, 1976


High Powers: "Non-linear Optics"
Electron heating - Power broadening - Absorption bleaching – Coulomb shift - Bistability (Denis Konstantinov)
The simplest conductor?

Non-degenerate conductor
AC measurements, phase shift $\varphi$

Non-degenerate conductor
AC measurements, phase shift $\varphi$

Sommer-Tanner
$\varphi \approx \rho_{xx}$

Highest mobility $\mu$

Conductivity $\sigma_0 = n e \mu = 1/\rho_0$

Corbino disk
$\varphi \approx 1/\sigma_{xx}$

Scattering from Thermal ripplons Gas-atoms

Scattering from Thermal ripplons Gas-atoms

Depends on $E_z$, $T$

Agreement with theory

The in-plane mobility is record high, $\mu \leq 2 \times 10^8$ cm$^2$/V·s


K. Shirahama et al., JLTP (1995)
Electron Liquid

The 2D electron system on bulk helium is *nondegenerate*.

\[ \varepsilon_F [n_s = 10^{12} \text{ m}^{-2}] \approx 3 \text{ mK} \]

Plasma parameter

\[ \Gamma = \frac{\text{Coulomb energy}}{\text{Kinetic energy}} = \frac{e^2 (\pi n_s)^{1/2}}{4\pi \varepsilon_0 kT} \]

Strong electron correlations

\[ T_m = 2.25 \times 10^{-7} n^{1/2} \text{ K} \]

Nondegenerate liquid

\[ \Gamma < \Gamma_m \approx 130 \]

Phase diagram

W.T. Sommer & D.G. Tanner, PRL 27, 1345 (1971)
The 2D electron system on bulk helium is *nondegenerate* $\varepsilon_F [n_s = 10^{12} \text{ m}^{-2}] \approx 3 \text{ mK}$

Plasma parameter

$$\Gamma = \frac{\text{Coulomb energy}}{\text{Kinetic energy}} = \frac{e^2 (\pi n_s)^{1/2}}{4\pi \varepsilon_0 kT}$$

**Wigner crystal**

$$\Gamma > \Gamma_m \approx 130$$

**Strong electron correlations**

$$T_m = 2.25 \times 10^{-7} n^{1/2} \text{ K}$$

**Nondegenerate liquid**

$$\Gamma < \Gamma_m \approx 130$$

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"A Repulsive Crystal"

C.C. Grimes & G. Adams, PRL 42, 795 (1979)

W.T. Sommer & D.G. Tanner, PRL 27, 1345 (1971)
Magnetoconductivity - Drude model

Drude model

\[
\frac{\sigma_0}{\sigma_{xx}} = 1 + \mu^2 B^2
\]

\[
\sigma_0 = ne\mu = \frac{ne^2\tau_0}{m}
\]

Einstein relation

\[
\sigma_{xx} = \frac{ne^2}{kT} \frac{L^2}{\tau_B} \equiv \frac{\sigma_0}{\mu^2 B^2} \frac{\tau_0}{\tau_B}
\]

\[
R_c = \frac{(2mkT)^{1/2}}{eB}
\]

\[
\frac{\tau_0}{\tau_B} = 1
\]

Classical cyclotron orbit

\[
\mu B = \omega_c \tau_0
\]

\[
\mu B = 1
\]

\[
\hbar \omega_c = kT
\]

\[
n = 0.64 \times 10^2 \text{ m}^{-2}
\]

Dykman et al. PRB 55, 16249 (1997); 55, 16272 (1997); 55, 16280 (1997)
Magneetoconductivity - Drude model

Drude model
\[ \frac{\sigma_0}{\sigma_{xx}} = 1 + \mu^2 B^2 \]
\[ \sigma_0 = n e \mu = \frac{n e^2 \tau_0}{m} \]

Einstein relation
Diffusion
\[ \sigma_{xx} = \frac{n e^2 L^2}{kT \tau_B} \approx \frac{\sigma_0}{\mu^2 B^2} \frac{\tau_0}{\tau_B} \]

Classical cyclotron orbit
\[ R_c = \frac{(2mkT)^{1/2}}{eB} \]
\[ \frac{\tau_0}{\tau_B} = 1 \]

Single particles
Collision broadened (SCBA):
Enhanced scattering?

\[ \mu B = \omega_c \tau_0 \]
\[ \omega_c = \frac{eB}{m} \]

Non-degenerate

BUT \[ \omega_c \tau_0 = \mu B \gg 1 \]

Landau levels
\[ E_N = (N + 0.5) \hbar \omega_c \]

\[ \frac{\tau_0}{\tau_B} \approx \frac{\hbar \omega_c}{\Delta} \]

\[ \frac{\tau_0}{\tau_B} \approx \frac{\hbar \omega_c}{\hbar / \tau_B} = (\omega_c \tau_0)^{1/2} = (\mu B)^{1/2} \]
Fluctuational Many-electron Field

Electrons in a potential well, plasma frequency
KE = kT

Energy fluctuation across a cyclotron orbit

\[ \Delta_c = eE_f R_c \]

Fluctuating electric field \( E_f \approx n^{3/4} T^{1/2} \)

Wigner solid \( F = 8.91 \)

Dykman and Khazan, JETP (1979) – prehistory?
Many-electron Magnetoconductivity

Landau levels

\[ \Delta_c = eE_c R_c > \hbar \omega_c \]

Many electron effects:
Electron liquid
Restores Drude model

\[ \frac{\tau_0}{\tau_B} = 1 \]

\[ \sigma_{xx}^{-1} \propto B^2 \]

\[ \sigma_{xx} \propto B^{3/2} \]

\[ \frac{\tau_0}{\tau_B} \propto (\mu B)^{1/2} \]

Single particles
(SCBA)

\[ \hbar \omega_c < kT \]

\[ \frac{\sigma_0}{\sigma_{xx}} \propto \frac{\mu^2 B^2}{\tau_0/\tau_B} \]

\[ \hbar \omega_c = kT \]

\[ n = 0.55 \times 10^{12} \text{ m}^{-2} \]
Many-electron Magnetoconductivity

Landau levels

\[ \Delta_c = eE_c R_c < \hbar \omega_c \]

\[ B > B_0 = A n^{3/8} T^{1/2} \approx 0.1 \text{ to } 1 \text{T} \]

Many electron effects:
Electron liquid

\[ \tau_0 \propto \frac{\hbar \omega_c}{eE_c R_c} \propto \frac{B^2}{B_0^2} \]

\[ \sigma_{xx}^{-1} \propto \mu B_0^2 \]

\[ \hbar \omega_c < kT \]

\[ \frac{\sigma_0}{\sigma_{xx}} \propto \frac{\mu^2 B^2}{\tau_0 / \tau_B} \]

Measure \( E_f \)

\[ n = 0.54, 0.85, 1.39, 1.88 \times 10^{12} \text{ m}^{-2} \]
Quantum Magnetoconductivity

Landau levels

\[ l_B = \left( \frac{\hbar}{eB} \right)^{1/2} \]

\[ \Delta_q = eE_t l_B < \hbar \omega_c \]

Many electron effects:
Electron liquid
Density dependent

\[ \frac{\tau_0}{\tau_B} \propto \frac{\hbar \omega_c}{eE_t l_B} \propto B^{3/2} \]

\[ \sigma_{xx}^{-1} \propto B^{-1/2} \]

\[ \sigma_0 \propto \frac{\mu^2 B^2}{\tau_0 \tau_B} \left( \frac{2 \hbar \omega_c}{kT} \right) \]

\[ \hbar \omega_c = kT \]

\[ \hbar \omega_c > kT \]

\[ \tau_0 \propto (\mu B)^{1/2} \]

\[ \sigma_{xx}^{-1} \propto B^{1/2} \]

Highest fields:
Single particles

\[ n = 0.55 \times 10^{12} \text{ m}^{-2} \]
Microwave-induced Magnetoresistance Oscillations

Denis Konstaninov and Kimitoshi Kono, PRL 103, 266808 (2009); PRL 105, 226801 (2010)

Novel Radiation-Induced Magnetoresistance Oscillations

Drude model
no oscillations

Many electron

Single particle

Low density
SCBA at high fields
Higher densities
Oscillations smeared
Wigner crystal


$\sigma_{xx}^{-1}$ depends on $V_0$

Jumps, hysteresis
Collective sliding from dimples

Non-linear transport:

a) Resonant Bragg-Cherenkov scattering

b) Decoupling of the Wigner solid from the helium surface


Non-linear magnetoconductivity

$n = 1.08 \times 10^{12} \, \text{m}^{-2}$
$T_m = 220 \, \text{mK}$
$B = 0.0289 \, \text{T}$
$T = 80 \, \text{mK}$

Ripplon phase velocity

$\alpha$ - surface tension
$\rho$ – helium density

Dynamical pinning for $\lambda = \alpha$

Bragg-Çerenkov scattering

$$
\omega_r^2 = \left( \frac{\alpha}{\rho} \right)^{1/2} q^3
$$

$$
\nu_r(\lambda) = \left( \frac{2\pi\alpha}{\lambda \rho} \right)^{1/2}
$$

Ripplon barrier

$$
\nu_r(a) = \left( \frac{2\pi\alpha}{a \rho} \right)^{1/2} \propto n^{1/4}
$$
Wigner crystal  Non-linear magnetoconductivity – Ripplon Barrier

A. Kristensen et al, PRL 77, 1350 (1996)

\[ \sigma(B) = 10^{-7} \Omega^{-1} \]

Corbino disk
Capacitively coupled current
Radial electric field
Aximuthal Hall Velocity
\[ v_H = E_x / B \]

"Simple-minded model"
No fitting parameters
Polycrystalline domains?

Restricted geometries Helium microchannels

Microchannels filled by capillary action can be used for experiments on small numbers of surface-state electrons:

Glasson et al, PRL 87, 176802 (2001)

[History: D. Marty (1986)]
A ‘Wigner wire’: across the channel

\[ v_r = 6.9 \, \text{m/s} \]

Quasi 1D: \( \Gamma_0 < 130 \)

Dislocations?

Ikegami et al, PRL 102, 046807 (2009); PR B 82, 201104 (2010)
Current work at RIKEN - split-gate device

Point-Contact Transport Properties of Strongly Correlated Electrons on Liquid Helium

David Rees et al, PRL 106, 026803 (2011)

A classical analogue to the quantum point contact . . .

Physics Focus: Electrons Take Turns Like Pedestrians
Kimitoshi Kono

Piacente & Peeters PRB 72, 205208 (2005)
Quantum Computing with Electrons Floating on Liquid Helium

P. M. Platzman and M. I. Dykman

A quasi-two-dimensional set of electrons ($1 < N < 10^9$) in vacuum, trapped in one-dimensional hydrogenic levels above a micrometer-thick film of liquid helium, is proposed as an easily manipulated strongly interacting set of quantum bits. Individual electrons are laterally confined by micrometer-sized metal pads below the helium. Information is stored in the lowest hydrogenic levels. With electric fields, at temperatures of $10^{-2}$ kelvin, changes in the wave function can be made in nanoseconds. Wave function coherence times are 0.1 millisecond. The wave function is read out with an inverted dc voltage, which releases excited electrons from the surface.

Requires:
- Quantum States
- Quantum Dots
- Electron Manipulation

(Lian-Fu Wei, Steve Lyon, David Schuster)
Quantum Computing with Electrons Floating on Liquid Helium

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G. Sabouret et al, APL, (2008)
Qubits are made by confining electrons with electrodes into quantum dots.

level spacing \( E_2 - E_1 \) is tuned by the electrode field via Stark effect

\[
\delta(E_2 - E_1) \sim 1 \text{ GHz for } \delta V_{el} \sim 0.3 \text{ mV}
\]

\[
h \sim 0.5 \mu\text{m}
\]

\[
\omega_{\parallel} = 20 \text{ GHz for } V_{el} \sim 10 \text{ meV}
\]
Experimental objective - scalable many-qubit system?

\[ h \approx 0.5 \, \mu m, \quad d \approx 1 \, \mu m \]

Goodkind & Pilla, Quant. Inf. Comp. (2001)

Speed of a single-qubit gate: \(~ 1 \, \text{GHz}\) for the microwave field \( E = 1 \, \text{V/cm}\)

Dipolar near-neighbors interaction : \( V_{ee} \)

Typical qubit parameters

\[ E_2 - E_1 \approx 1.6 \times 10^{11} \, \text{Hz}, \quad V_{ee} \geq 10^8 \, \text{Hz} \]

speed of two-qubit gates
Classical dots

Guard

Reservoir

Helium Pool
0.7 µm deep

Pool

Gate

SET

Injector

550 nm

40 nm
Single electronics on liquid helium

Coulomb Blockade Oscillations
Single Electron Transistor

SCPT – periodicity 2e

SET – periodicity 1e

Papageorgiou et al, APL, 86, 153106 (2005)
Experiment at Saclay
Excellent reproducibility ("My Precious")

Allows the investigation of the *classical* addition spectrum…

Single electronics

Electron dots

Yury Mukharsky, CEA, Saclay, France
E. Rousseau et al. PRB 79, 045406 (2009)

Viewpoint: Electron crystallites floating on superfluid helium
François Peeters
V. M. Bedanov and F. M. Peeters, PRB 49, 2667 (1994)
Clocking Electrons on an array of pixels

Princeton

Electron transfer (clocking) along a charge-coupled-array (CCD)-like array of gates.

Previously . . .

- 60 parallel channels
- CTE = 0.999 999 92
  G. Sabouret *et al.*, *APL* 92, 082104 (2008)

(Extremely) Efficient Clocked Electron Transfer on Superfluid Helium
(It works perfectly!) F.R. Bradbury *et al.* *PRL* 107, 266803 (2011)

- Clocking on a 2D array of pixels
- 120 channels
- Efficiency of 99.99999999%
- Down to one electron per pixel

Physics Viewpoint by Kimitoshi Kono:
Electrons take their places on a liquid helium grid
Proposal for Manipulating and Detecting Spin and Orbital States of Trapped Electrons on Helium Using Cavity Quantum Electrodymanics

D.I. Schuster,¹ A. Fragner,¹ M. I. Dykman,² S. A. Lyon,³ and R. J. Schoelkopf¹
PRL 105, 040503 (2010)

Single electron trap – quantised lateral motion
Resonate with microwave cavity ≈ 5 GHz
Magnetic field $B_x$ gives spin quantisation
Inhomogeneous $B_z$ gives spin-orbit coupling
Manipulation and readout via RF input
Electron-electron coupling via single cavity photons

Cavity to charge coupling 10 ns; spin coupling 1 μs

David Schuster,
Andreas Fragner,
Ge Yang

Progress: APS March Meeting 2012
**Conclusions**

- **Electrons on helium** - “ideal” 2D conductor  
  Correlated Coulomb liquid - Magnetoconductivity  
  Classical Wigner solid - Dynamic pinning

- **Quantum states**  
  Rydberg states - Landau levels - non-linear optics - zero conductance states

- **Microchannel devices** offer precise control over small dimensions  
  Used to study Novel mesoscopic phenomena

- **Wigner wires**  
  Non-linear transport  
  2D melting in confined geometries  
  Transport through a constriction

- **SET detectors** - ultra-sensitive charge detectors  
  Single electronics - ‘Classical dot’ addition spectra

- **Electron Transport and Manipulation** - clocking of electrons in arrays

- **Cavity Quantum Electrodynamics**

- **Qubits** may be possible…

*Thank you*
Electrons on Helium

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- Rydberg states
- Electron mobility
- Many electron effects
- Wigner crystal
- Restricted geometries
- Single electronics
- Electron manipulation
- Qubits?

Many research groups