

Magneto-Oscillations in intensely irradiated two-dimensional electron gas



Maxim Khodas

Collaboration:

M. Khodas, H.-S. Chiang, A.T. Hatke, M.A. Zudov M.G. Vavilov,
L.N. Pfeiffer, K.W. West,
Phys. Rev. Lett. 104, 206801 (2010).

I.A. Dmitriev, M. Khodas, A.D. Mirlin, D.G. Polyakov, M.G. Vavilov, Phys. Rev. B 80, 165327 (2009).

M. Khodas and M.G. Vavilov,
Phys. Rev. B 78, 245319 (2008)

M.G. Vavilov U of W, Madison
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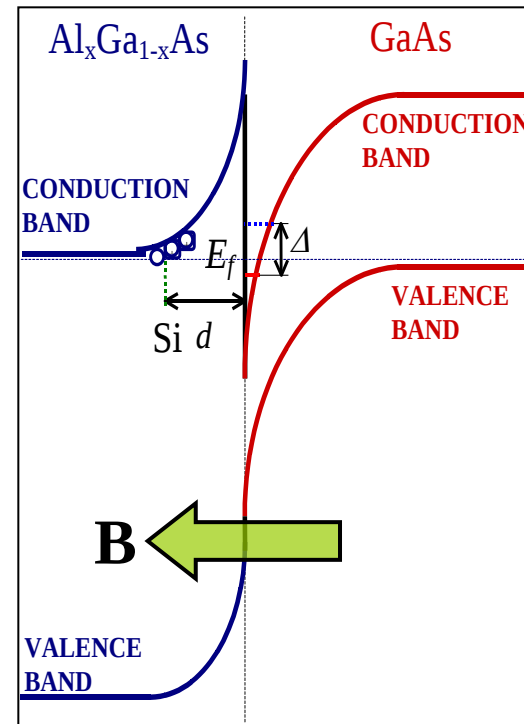
I.A. Dmitriev,
A.D. Mirlin, D.G. Polyakov
Karlsruhe, Germany

Outline

- 
1. Experimental motivation
 2. Hall induced resistance oscillation
 3. Combined action of AC and DC excitations
 4. MIRO at high power and at strong dc bias
 5. MIRO: Temperature dependence
 6. Conclusions

2D electron systems

- Two lattice-matched semiconductors:
 - GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$
- Realizations
 - 1). Heterostructure (HS)
 - single interface
 - 2). Quantum Well (QW)
 - double interface
 - highest mobility
- 2D electrons
 - $\text{Al}_x\text{Ga}_{1-x}\text{As}$: Si *d*-doping
 - $E_F \ll D$
 - At low T, only the lowest subband is occupied

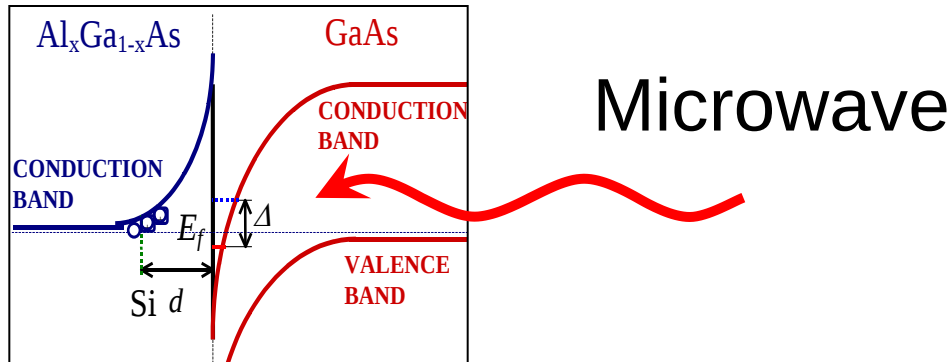


Heterostructure

$$1/\tau_{tr} \ll \omega_c \lesssim 1/\tau_q$$

Hall angle $\approx \pi/2$ levels unresolved

Microwave induced resistance oscillations (MIRO)



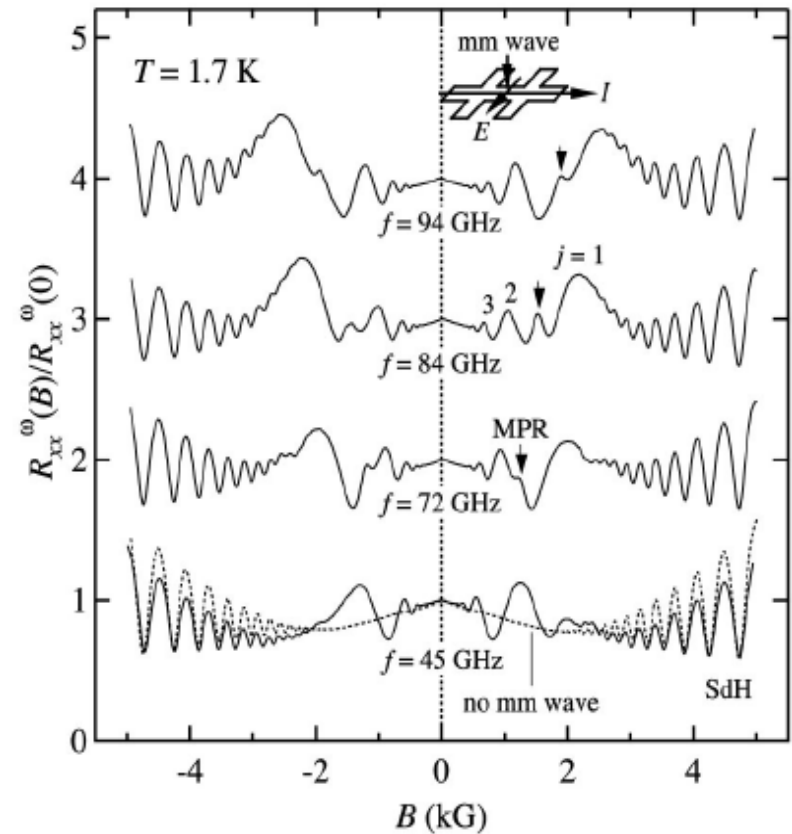
Novel Oscillations!

Parameter: $\epsilon_{ac} \equiv \frac{\omega}{\omega_C}$

(Not ϵ_F/ω_C)

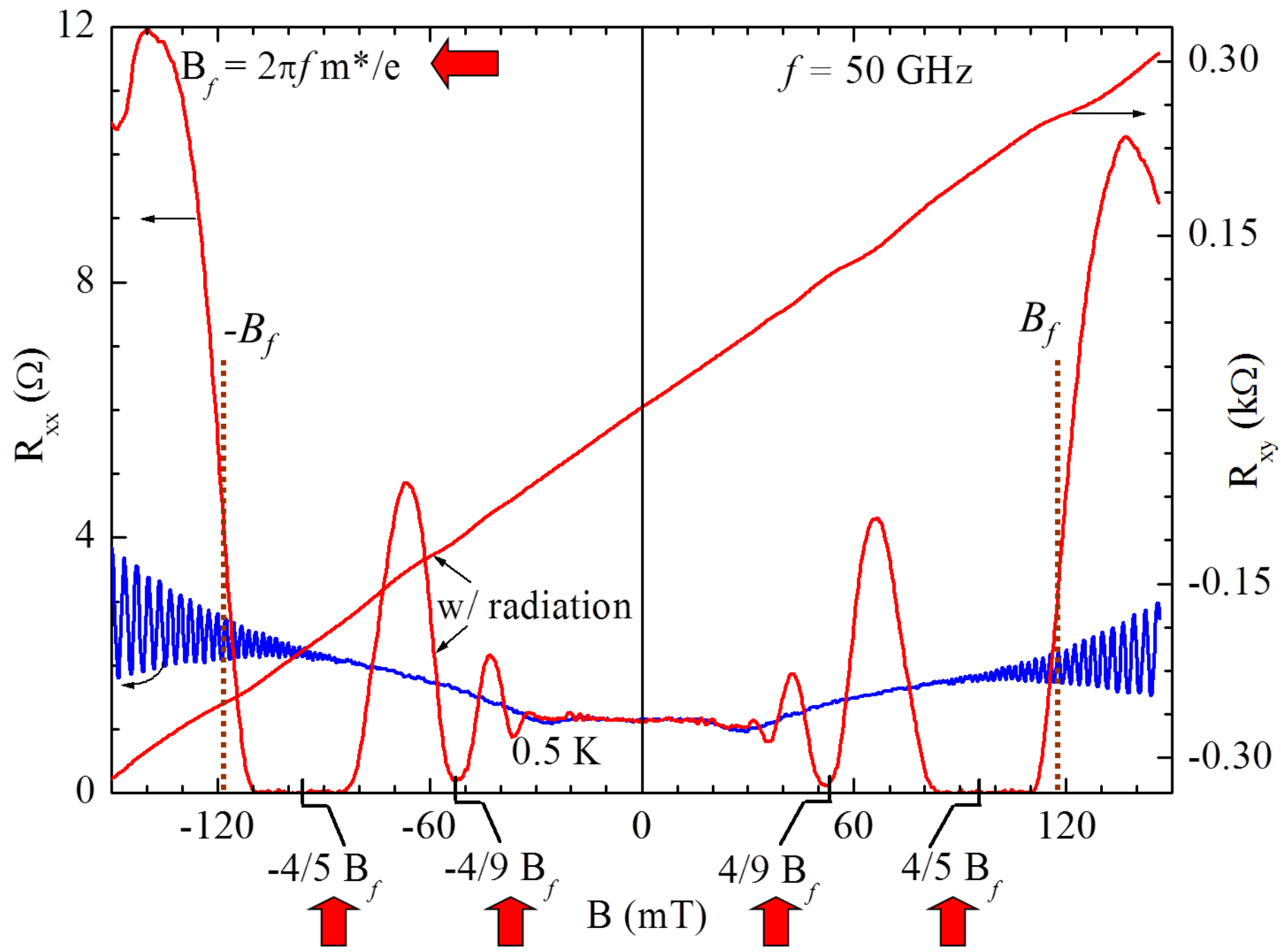
Maxima/Minima: $\epsilon_{ac}^{\pm} \simeq n \mp \frac{1}{4}$

Theory: $\Delta R \propto \epsilon_{ac} \sin(2\pi\epsilon_{ac}), 2\pi\epsilon_{ac} \gg 1$



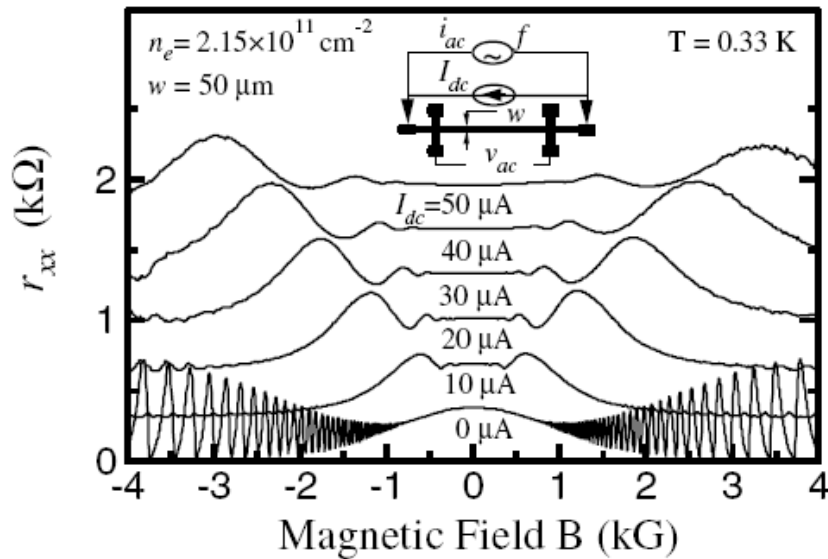
Zudov, Du, Simmons, Reno, Phys. Rev. B **64**, 201311(R) (2001)

Vavilov and Aleiner, Phys. Rev. B **69**, 035303 (2004)



Hall field induced resistance oscillations (HIRO)

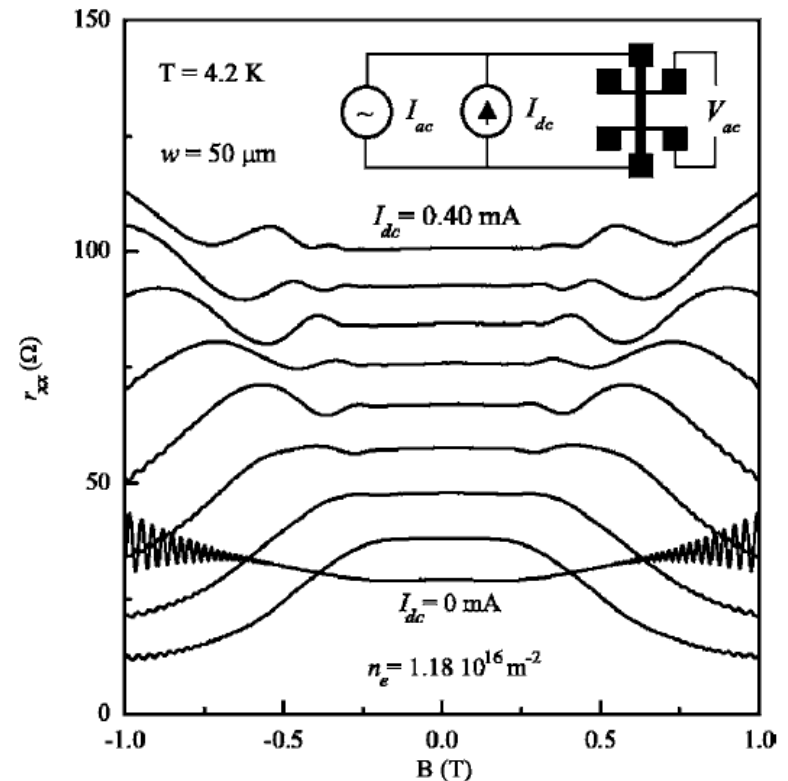
C.L. Yang, J. Zhang, R.R. Du,
J.A. Simmons, J.L. Reno;
PRL 89, 076801



As current increases:

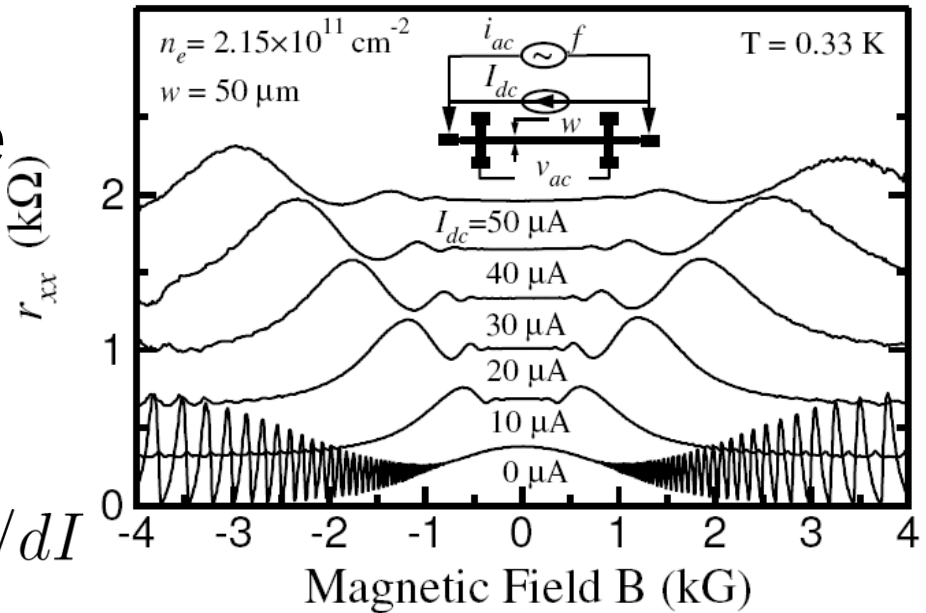
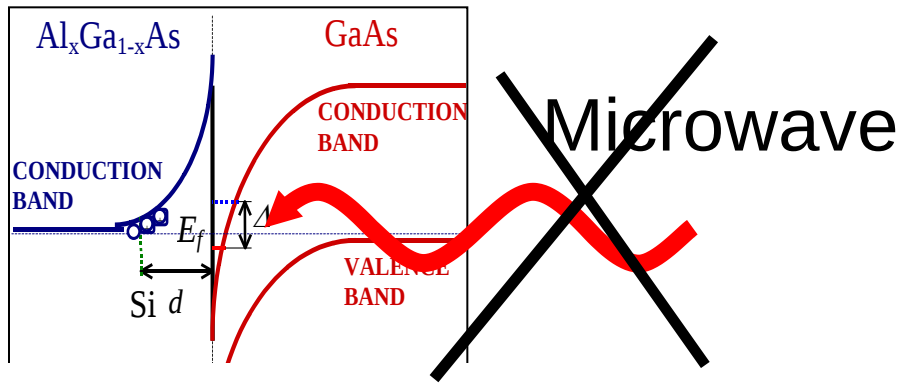
1. SdH oscillations are suppressed
2. New oscillations develop

A.A. Bykov, J-q Zhang, S. Vitkalov,
A.K. Kalagin, A.K. Bakarov
PRB 72, 245307

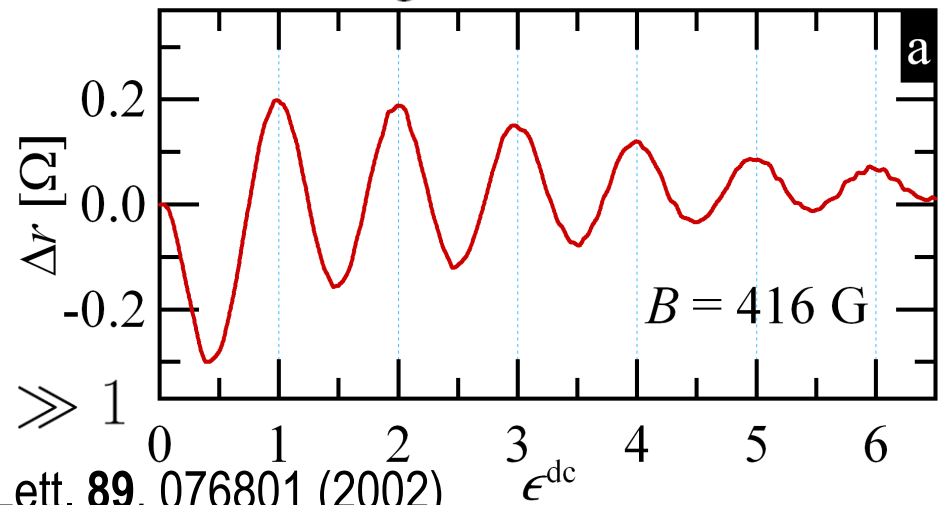


$$E \approx \rho_H j$$

Hall field induced resistance oscillations (HIRO)



- Differential resistance: $r = dV/dI$
- Parameter: $\epsilon^{dc} \equiv \frac{eE(2R_C)}{\hbar\omega_C}$
- Maxima: $\epsilon^{dc}_+ \simeq n$
- Theory: $\Delta r \propto \cos(2\pi\epsilon^{dc})$, $2\pi\epsilon^{dc} \gg 1$



Yang, Zhang, Du, Simmons, Reno, Phys. Rev. Lett. **89**, 076801 (2002)
 Zhang, Chiang, Zudov, Pfeiffer, West, Phys. Rev. B **75**, 041304(R) (2007)
 Vavilov, Aleiner, Glazman, Phys. Rev. B **76**, 115331 (2007)

Outline

1. Experimental motivation

→ 2. Hall induced resistance oscillations (HIRO)

3. AC and DC excitation: low power

4. AC and DC excitation: high power

5. MIRO: Power dependence

6. Conclusions

Classical picture

$\mathbf{B}^\odot$

$$\omega_c \gg 1/\tau_{tr}$$

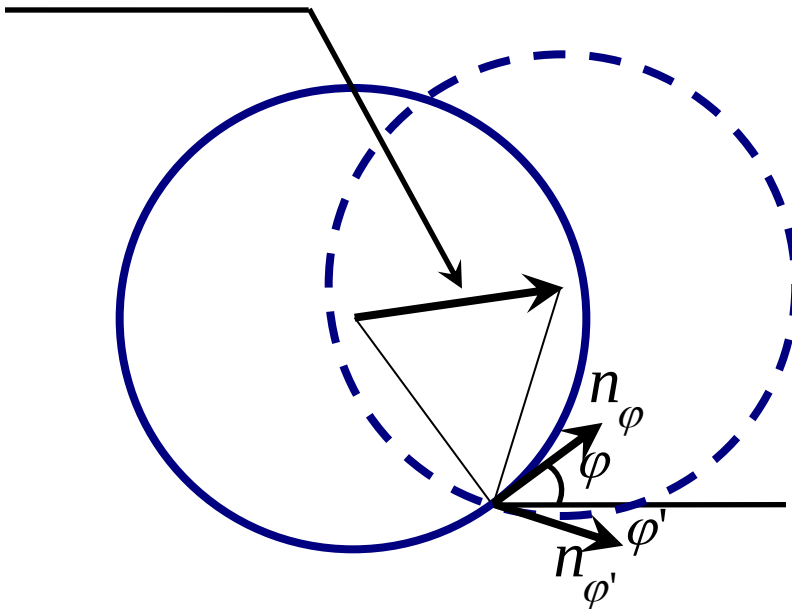
Diffusion of guiding centers

$$\sigma_{xx} = e^2 2\nu_0 \mathbf{D}$$

$$\mathbf{D} \approx \frac{(\sum_i \Delta R_i)^2}{2T}$$

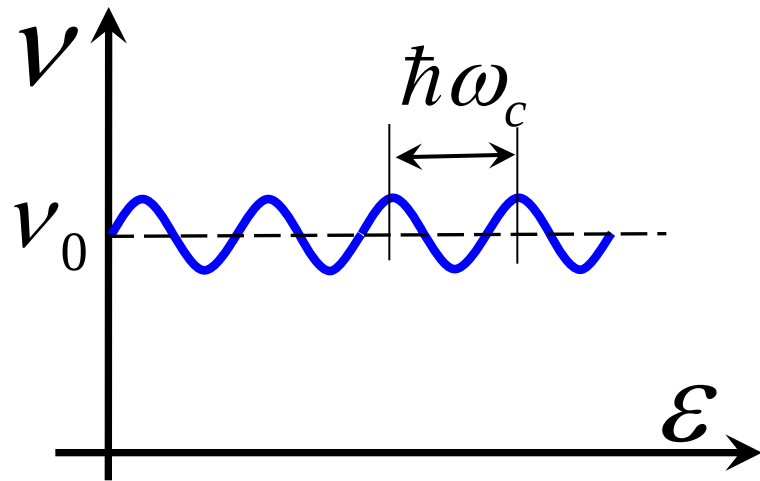
$$\mathbf{D} = \left\langle \frac{\Delta \mathbf{R}_{\varphi \rightarrow \varphi'}^2}{\tau_{\varphi \rightarrow \varphi'}} \right\rangle_{angles}$$

$$\Delta \mathbf{R}_{\varphi \rightarrow \varphi'} = R_c \hat{\mathbf{B}} \times (\mathbf{n}_\varphi - \mathbf{n}_{\varphi'})$$



$$\sigma_D = e^2 \nu_0 \frac{R_c^2}{\tau_{tr}}$$

Quasi-classical picture



$$\nu(\varepsilon) = \nu_0 \left(1 - 2\lambda \cos \frac{2\pi\varepsilon}{\omega_c} \right)$$

Add constant **electric** field

$$E_F / \omega_c \sim 100$$

$$1/\tau_{tr} \ll \omega_c \lesssim 1/\tau_q$$

Probability to make a circle

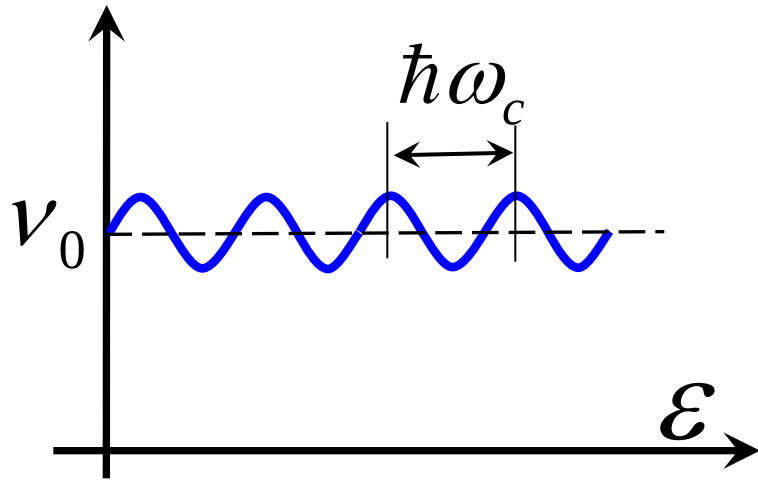
$$\lambda = \exp(-\pi / \omega_c \tau_q) \lesssim 1$$

$$\varepsilon \rightarrow \varepsilon + eEx$$

Energy and **space** dependent DOS $\nu(\varepsilon) \rightarrow \nu(\varepsilon + eEx)$

Dissipative current **?**

Contact with electrons on He (M.J. Lea)

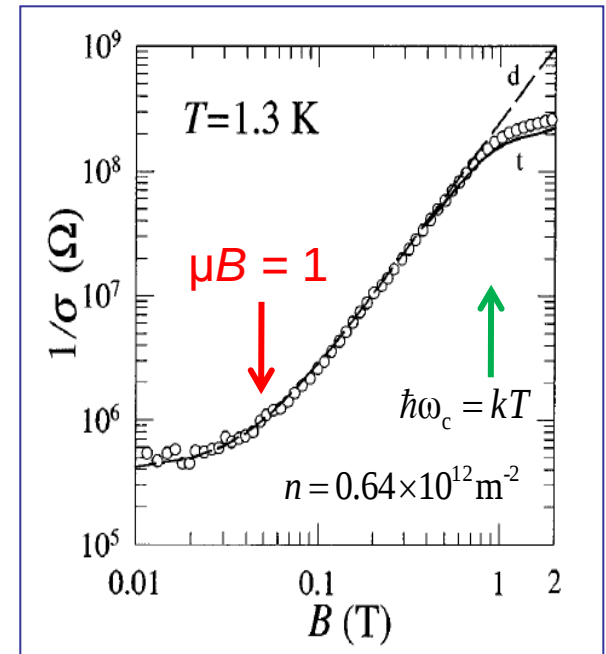
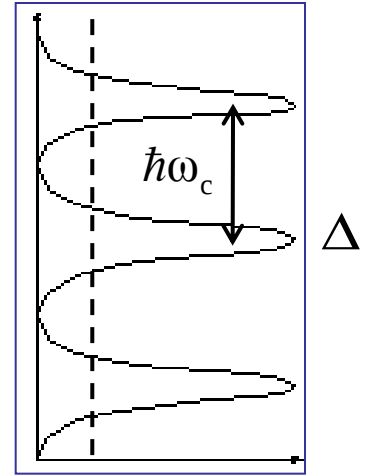


$$\nu(\varepsilon) = \nu_0 \left(1 - 2\lambda \cos \frac{2\pi\varepsilon}{\omega_c} \right)$$

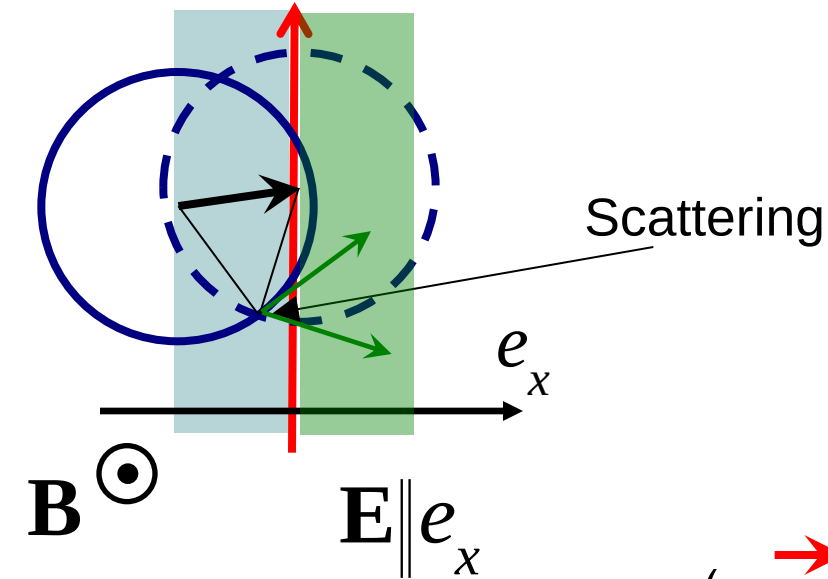
$$\lambda = \exp \left(-\frac{\pi}{\omega_c \tau_0} \right) \rightarrow \frac{\hbar\omega_c}{\Delta}$$

$$\frac{\tau_0}{\tau_B} \approx \frac{\hbar\omega_c}{\Delta}$$

$$\frac{\tau_0}{\tau_B} \approx \frac{\hbar\omega_c}{\hbar/\tau_B} = (\omega_c \tau_0)^{1/2} = (\mu B)^{1/2}$$



HIRO: Current



Red line crossings rate

$$j = j^+ - j^-$$

$$j = \left(\int d\phi d\phi' \int_{\text{light blue}} dx - \int d\phi d\phi' \int_{\text{green}} dx \right) \int d\varepsilon$$

$$\times v(\varepsilon, x) f(\varepsilon, x) (1 - f(\varepsilon, x + \Delta X)) \Gamma_{\phi \rightarrow \phi'}$$

Golden
Rule:

$$\text{Scattering rate: } \Gamma_{\phi \rightarrow \phi'} = \frac{1}{\tau_{\phi \rightarrow \phi'}} \frac{v(\varepsilon, x + \Delta X)}{v_0}$$

HIRO: Current (cont)

$$j \propto \int dx \int d\epsilon [f(\epsilon, x) - f(\epsilon, x + \Delta X_{\varphi \rightarrow \varphi'})] \frac{1}{\tau_{\varphi \rightarrow \varphi'}} \times \nu(\epsilon, x) \nu(\epsilon, x + \Delta X_{\varphi \rightarrow \varphi'})$$

$$\left(1 - 2\lambda \cos \frac{2\pi\epsilon}{\omega_c}\right) \times \left(1 - 2\lambda \cos \frac{2\pi(\epsilon + eE\Delta X_{\varphi \rightarrow \varphi'})}{\omega_c}\right)$$

Classics

SdHO
small: $T \gg \omega_c$

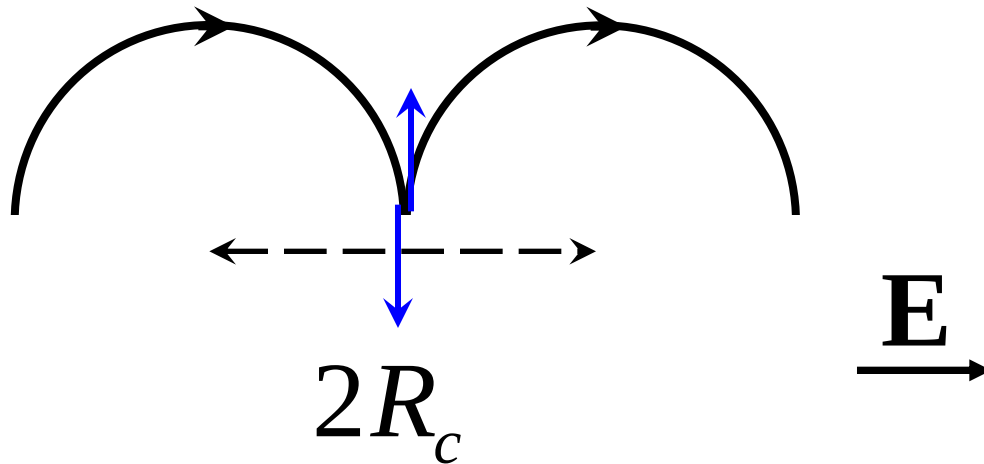
?!

HIRO: Current (cont)

Scale separation

$$T \gg \omega_c$$

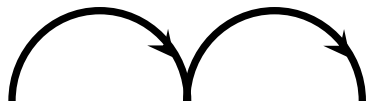
$$j \propto \lambda^2 E \left\langle \frac{\Delta X_{\varphi \rightarrow \varphi'}^2}{\tau_{\varphi \rightarrow \varphi'}} \cos [2\pi \epsilon_{dc} (\sin \varphi - \sin \varphi')] \right\rangle_{\text{angles}}$$



$$\epsilon_{dc} = \frac{eE(2R_c)}{\hbar\omega_c} \geq 1$$

HIRO: Current (cont)

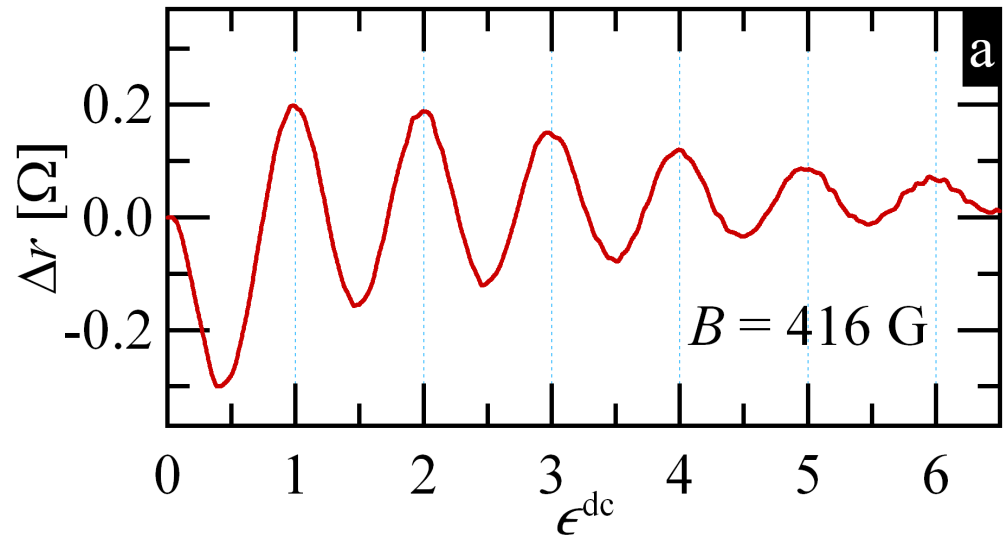
Backscattering



$$\epsilon_{dc} = \frac{eE(2R_c)}{\hbar\omega_c} \geq 1$$

$$\Delta r \propto \lambda^2 \frac{\cos 2\pi\epsilon_{dc}}{\tau_{\pi}}$$

Parameter: ϵ_{dc}

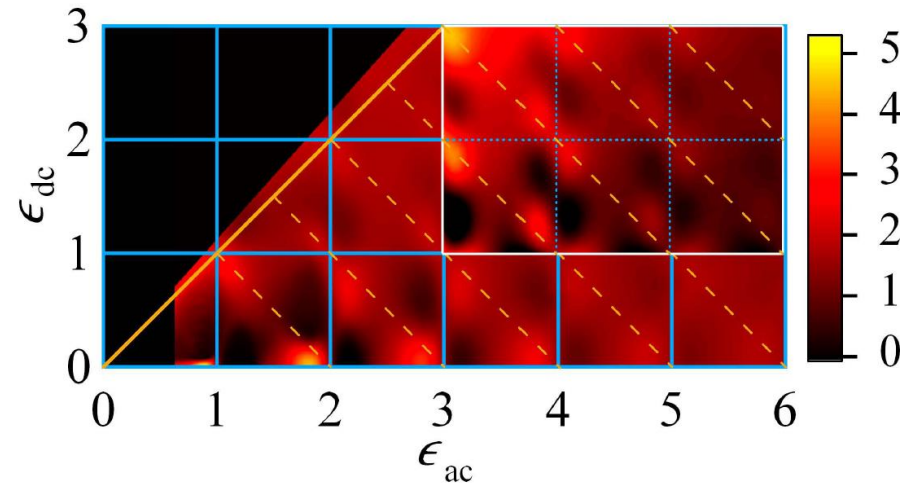
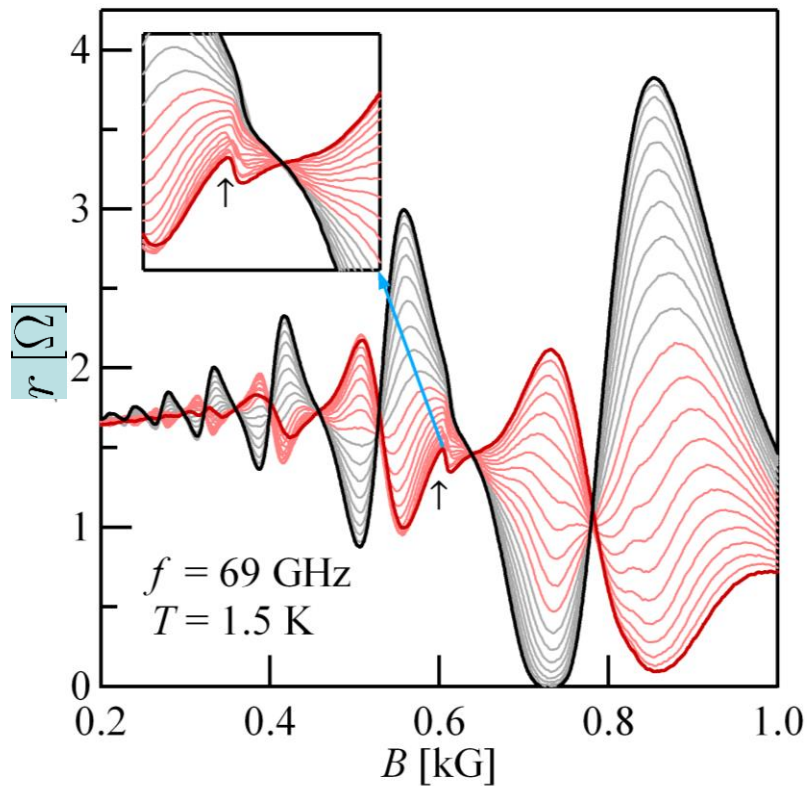


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1. Experimental motivation
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- ➔ 3. AC and DC excitation: low power
4. AC and DC excitation: high power
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AC+DC

Combined (dc) current and (ac) excitation



Strong effect of current excitation

Zhang, Zudov, Pfeiffer, West, Phys. Rev. Lett. **98**, 106804 (2007)

Zhang, Zudov, Pfeiffer, West, Physica E **40**, 982 (2008)

AC+DC

Hall Field **DC** parameter

Microwave **AC** parameter

$$\epsilon_{dc} = \frac{eE(2R_c)}{\hbar\omega_c} \geq 1$$

$$\epsilon_{ac} = \frac{\omega}{\omega_c}$$

$$\Delta r_{xx} \propto (1 - 2\mathcal{P}_\omega) \cos \epsilon_{dc}$$

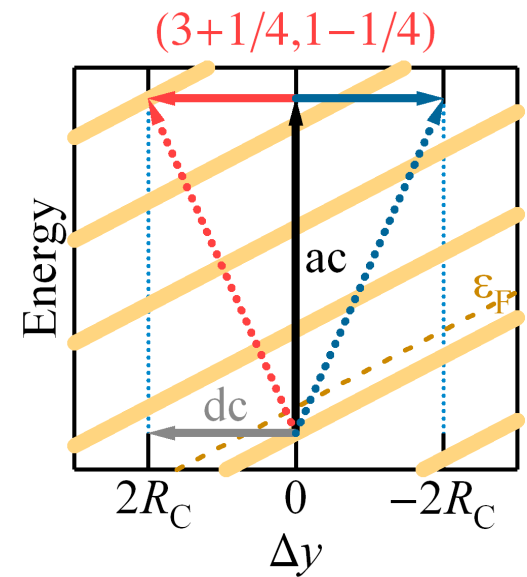
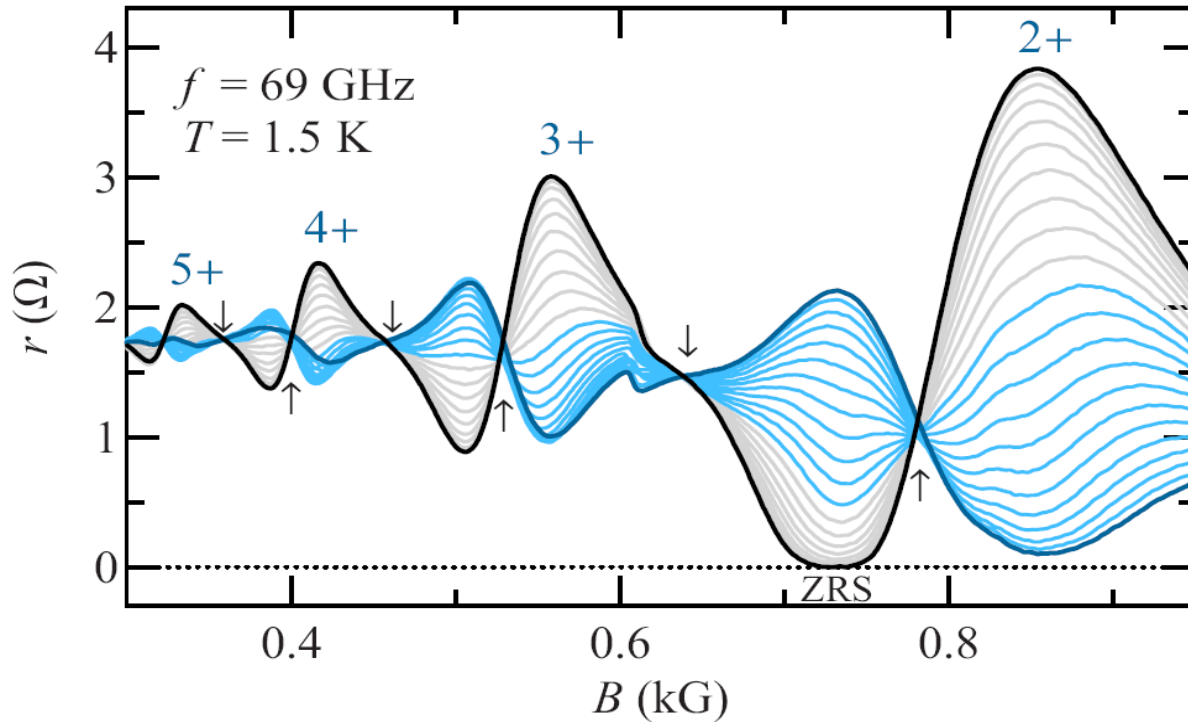
$$+ \mathcal{P}_\omega \frac{\epsilon_{dc} + \epsilon_{ac}}{\epsilon_{dc}} \cos(\epsilon_{dc} + \epsilon_{ac}) + \mathcal{P}_\omega \frac{\epsilon_{dc} - \epsilon_{ac}}{\epsilon_{dc}} \cos(\epsilon_{dc} - \epsilon_{ac})$$

Absorption

Emission

$$\mathcal{P}_\omega = \frac{v_F^2 e^2 E_\omega^2}{\omega^2 (\omega \mp \omega_c)^2}$$

ac/dc-induced oscillations in differential resistivity



- Explained by a **single-photon** “displacement” mechanism:

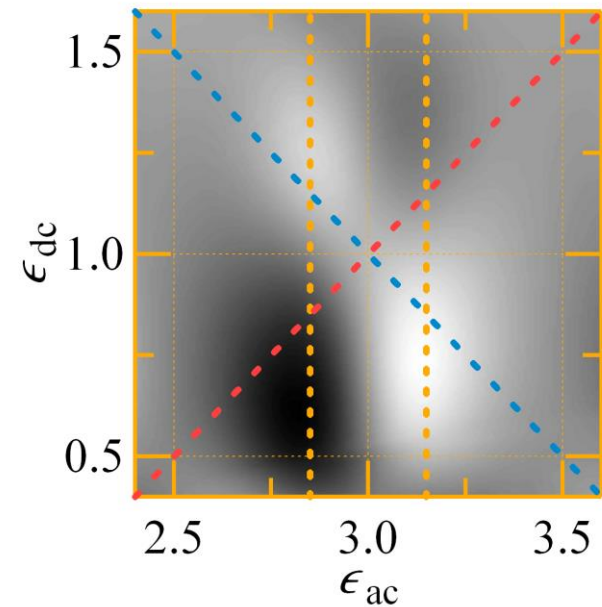
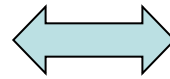
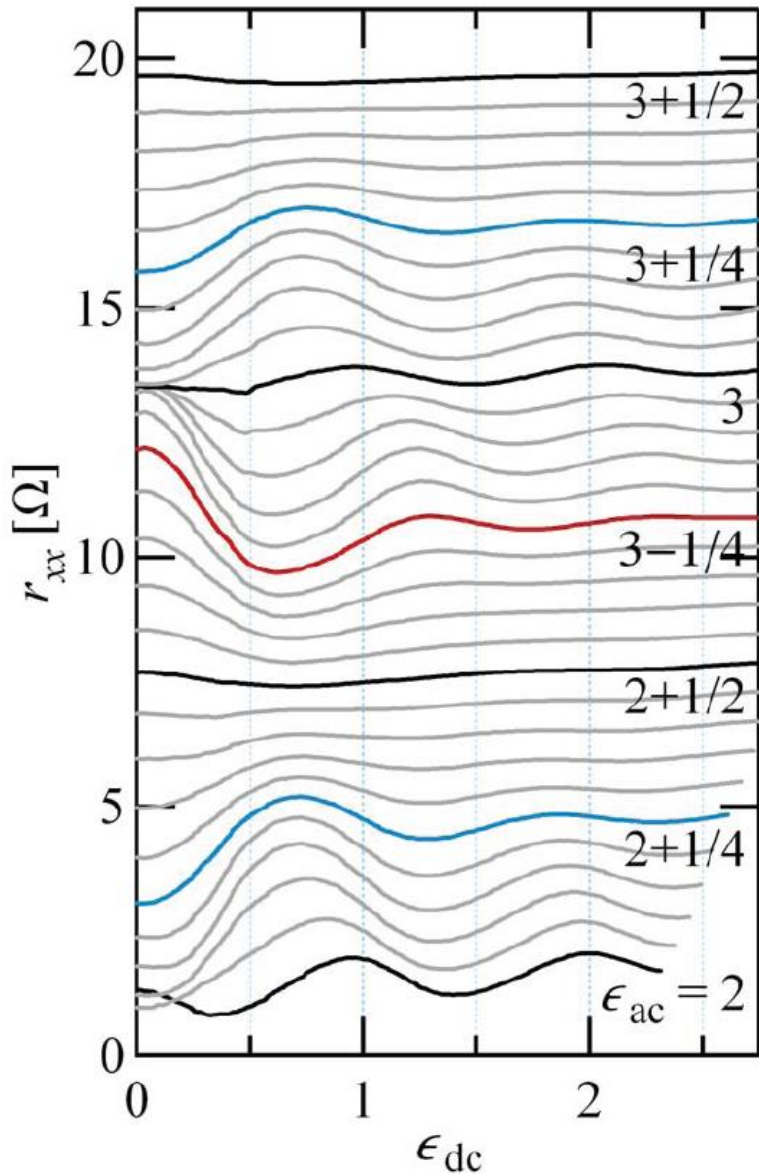
$$\Delta r \propto \epsilon_{\text{dc}}^{-1} [(\epsilon_{\text{ac}} + \epsilon_{\text{dc}}) \cos(\epsilon_{\text{ac}} + \epsilon_{\text{dc}}) - (\epsilon_{\text{ac}} - \epsilon_{\text{dc}}) \cos(\epsilon_{\text{ac}} - \epsilon_{\text{dc}})]$$

Zhang, Zudov, Pfeiffer, West, Phys. Rev. Lett. **98**, 106804 (2007)

Hatke, Chiang, Zudov, Pfeiffer, West, Phys. Rev. B **77**, 201304(R) (2008)

Khodas and Vavilov, PRB (2008)

Experiment (closer look)



Theoretical prediction

$$(\epsilon_{ac}, \epsilon_{dc})^+ \simeq (n \pm 1/4, m \mp 1/4)$$

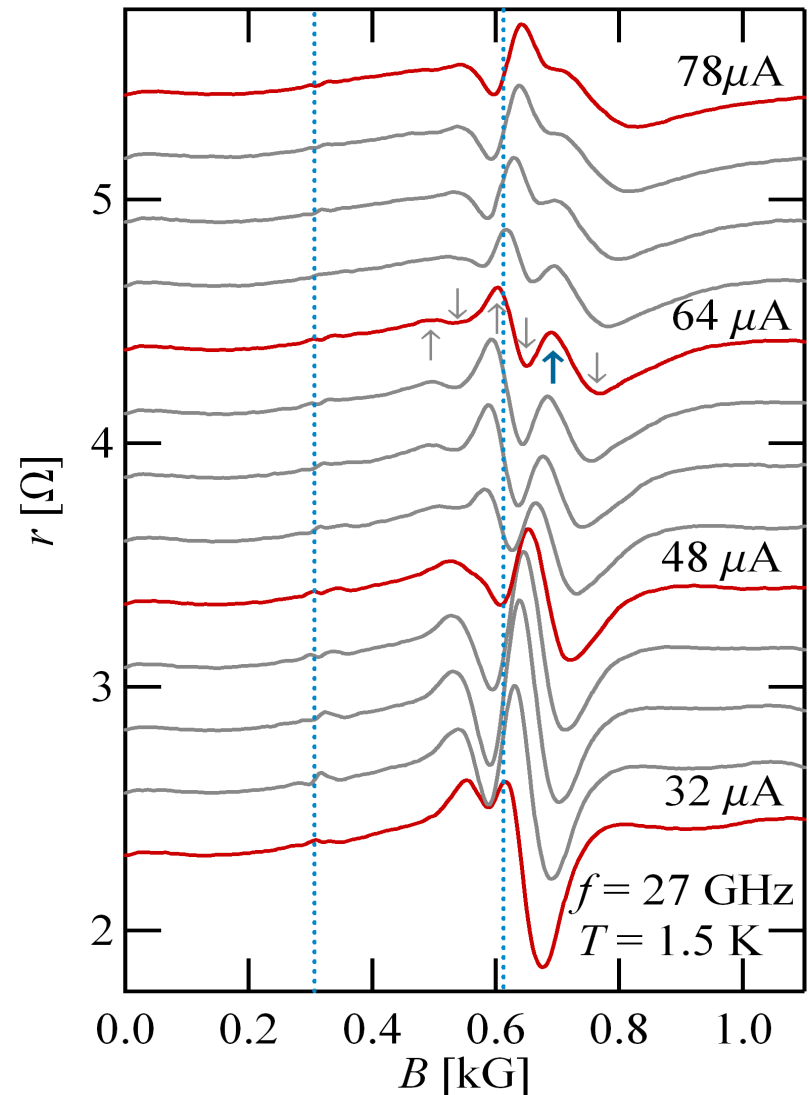
$$(\epsilon_{ac}, \epsilon_{dc})^- \simeq (n \pm 1/4, m \pm 1/4)$$

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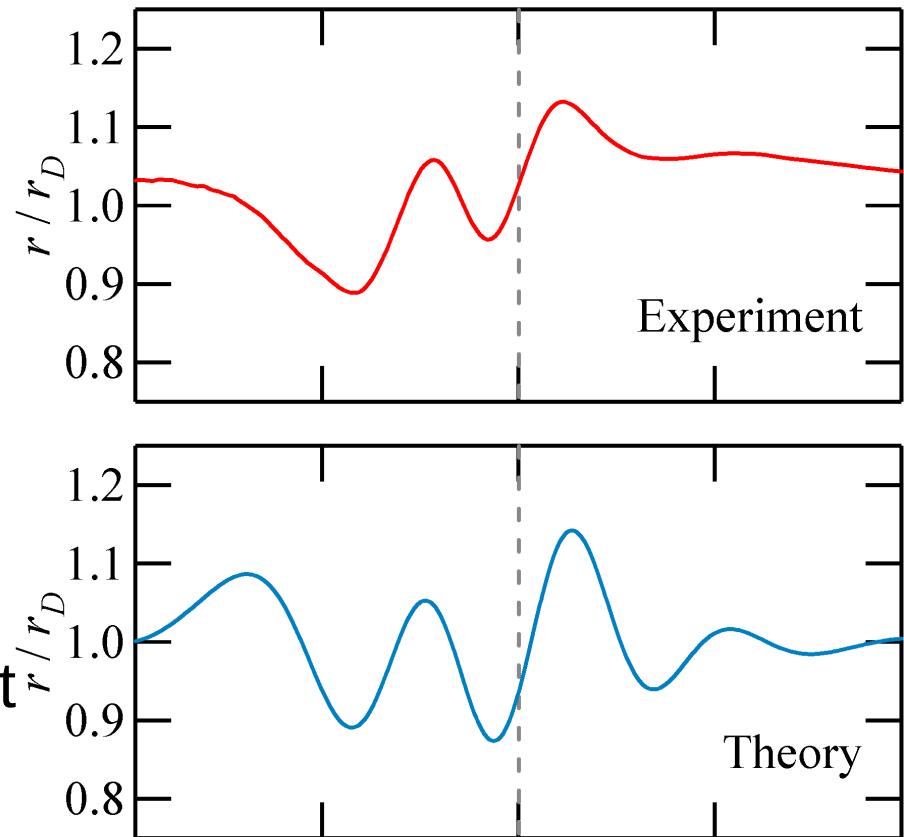
High power: new oscillations

- Non-zero current:
new oscillations!
 - **Multiple** maxima and minima in the narrow range around cyclotron resonance which move to higher B with increasing I
- **Qualitatively different** from all magneto-resistance oscillations reported earlier!



Experiment vs. Theory

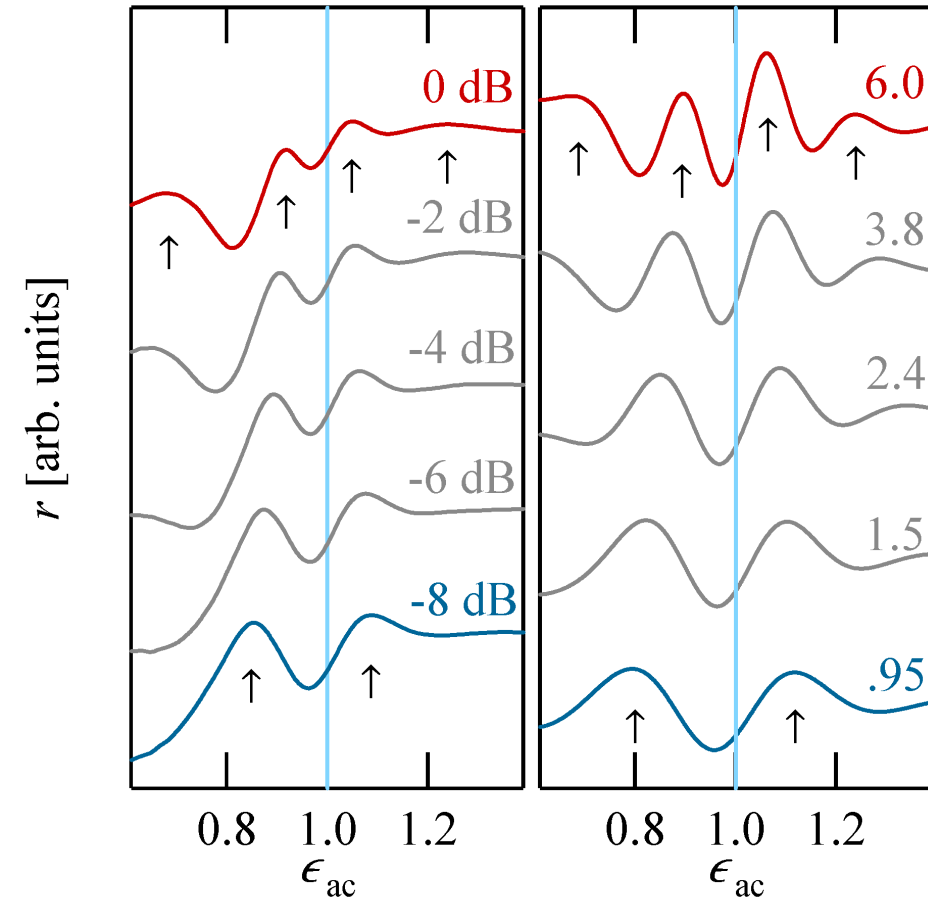
- Qualitative agreement
 - Number of oscillations and their positions
- Quantitative agreement
 - Amplitude of oscillations
 - We use $\tau_{\text{tr}}/\tau_{\pi} \approx 0.18$ obtained from Dingle plot analysis of HIRO



$$\frac{\delta r}{r_D} = \frac{(4\delta)^2 \tau_{\text{tr}}}{\pi \tau_{\pi}} \left[\cos(2\pi \epsilon_{\text{dc}}) J_0 \left(4\sqrt{\mathcal{P}_{\omega}} \sin(\pi \epsilon_{\text{ac}}) \right) - \frac{2\epsilon_{\text{ac}}}{\epsilon_{\text{dc}}} \sqrt{\mathcal{P}_{\omega}} \sin(2\pi \epsilon_{\text{dc}}) \cos(\pi \epsilon_{\text{ac}}) J_1 \left(4\sqrt{\mathcal{P}_{\omega}} \sin(\pi \epsilon_{\text{ac}}) \right) \right]$$

M. K., H.-S. Chiang, A.T. Hatke, M.A. Zudov M.G. Vavilov, L.N. Pfeiffer, K.W. West, Phys. Rev. Lett. 104, 206801 (2010).

Dependence on Microwave Intensity



Microwave power affects:

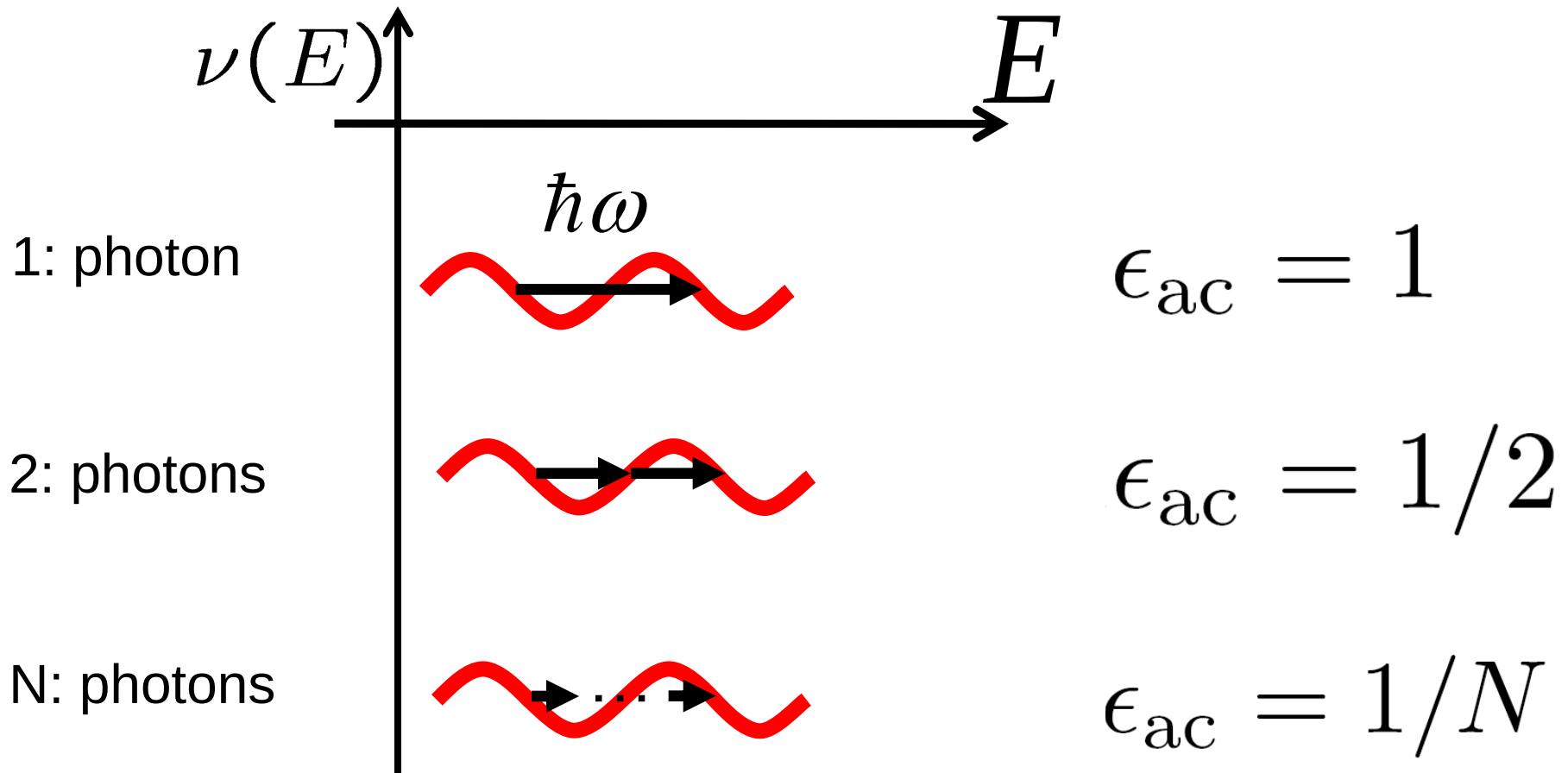
- Number of oscillations
- Period of oscillations
- Position of oscillations

Theory captures all three reasonably well!

M.K., H.-S. Chiang, A.T. Hatke, M.A. Zudov M.G. Vavilov,
L.N. Pfeiffer, K.W. West,
Phys. Rev. Lett. 104, 206801 (2010).

Multi-photon processes

Why multi-photon ?



Experimental motivation

- High power, near CR: $\epsilon_{ac} = 1 + \delta$, $\delta \ll 1 \Rightarrow \sin(\pi\epsilon_{ac}) \simeq -\pi\delta$

$$\frac{\delta r}{r_D} \propto \cos \left(4\sqrt{\mathcal{P}_\omega} \sin(\pi\epsilon_{ac}) - \frac{1}{4} - 2\pi\epsilon_{dc} \right) \simeq \cos \left[2\pi \left(2\sqrt{\mathcal{P}_\omega}\delta + \frac{1}{8} + \epsilon_{dc} \right) \right]$$

- Maxima positions:

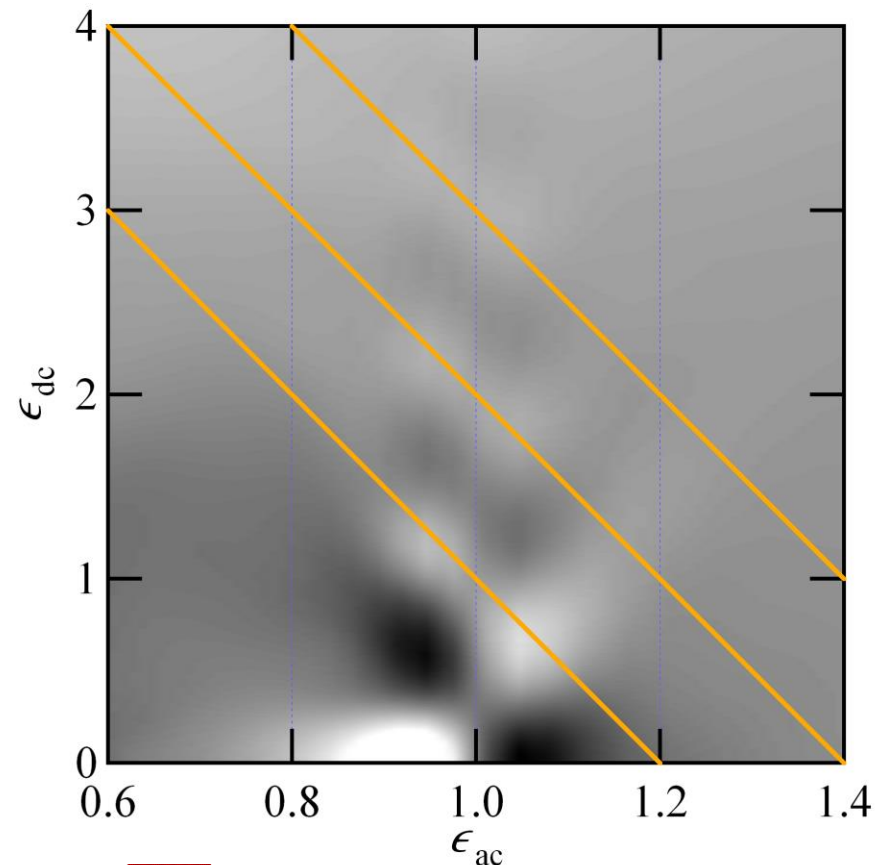
$$2\sqrt{\mathcal{P}_\omega} \cdot \epsilon_{ac} + \epsilon_{dc} \simeq n + \text{const}$$

- Effective number of participating photons:

$$N^* \simeq 2\sqrt{\mathcal{P}_\omega}$$

- Low power (single photon):

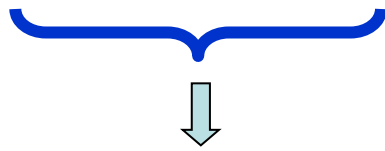
$$\epsilon_{ac} + \epsilon_{dc} \simeq n$$



$$2\sqrt{\mathcal{P}_\omega} \simeq 5 \Rightarrow \mathcal{P}_\omega \simeq 6.3$$

Theory: approach

$$\hat{H} = \frac{1}{2m} [-i\nabla - e\mathbf{A} - e\mathbf{A}_{el}(t)]^2$$



$$\hat{U}_t = \exp(-i\hat{\mathbf{p}}\zeta_t) \rightarrow \text{Guiding center trajectory}$$

A blue arrow points downwards from the text "Guiding center trajectory" to the equation below.

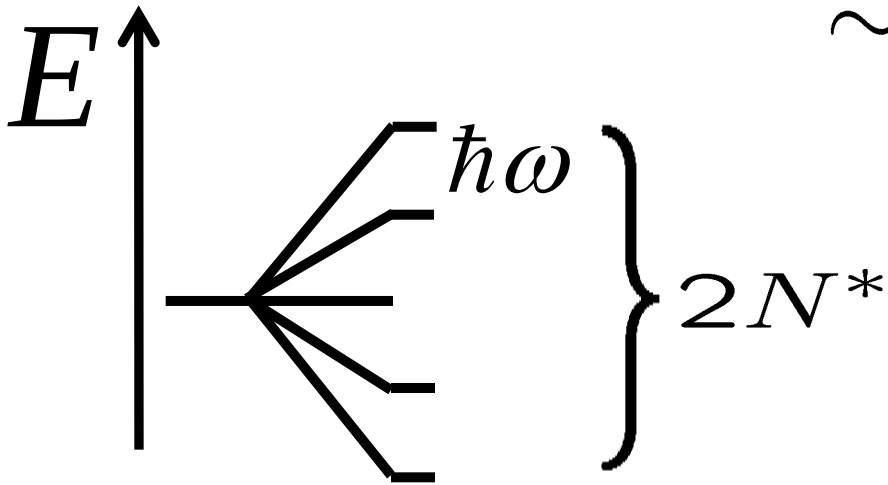
$$\hat{H}' = \frac{\mathbf{p}^2}{2m}$$

Floquet subbands

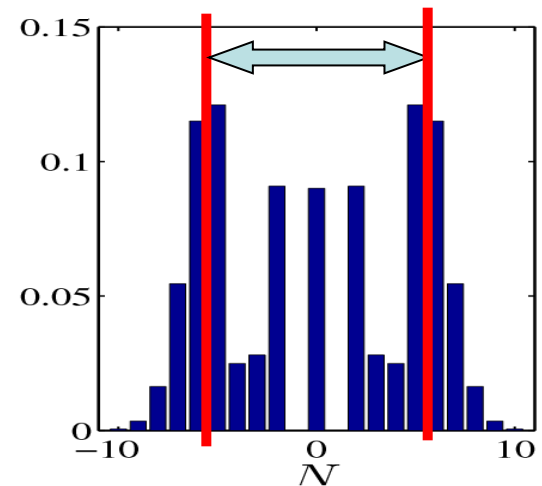
$$\omega_c \ll E_F \longrightarrow \text{Quasi-classics}$$

$$\psi_F = \psi_{\mathbf{p}} e^{-i p \mathbf{n}_{\varphi} \zeta_t - i E t}$$

$$\sim \sum_N J_N(\sqrt{\mathcal{P}_{\omega}}) e^{-i N \omega t}$$



$$N^* \sim \sqrt{\mathcal{P}_{\omega}} \propto \frac{e \mathcal{E}_{acl}}{\hbar \omega}$$



N^*

Impurity potential as a perturbation

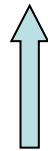
1) Add impurity potential

2) Floquet basis

$$\psi = \sum_{\mathbf{k}} c_{\mathbf{k}}(t) \psi_{\mathbf{k},\varphi} e^{-iEt - i\mathbf{k}\mathbf{n}_{\varphi}\zeta t}$$

3) Transition amplitudes

$$i\hbar\dot{c}_{\mathbf{k}} = \sum c_{\mathbf{k}'} \hat{V}_{\mathbf{k}\mathbf{k}'} e^{i(E-E')t + i\mathbf{k}(\mathbf{n}'_{\varphi} - \mathbf{n}_{\varphi})\zeta t}$$



Impurity potential = perturbation

Impurity potential as a perturbation (cont)

1) Transition *probability*

$$\overline{|c_{\mathbf{k}}(t_f)|^2} = n \sum_{\mathbf{k}} |V_{\mathbf{k}'\mathbf{k}}|^2 \int_{t_i}^{t_f} dt' \int_{t_i}^{t_f} dt e^{i(E-E')(t'-t)} \\ \times \overline{e^{ip(\mathbf{n}'_{\varphi} - \mathbf{n}_{\varphi})\zeta_{t',t}}} \Rightarrow \sum_{\mathbf{N}} a_{\varphi\varphi'}^{(\mathbf{N})} e^{-i\mathbf{N}\omega(t-t')}$$

2) Transition *rate*

$$\frac{d}{dt} \overline{|c_{\mathbf{k}}|^2} = \sum_{\mathbf{N}} \int \frac{d\varphi'}{2\pi} a_{\varphi\varphi'}^{(\mathbf{N})} \frac{\nu(\epsilon + \mathbf{N}\omega)}{\tau_{\varphi\varphi'}}$$

Golden rule
(in disguise)

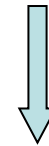
Kinetic equation

$$\omega_c \ll E_F \longrightarrow \text{Quasi-classics}$$

$$\omega_c \partial_\varphi f_{\varepsilon;\varphi} = \text{St}\{f\}_{\varepsilon;\varphi}$$

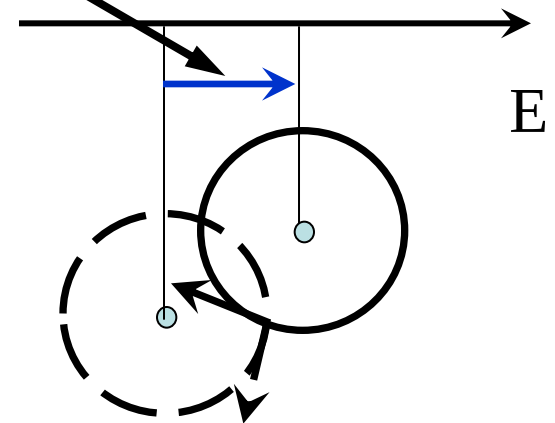
$$\text{St}\{f\} = \sum_N \int \frac{d\varphi'}{2\pi} \Gamma_{\varphi\varphi'}^{(N)} [f_{\varphi'}(\varepsilon + \boxed{W_{\varphi\varphi'}} + \boxed{N\hbar\omega}) - f_\varphi(\varepsilon)]$$

N -photon processes

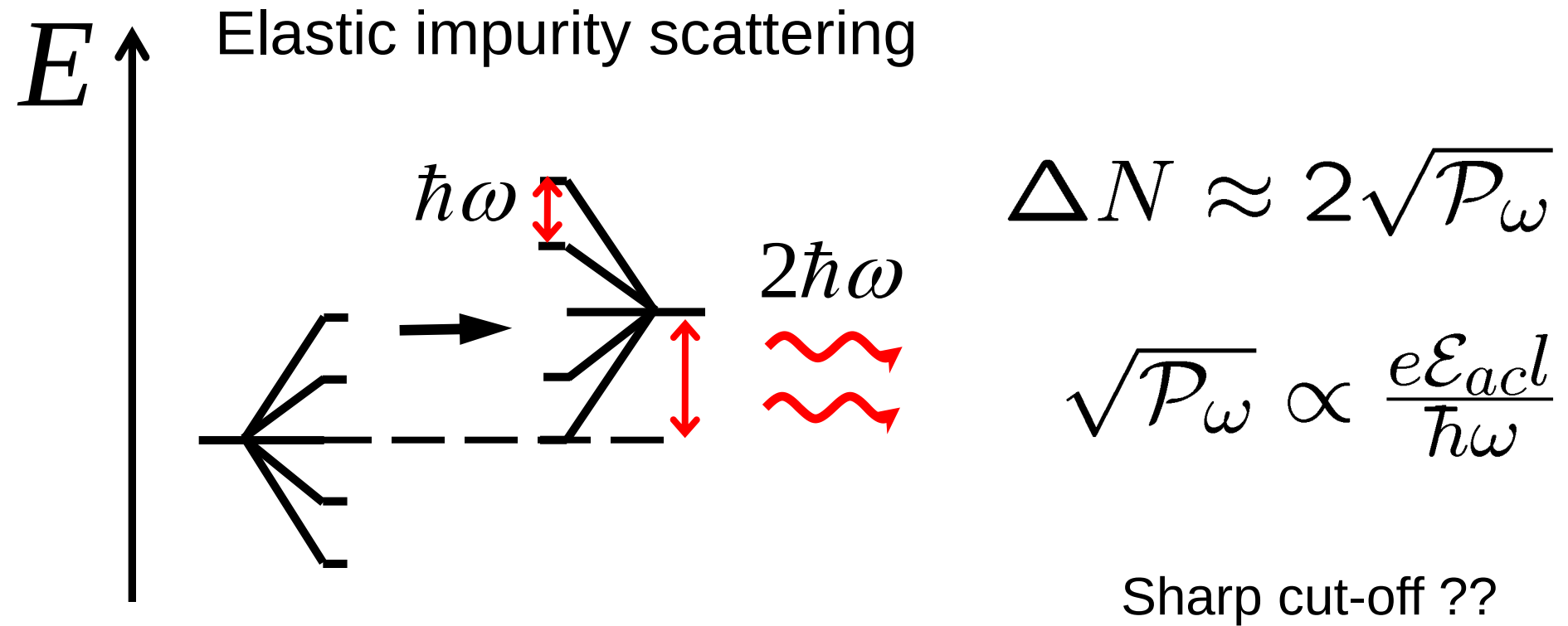


$$\Gamma_{\varphi\varphi'}^{(N)} = a_{\varphi\varphi'}^{(N)} \frac{\nu(\varepsilon + \textcolor{blue}{W}_{\varphi\varphi'} + \textcolor{red}{N}\hbar\omega)}{\tau_{\varphi\varphi'}}$$

of photons ??



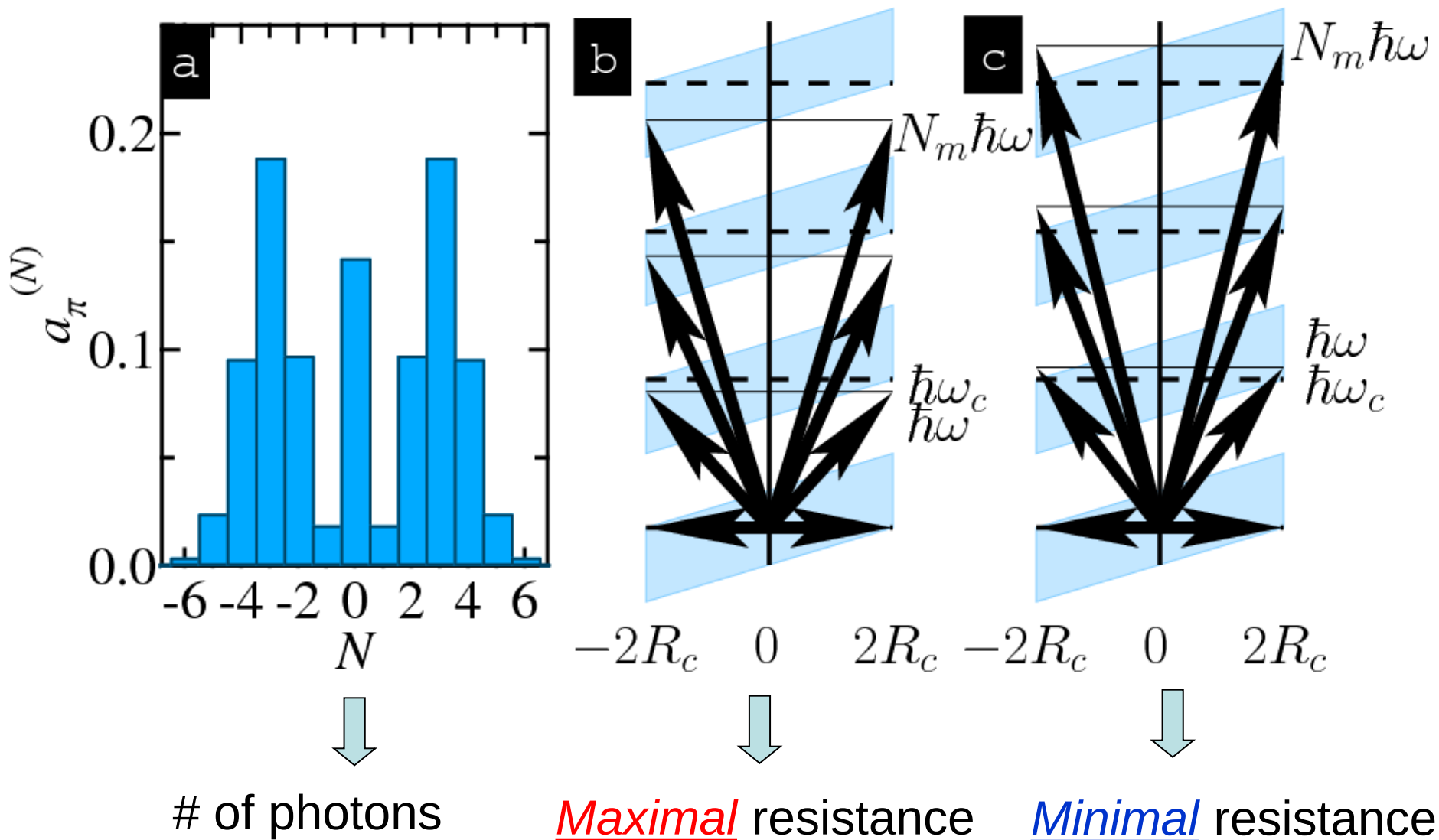
Multi-photon processes



$$a_{\varphi\varphi'}^{(N)} = J_N^2(\sqrt{\mathcal{P}_\omega}(1 - \cos(\varphi - \varphi')))$$

Yes !!

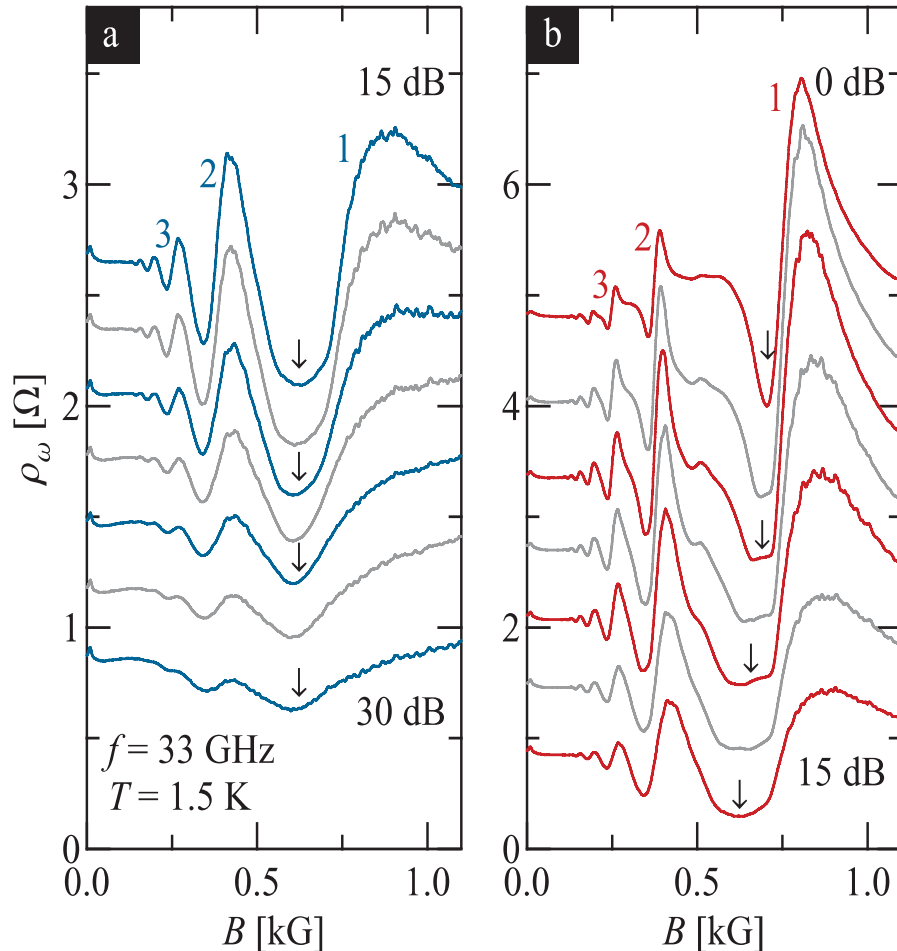
Multi-photon processes at work



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MIRO: Power dependence



MIRO:
Both amplitude and phase
are affected by
power increase

Displacement mechanism
is insufficient

Inelastic mechanism

I.A. Dmitriev, A.D. Mirlin, and D.G. Polyakov, Phys. Rev. Lett. **91**, 226802 (2003)
I.A. Dmitriev, M.G. Vavilov, I.L. Aleiner, A.D. Mirlin, and D.G. Polyakov,
Phys. Rev. B **71**, 115316 (2005)

MIRO: Inelastic Mechanism

Non-equilibrium occupation
isotropic



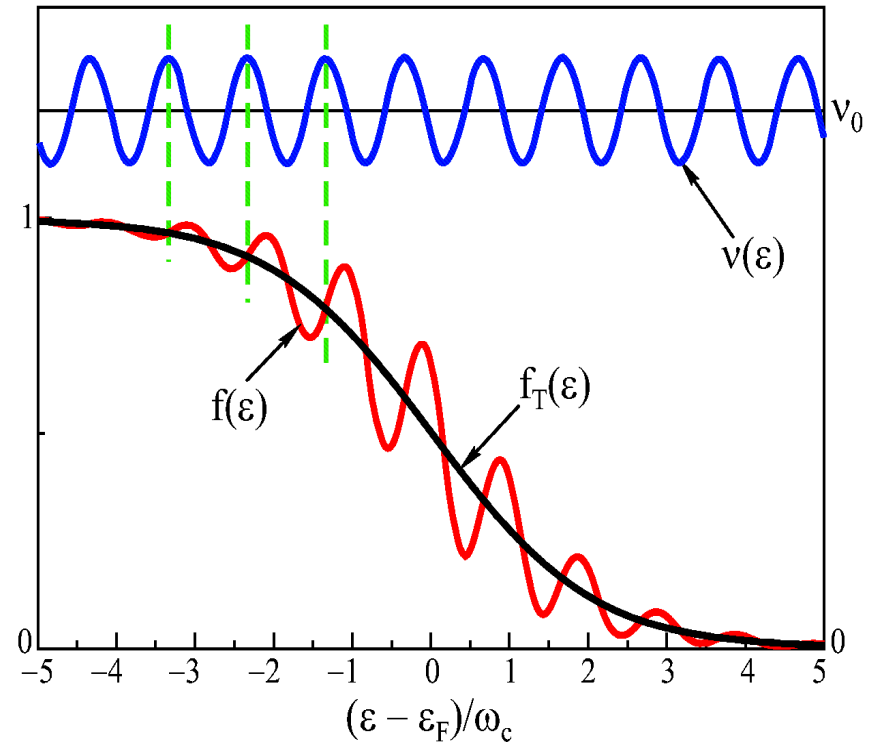
Stabilized by
inelastic processes



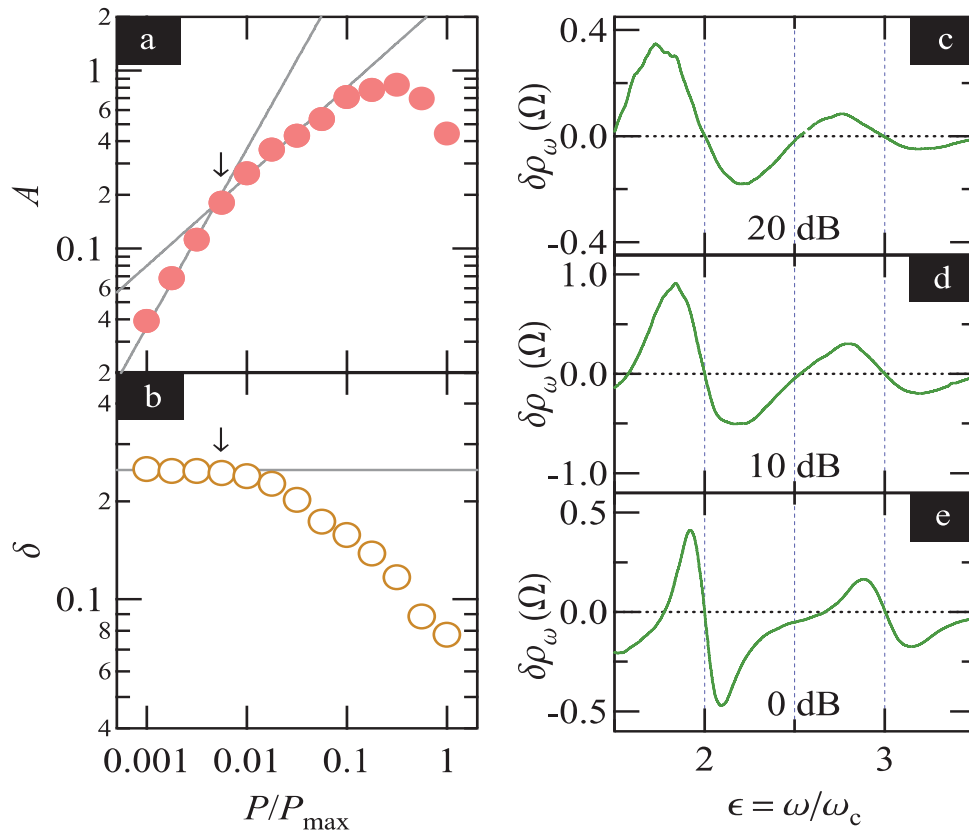
$$\Delta r \propto \frac{\tau_{\text{inelastic}}}{\tau_{tr}}$$



Dominates at low T



MIRO: Power Crossover



Maxima found at

$$\epsilon_{ac}^+ = n - \delta$$

$\uparrow$
 phase

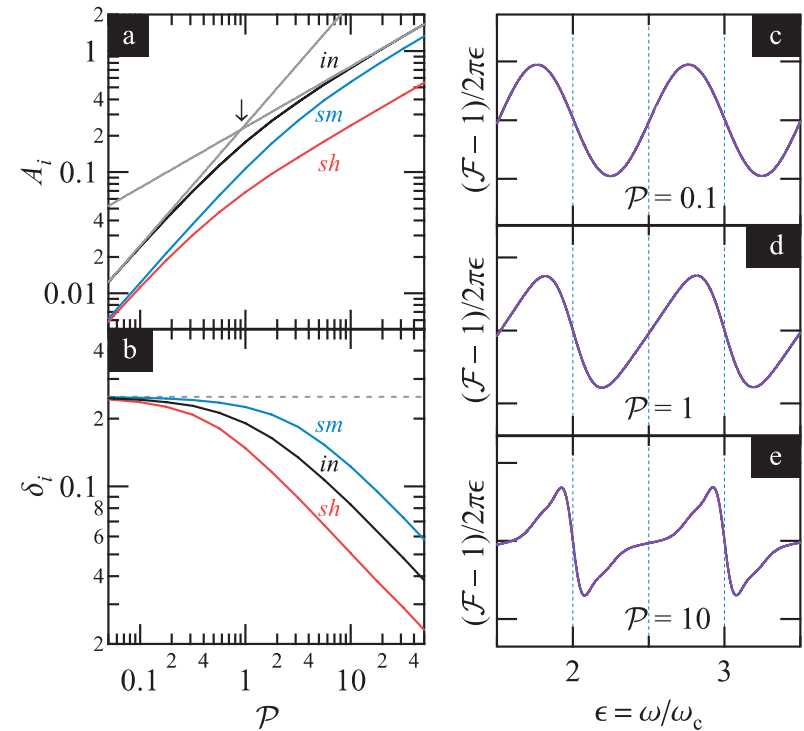
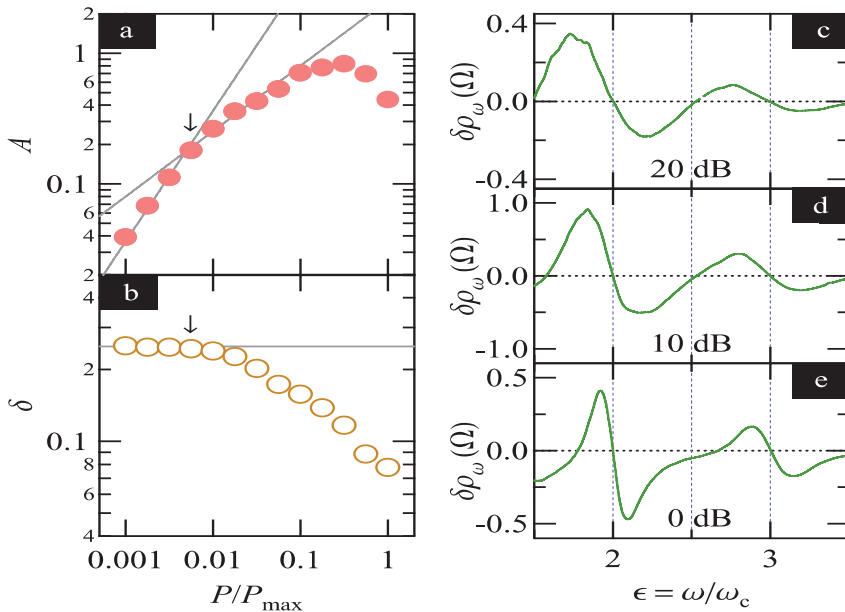
$$A \propto \frac{\delta\rho(\epsilon_{ac}^+)}{\lambda^2 \epsilon_{ac}^+}$$

$\uparrow$
 Amplitude

$$\mathcal{P} \ll 1 \quad \xrightarrow{\text{Crossover}} \quad \mathcal{P} \gg 1$$

MIRO: Power Crossover (cont)

	$\mathcal{P} \ll 1$	$\mathcal{P} \gg 1$
$\delta \sim$	$1/4$	$1/\sqrt{\mathcal{P}}$
$A \sim$	$\mathcal{P}$	$\sqrt{\mathcal{P}}$



Inelastic and Displacement mechanisms



Similar behavior

Conclusions

- Quasi-classical transport theory:
migrating guiding centers
- Strong currents favor backscattering and displacement mechanism
- High power increases the number of emitted photons and leads to crossover in amplitude and phase of oscillations
- MIRO: inelastic mechanism is as important as displacement and is expected to dominate at low T
- The amplitude for both mechanisms cross over from linear to sublinear in power concomitantly with the decrease in phase

Thank you!

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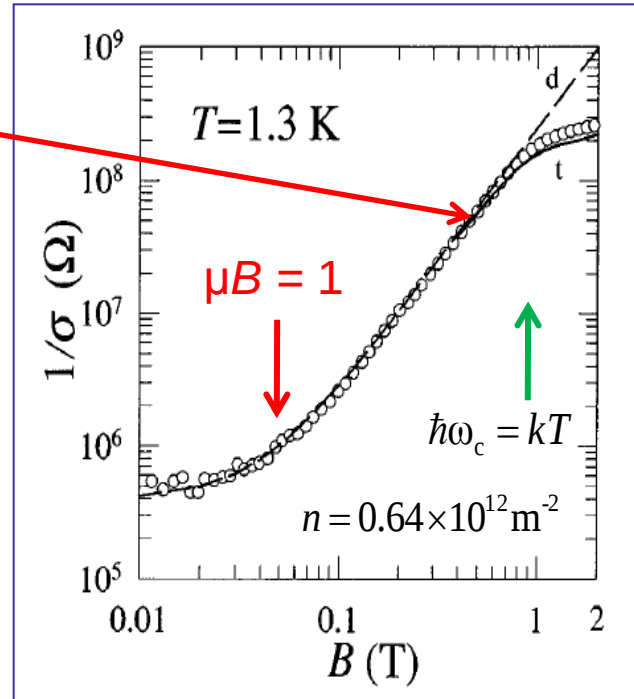
Drude model

$$\frac{\sigma_0}{\sigma_{xx}} = 1 + \mu^2 B^2$$

$$\sigma_0 = ne\mu = \frac{ne^2\tau_0}{m}$$

Einstein relation
Diffusion

$$\sigma_{xx} = \frac{ne^2}{kT} \frac{L^2}{\tau_B} \cong \frac{\sigma_0}{\mu^2 B^2} \frac{\tau_0}{\tau_B}$$



$$\mu B = \omega_c \tau_0$$

$$\omega_c = \frac{eB}{m}$$

Non-degenerate

$$R_c = \frac{(2mkT)^{1/2}}{eB} \quad \frac{\tau_0}{\tau_B} = 1$$

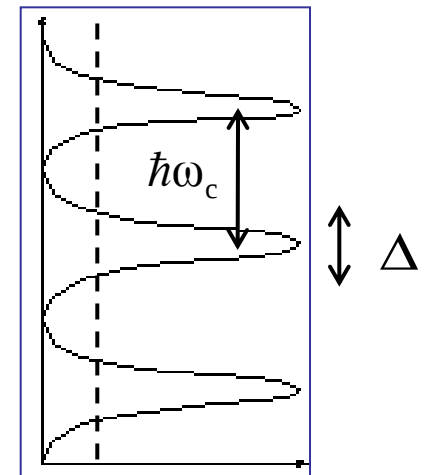
Classical cyclotron orbit

BUT $\omega_c \tau_0 = \mu B \gg 1$

Landau levels $E_N = (N + 0.5)\hbar\omega_c$

$$\frac{\tau_0}{\tau_B} \approx \frac{\hbar\omega_c}{\Delta}$$

$$\frac{\tau_0}{\tau_B} \approx \frac{\hbar\omega_c}{\hbar/\tau_B} = (\omega_c \tau_0)^{1/2} = (\mu B)^{1/2}$$



Single particles
Collision broadened
(SCBA):
Enhanced scattering?

AC+DC (cont)

$$f \approx f_{FD} \text{ ?}$$

Inelastic processes?

$$\omega_c \partial_\varphi f = \text{St}_{dc} \{f\} + \text{St}_{mw} \{f\} + \text{St}_{ee} \{f\}$$

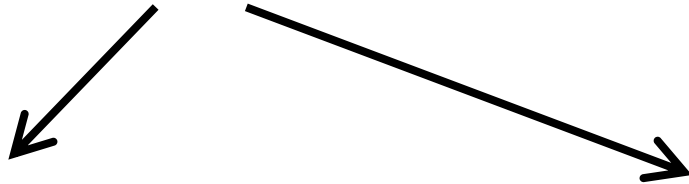
$$\frac{\Delta r_{in}}{\Delta r_{disp}} \approx \frac{\tau_0}{\tau(\pi) \varepsilon_{dc}} \leq 1$$

$$\varepsilon_{dc} = \frac{eE(2R_c)}{\hbar \omega_c} \geq 1$$

Displacement mechanism dominates

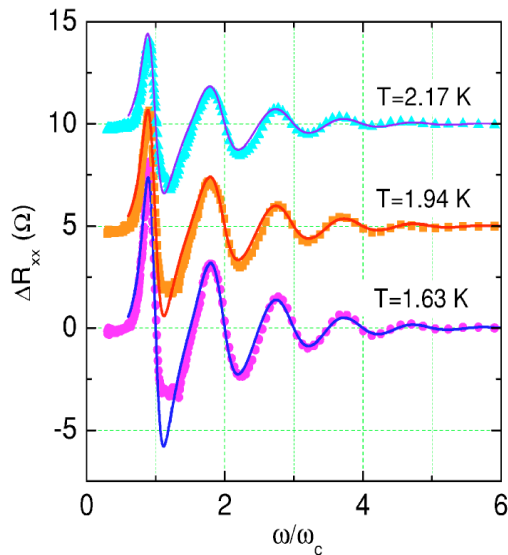
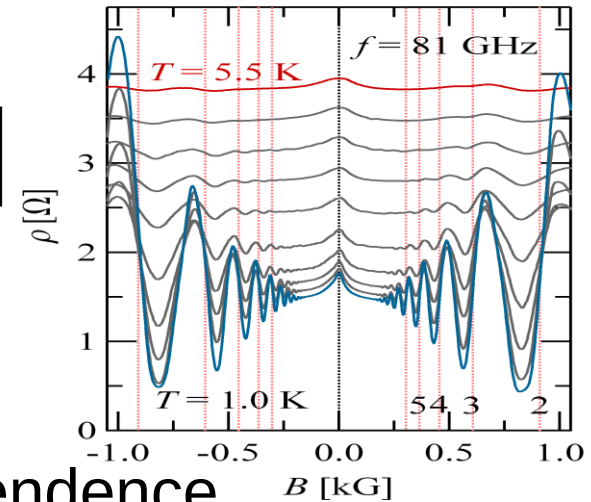
MIRO: Temperature dependence

$$\delta\rho \simeq A(T)\epsilon_{ac} \exp[-2\pi/\omega_c\tau_q]$$

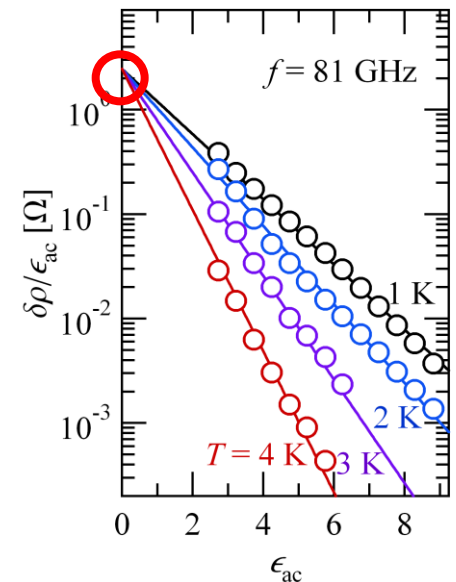


stronger T dependence

weak T dependence

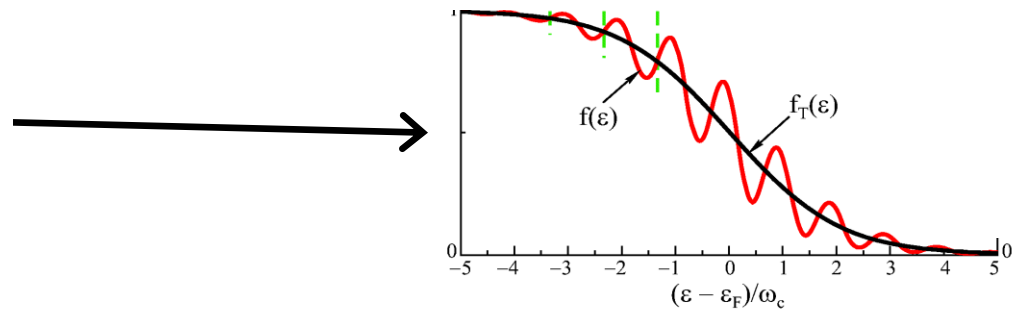
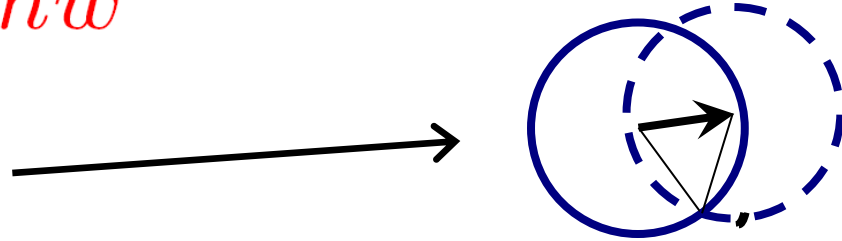


$A(T) ?$



Results: mechanisms of MIRO

$$\delta \mathbf{j} \propto \mathbf{E}_{dc} E_{mw}^2$$



$E_{dc} + E_{mw} \Rightarrow$ Time dependent occupation

$E_{mw} \Rightarrow$ Second angular harmonics of distribution function

Results (cont): Displacement

$$\delta\rho \simeq \textcolor{red}{A}(T)\epsilon_{ac} \exp[-2\pi/\omega_c\tau_q]$$

Displacement

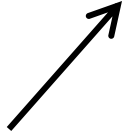
$$\textcolor{red}{A} \propto \frac{\tau_{tr}}{\tau^*}$$

$$\frac{1}{\tau_{tr}} = \left\langle \frac{1 - \cos \varphi}{\tau_\varphi} \right\rangle_\varphi$$

$$\frac{1}{\tau^*} = \left\langle \frac{(1 - \cos \varphi)^{\textcolor{red}{2}}}{\tau_\varphi} \right\rangle_\varphi$$

MW assisted scattering

$$\frac{1}{\tau_{\textcolor{red}{assisted}}} \propto E_{mw}^2 \frac{1 - \cos \varphi}{\tau_\varphi}$$


Kohn's theorem

Results (cont)

$$\delta\rho \simeq A(T)\epsilon_{ac} \exp[-2\pi/\omega_c\tau_q]$$

Inelastic

$$A \propto \frac{\tau_{\text{inel}}}{\tau_{tr}}$$

→ Temperature dependence

Photovoltaic, Quadrupole

$$A \propto \frac{\tau_q^{\text{dis}}}{\tau_{tr}}$$

→ Disorder
Quantum Time

$$\frac{1}{\tau_q^{\text{dis}}} = \left\langle \frac{1}{\tau_\varphi} \right\rangle_\varphi$$

Results (cont)

$$\delta\rho \simeq A(T)\epsilon_{ac} \exp[-2\pi/\omega_c\tau_q]$$

Displacement

$$A \propto \frac{\tau_{tr}}{\tau^*}$$

Inelastic:

$$A \propto \frac{\tau_{inel}}{\tau_{tr}}$$

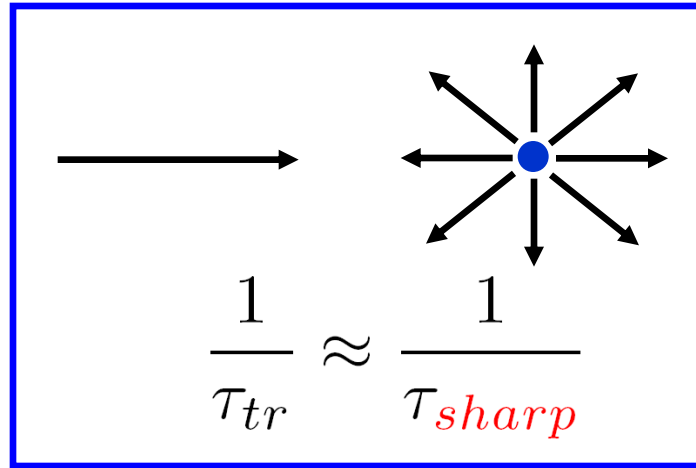
Photovoltaic
Quadrupole

$$A \propto \frac{\tau_q^{dis}}{\tau_{tr}}$$

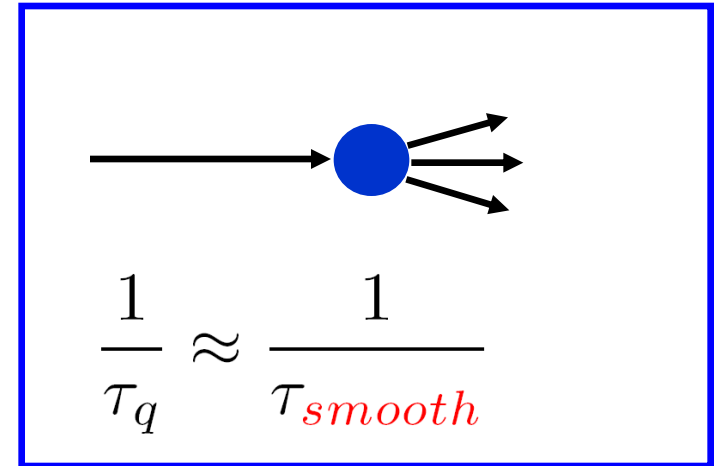
Dominant ?

Results: mixed disorder

Sharp disorder:



Smooth disorder:



$\ll$

What if $\frac{1}{\tau_{sharp}} \ll \frac{1}{\tau_{inel}(T)} \ll \frac{1}{\tau_{smooth}}$?

$$\boxed{A_{disp} \approx 1} \gg A_{inel} \approx \frac{\tau_{inel}}{\tau_{sharp}} \gg A_{quadr, phot} \approx \frac{\tau_{smooth}}{\tau_{sharp}}$$

Temperature **independent** MIRO !!!

Results: smooth vs. sharp disorder

Smooth disorder:

Inelastic contribution is dominant



Strong T dependence

Sharp disorder:

Displacement contribution is dominant



Weak T dependence

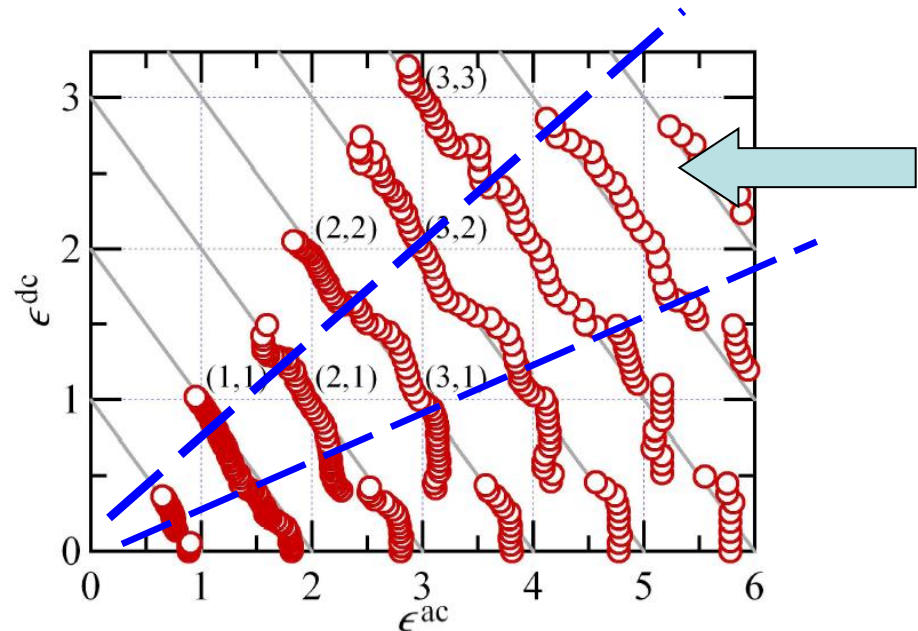
Results

Differential magnetoresistance

$$\rho_{xx} \propto (1 - 2\mathcal{P}_\omega) \cos \epsilon_{dc} + \mathcal{P}_\omega \frac{\epsilon_{dc} + \epsilon_{ac}}{\epsilon_{dc}} \cos(\epsilon_{dc} + \epsilon_{ac}) + \mathcal{P}_\omega \frac{\epsilon_{dc} - \epsilon_{ac}}{\epsilon_{dc}} \cos(\epsilon_{dc} - \epsilon_{ac})$$

Condition of maxima:

$$\epsilon_{ac} + \epsilon_{dc} \simeq n$$



Results (closer look)

Differential magnetoresistance **maxima**

$$\rho_{xx} \propto (1 - 2\mathcal{P}_\omega) \cos \epsilon_{dc} + \mathcal{P}_\omega \frac{\epsilon_{dc} + \epsilon_{ac}}{\epsilon_{dc}} \cos(\epsilon_{dc} + \epsilon_{ac}) + \mathcal{P}_\omega \frac{\epsilon_{dc} - \epsilon_{ac}}{\epsilon_{dc}} \cos(\epsilon_{dc} - \epsilon_{ac})$$

$\epsilon^{ac} > \epsilon^{dc}$

Maximize

Minimize

Condition for global **maxima**

$$\epsilon_{ac} + \epsilon_{dc} \simeq n$$

$$\epsilon_{ac} - \epsilon_{dc} \simeq m + 1/2$$



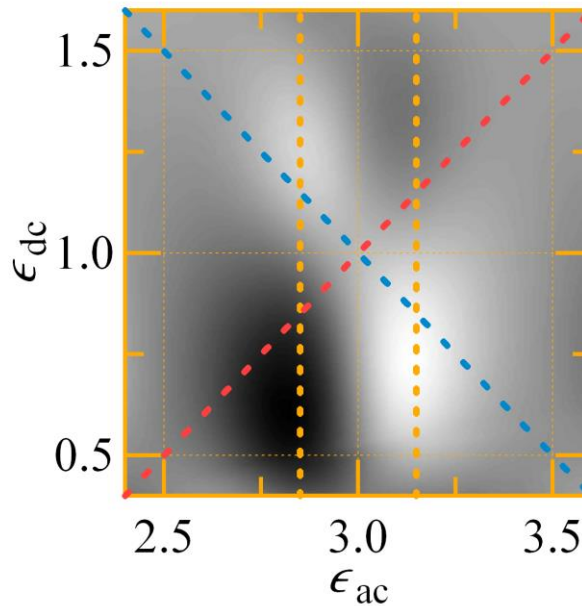
$$\epsilon^{dc} = n \pm 1/4$$

$$\epsilon^{ac} = m \mp 1/4$$

Experiment (closer look)

- **Minima** are found along:

$$\epsilon^- = \epsilon_{ac} - \epsilon_{dc} = 2$$



- **Maxima** are found along:

$$\epsilon^+ = \epsilon_{ac} + \epsilon_{dc} = 4$$

- Max (+) and min (-) are symmetrically offset by $\sim 1/4$ -cycle from $(\epsilon_{ac}, \epsilon_{dc}) \simeq (n, m)$
- Equations for the global extrema:
$$\begin{aligned} (\epsilon_{ac}, \epsilon_{dc})^+ &\simeq (n \pm 1/4, m \mp 1/4) \\ (\epsilon_{ac}, \epsilon_{dc})^- &\simeq (n \pm 1/4, m \pm 1/4) \end{aligned}$$

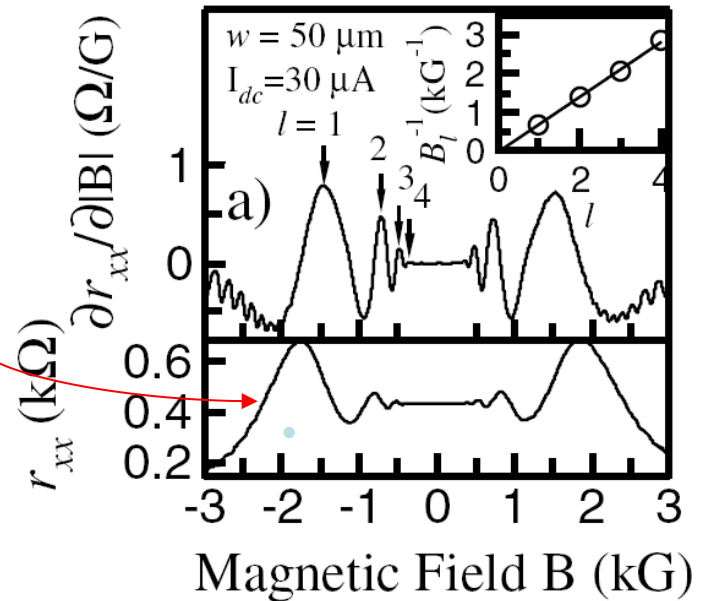
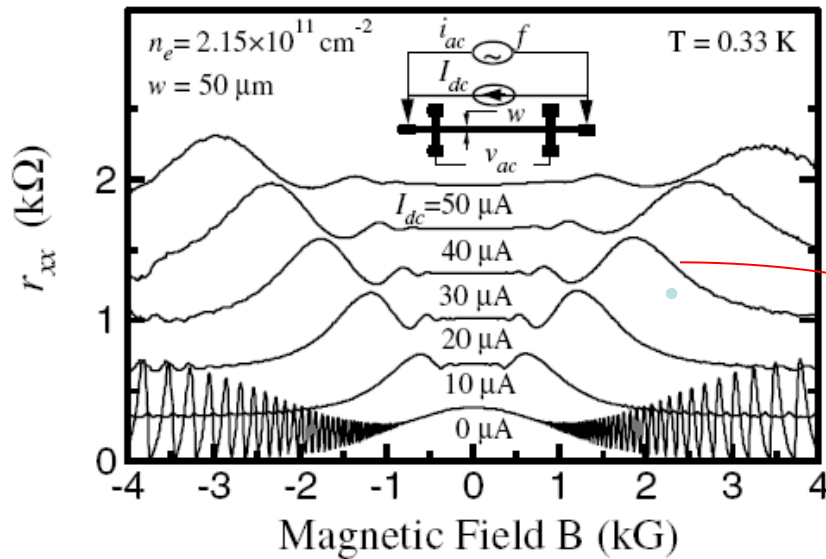
Conclusions

1. The theory of MIRO is constructed
2. Four different mechanisms were identified
3. The temperature dependence of magnetoresistance determined by the relative role played by the smooth and sharp part of the disorder potential
4. Understanding of the temperature dependence of MIRO in recent experiments (partial)

Thank you!

Oscillatory Differential DC Conductivity

C.L. Yang, J. Zhang, R.R. Du,
J.A. Simmons, J.L. Reno;
PRL 89, 076801

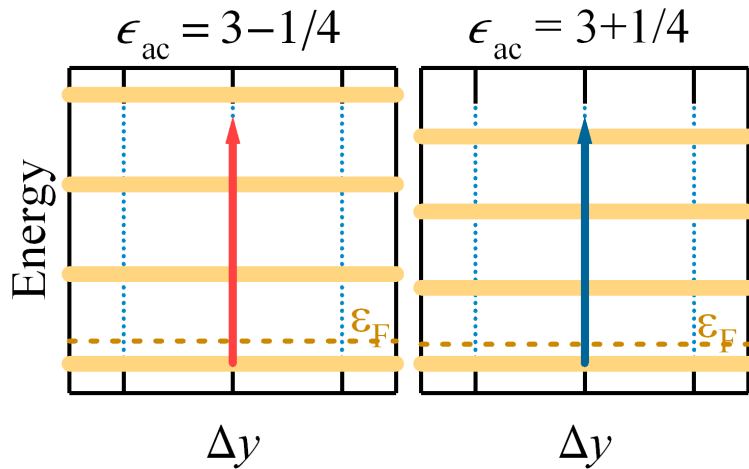


As current increases:

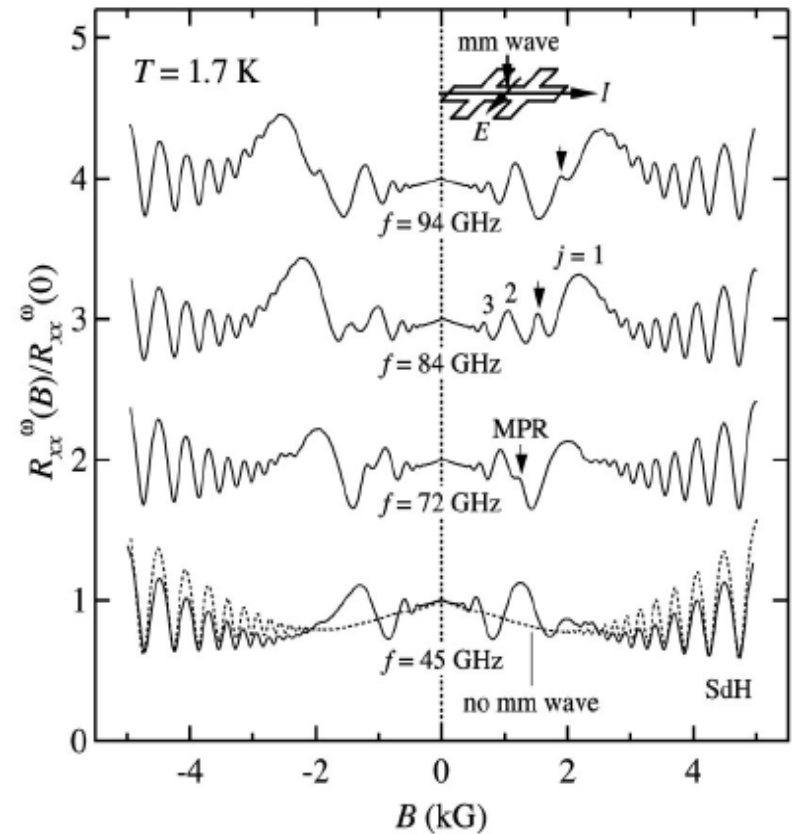
1. SdH oscillations are suppressed (heating?)
2. New oscillations develop

Oscillations in r_{xx} as
a function of $\frac{2R_c e E}{\hbar \omega_c}$.

Microwave induced resistance oscillations (MIRO)



- Parameter: $\epsilon_{ac} \equiv \frac{\omega}{\omega_C}$
- Maxima and minima: $\epsilon_{ac}^{\pm} \simeq n \mp \frac{1}{4}$
- Theory: $\Delta R \propto \epsilon_{ac} \sin(2\pi\epsilon_{ac})$, $2\pi\epsilon_{ac} \gg 1$



Zudov, Du, Simmons, Reno, Phys. Rev. B **64**, 201311(R) (2001)
 Vavilov and Aleiner, Phys. Rev. B **69**, 035303 (2004)