

# QUANTUM MANIPULATIONS OF TRAPPED ELECTRONS ON LIQUID HELIUM

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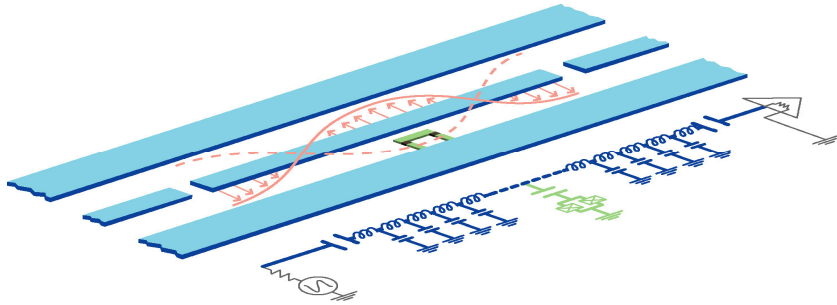
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Lhasa	拉萨	Lanzhou	兰州	Mount Everest	珠穆朗玛峰	Lanzhou	兰州
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# OUTLINE

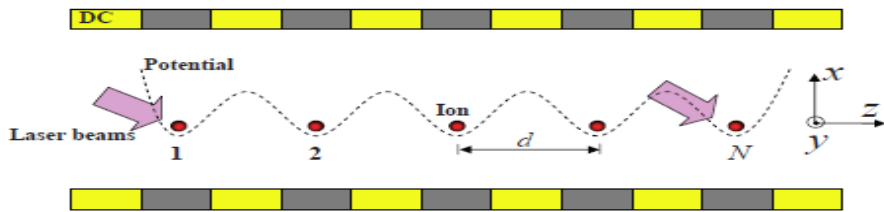
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- Backgrounds and Introduction
- Coherent manipulations of surface-electron states by controlling their evolution-times
- Quantum logic gate with surface-electron states by population passages
- Application to quantum devices
- Conclusions and Discussions

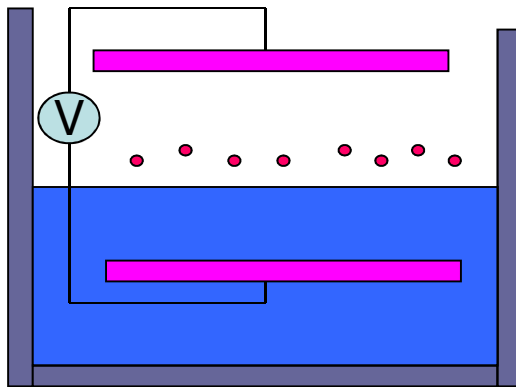
# BACKGROUNDS AND INTRODUCTION: We are working on



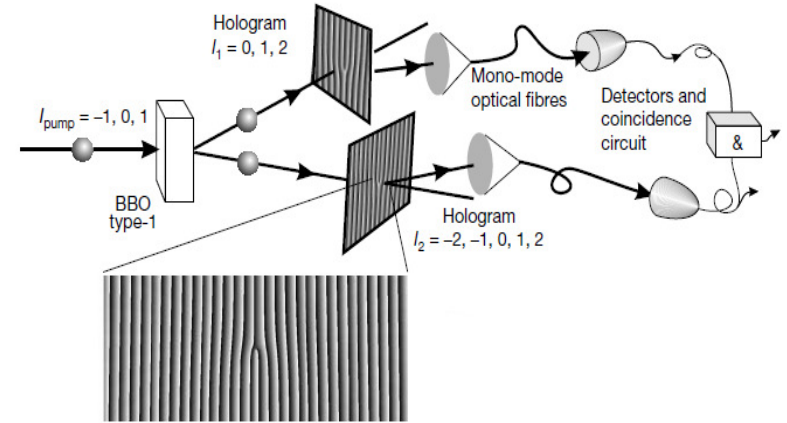
Circuit QEDs (Theory and Exp.)



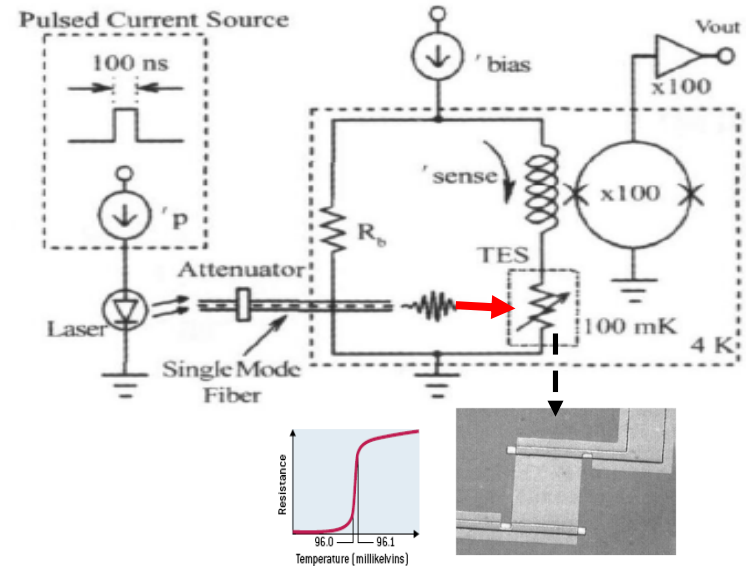
Trapped ions (Theory)



Trapped electrons on liquid Helium (Theory and future Exp.)



Single photons: generations and manipulations (Exp.)



Superconducting Single-photon detections (Exp.)

# Electrons trapped on Liquid Helium

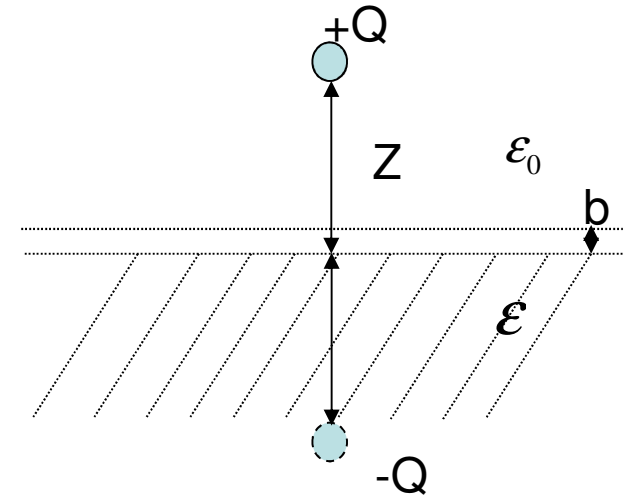
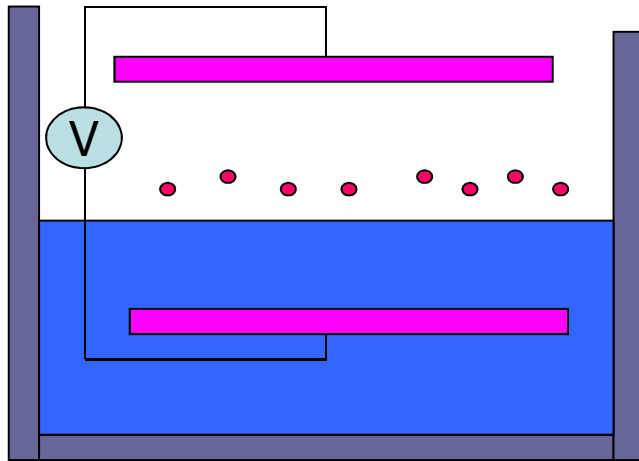


Image potential



Quantum Computing with Electrons Floating on Liquid Helium

P. M. Platzman, *et al.*

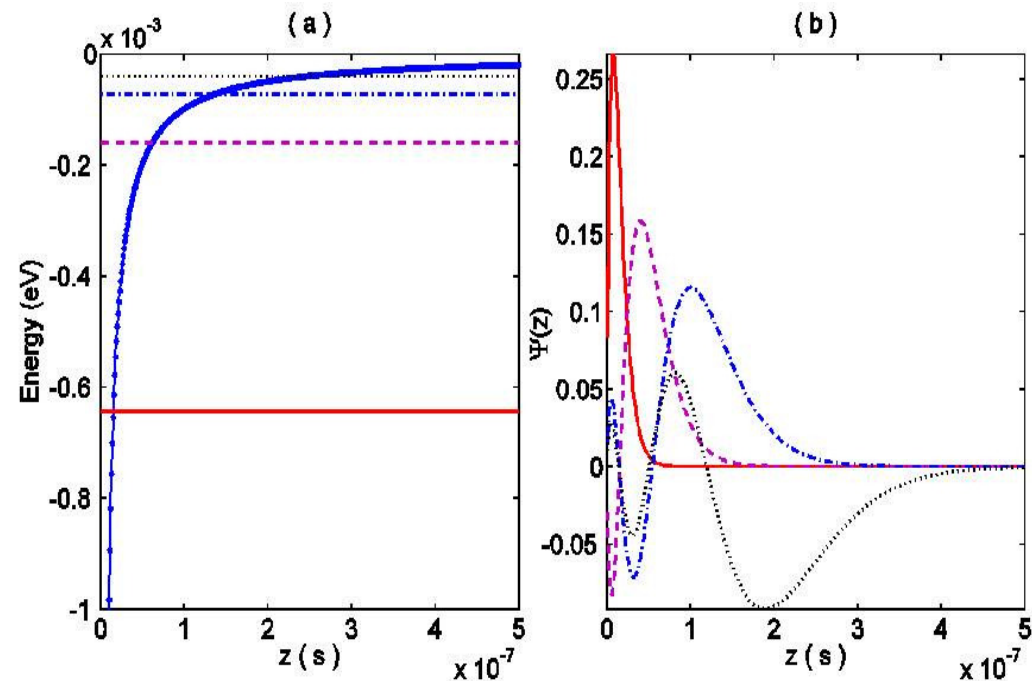
*Science* **284**, 1967 (1999);

DOI: 10.1126/science.284.5422.1967

# Single electrons on liquid Helium

A single electron is weakly attracted and trapped by an image potential of the form

$$V(z) = \begin{cases} -\frac{\Lambda e^2}{z}, & z > 0 \\ \infty, & z \leq 0 \end{cases}$$



The ground state and the lower two excited state energies are approximately

$E_0 = -0.65 \text{ meV}$ ,  $E_1 = -0.16 \text{ meV}$  and  $E_2 = -0.072 \text{ meV}$ , respectively.

The transition frequency between the ground and the first excited state is about  $117 \text{ GHz}$  and these transitions can be shifted with a Stark field applied normal to the surface.

$$100 \text{ GHz} \sim 5 \text{ K}$$

**Quantum coherent manipulation requires very low-temperature:  $< 1 \text{ K}$  !**

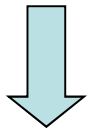
## Quantum computation with surface-state electrons:

### Platzman-Dykman Model

Single-qubit gates achieved by precisely controlling Rabi oscillations

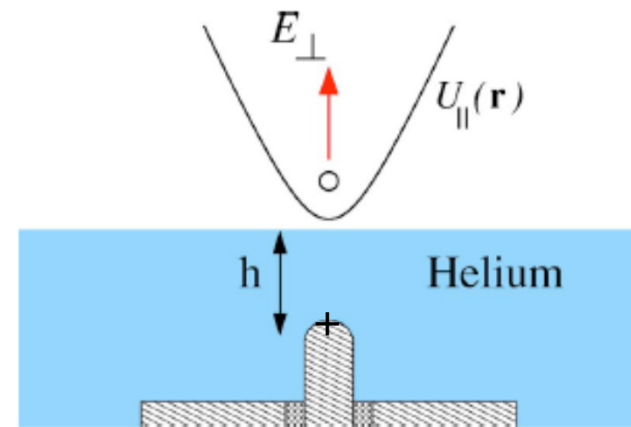
$$H(t) = \frac{\hbar\omega_0}{2}\sigma_z + \frac{\hbar\Omega(t)}{2}\sigma_x,$$

$$\Omega(t) = \Omega_0 \cos(\omega_0 t)$$



$$U(t) = \cos\left[\frac{A(t)}{2}\right] + i \sin\left[\frac{A(t)}{2}\right]\sigma_x,$$

$$A(t) = \Omega_0 t$$

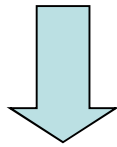


M. I. Dykman et al., PRB **67**, 155402 (2003)

# Quantum computation with surface-state electrons:

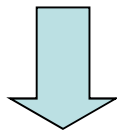
## Platzman-Dykman Model

$$\hat{H}_h = \sum_{j=1,2} \frac{\hbar\omega_j^0}{2} \hat{\sigma}_j^z + V_c(z_1, z_2),$$



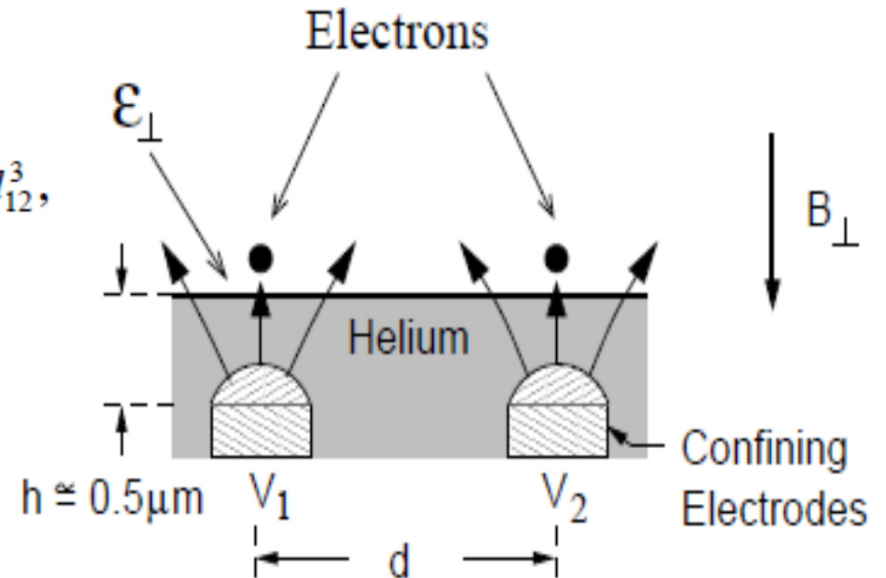
$$V_c(z_1, z_2) \approx e^2 z_1 z_2 / d_{12}^3,$$

$$\bar{H}_I = C_{12}^{zz} \hat{\sigma}_1^z \hat{\sigma}_2^z + C_{12}^{xx} (\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_1^- \hat{\sigma}_2^+).$$



$$\hat{U} = \begin{pmatrix} e^{-i\phi} & 0 & 0 & 0 \\ 0 & e^{i\phi} \cos \xi & -i \sin \xi & 0 \\ 0 & -i \sin \xi & e^{i\phi} \cos \xi & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{pmatrix}$$

$$\xi = C_{12}^{xx} t$$



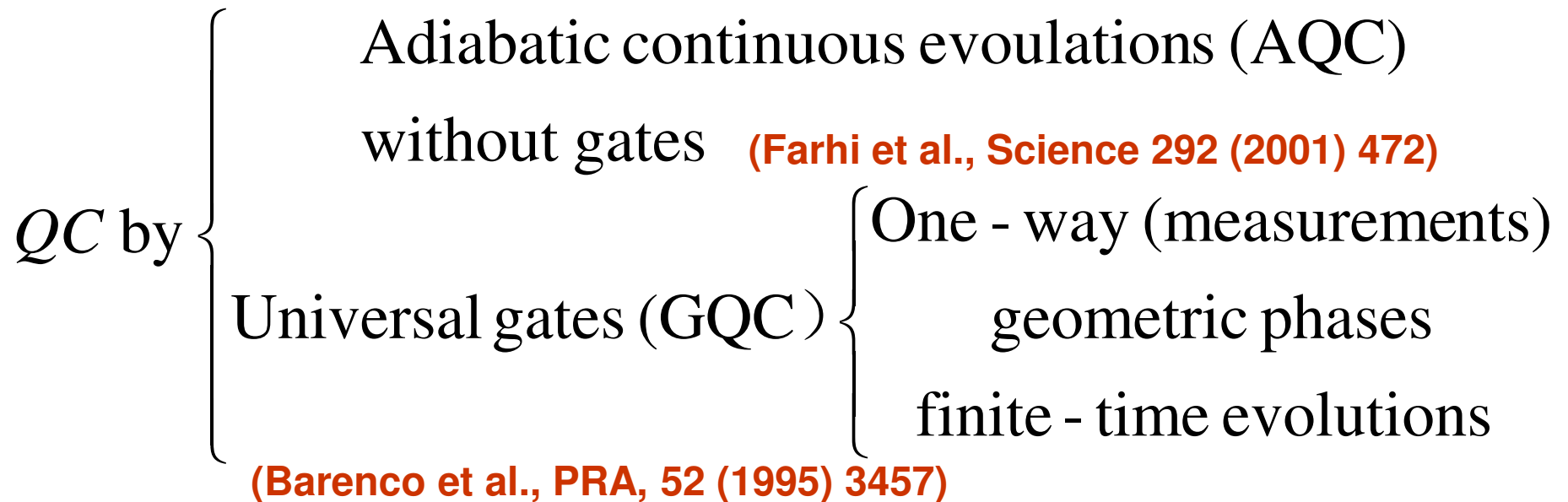
P. M. Platzman and M. I. Dykman, Science 284, 1967 (1999).  
 M. J. Lea et al., Fortschr. Phys. 48, 1109 (2000).



# BACKGROUNDS AND INTRODUCTION

*Two kinds of ways-four approaches to quantum computing (QC)*

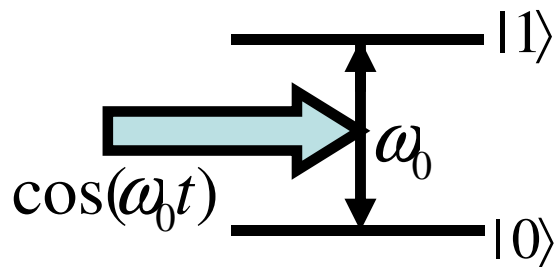
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# Evolution-time sensitive and insensitive QGs

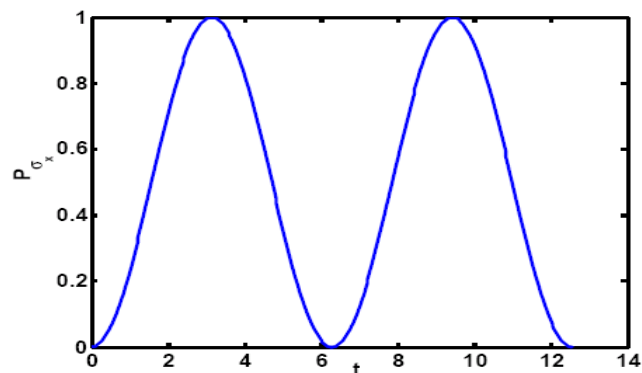
## Single-qubit gates

$\sigma_x$  - rotations by  
usual Rabi oscillations

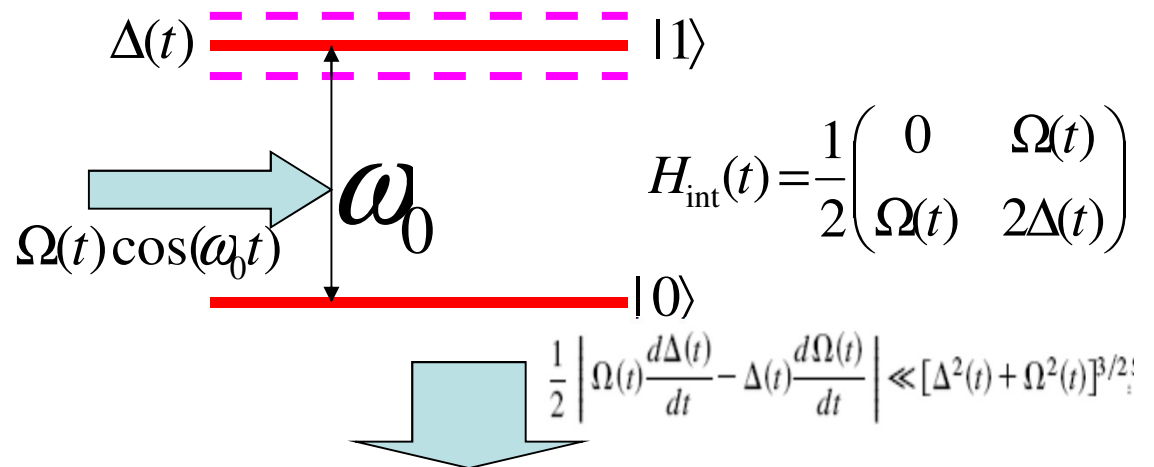


**Evolution-time sensitive  
(oscillation)**  
successful probability

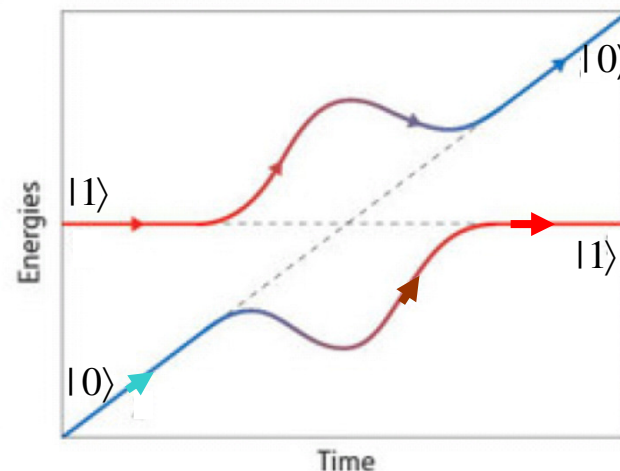
$$P_{\sigma_x}(t) = [1 - \cos(t)] / 2$$



**Our approach: Chirping qubit's level**



**Bidirectional adiabatic population transfers**



$$\sigma_x : \begin{cases} |1\rangle \rightarrow |0\rangle \\ |0\rangle \rightarrow |1\rangle \end{cases}$$

# Evolution-time sensitive and insensitive QGs

## Two-qubit gates

Interbit interaction time is usually required to be precisely set for implementing two-qubit quantum gates. For example

$$H = \sum_{i=1,2} \frac{\omega_0}{2} \sigma_i^z + \frac{K(t)}{2} \sum_{i \neq j} \sigma_i^+ \sigma_j^-$$

$$\int_0^T K(t) dt = \pi$$

i - SWAP gate

(evolution time sensitive) :

$$|00\rangle \rightarrow |00\rangle, |11\rangle \rightarrow |11\rangle$$

$$|01\rangle \rightarrow i|10\rangle, |10\rangle \rightarrow i|01\rangle$$

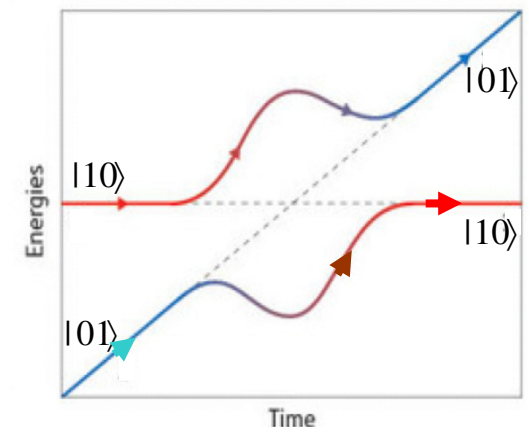
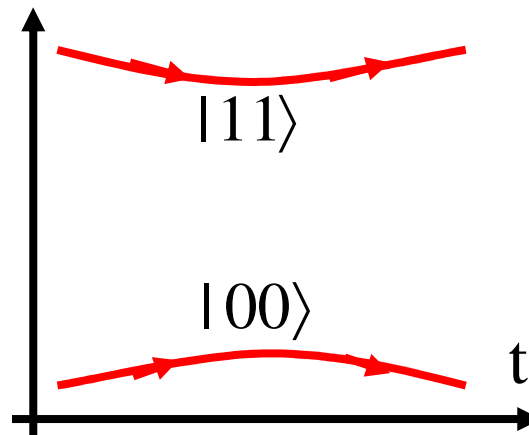
Our approach:

*chirping one qubit's level to implement ETIQG.*

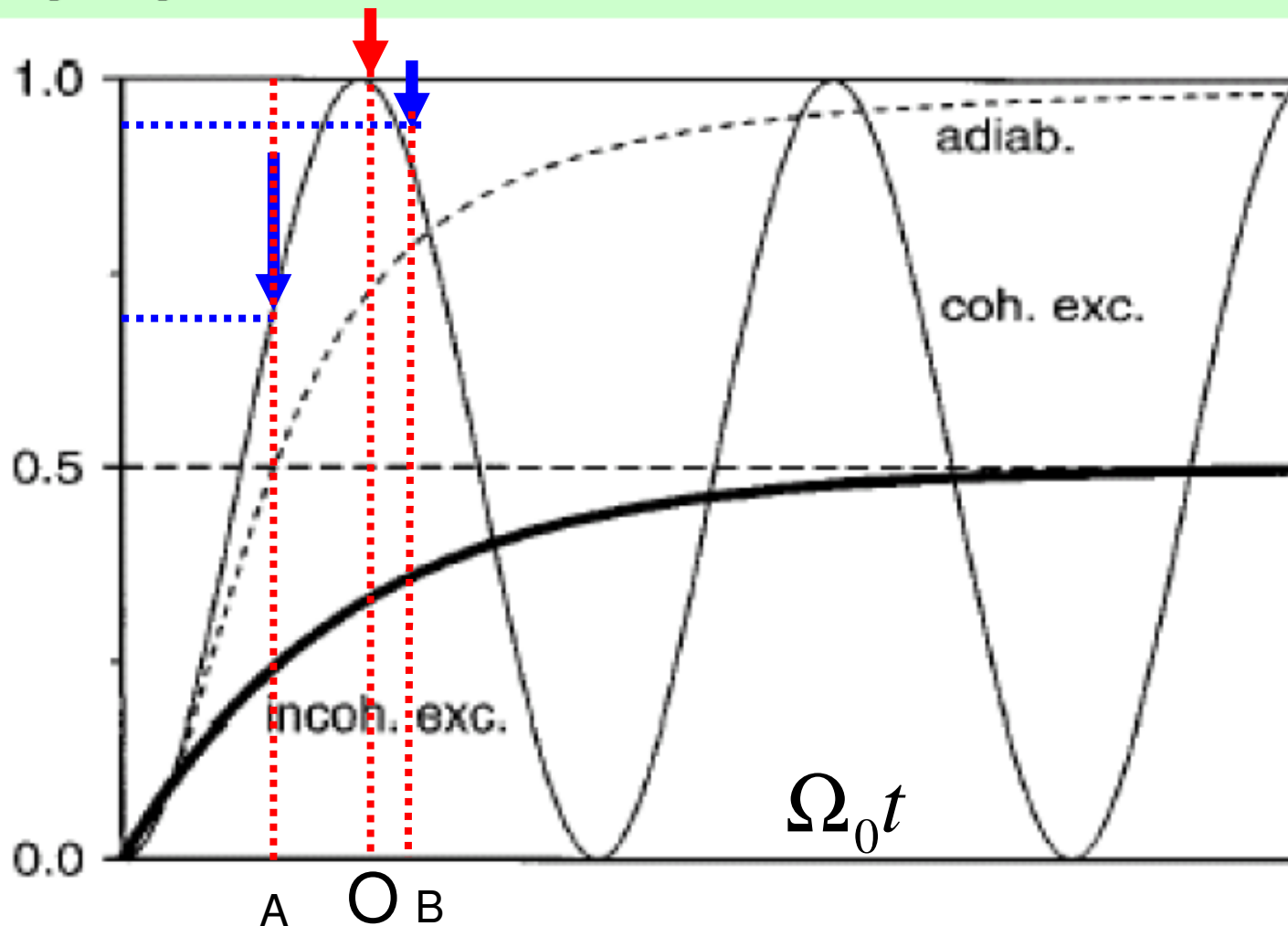
$$H = \sum_{i=1,2} \frac{\omega_0}{2} \sigma_i^z + \frac{K(t)}{2} \sum_{i \neq j} \sigma_i^+ \sigma_j^- + \frac{\Delta_2(t)}{2} \sigma_2^z$$

$$H_{\text{int}} = \frac{1}{2} \begin{pmatrix} -\Delta_2(t) & 0 & 0 & 0 \\ 0 & -\Delta_2(t) & K(t) & 0 \\ 0 & K(t) & \Delta_2(t) & 0 \\ 0 & 0 & 0 & \Delta_2(t) \end{pmatrix}$$

i - SWAP gate:  
evolution time  
insensitive:



# Comparison of the efficiency of population transfer methods.



\*From K. Bergmann et al., Rev. Of Mod. Phys., 17, 3 (1998)

# AQC

Ground state of  $H_i$  is easily accessible.

Ground state of  $H_f$  encodes the solution to *a hard computational problem*.

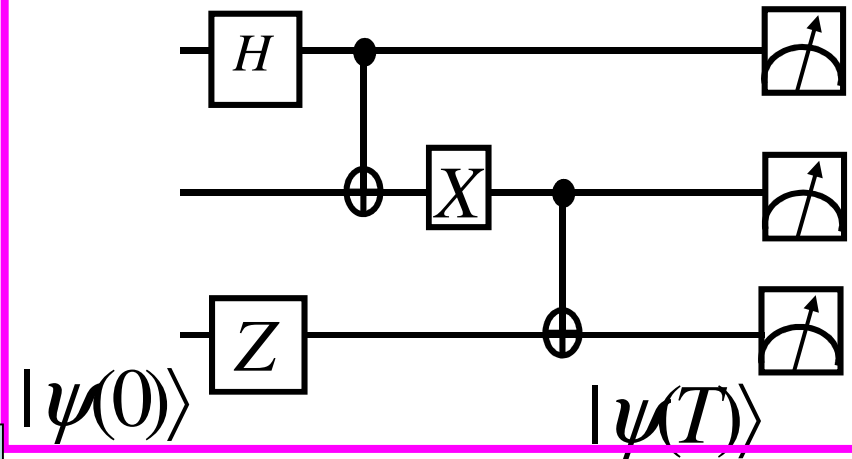
$$|0_i\rangle \Rightarrow |0_f\rangle$$

*Adiabatically,*

*AQC is evolution-time insensitive but not universal, in principle.*

# GQC

*Universal* computing  
Implemented by a series of *evolution-time sensitive* quantum gates



**Motivation: Proposes an *evolution-time insensitive* GQC with EH**

# Evolution-time dependent quantum manipulations

## Laser induced orbit-orbit couplings of an electron on the liquid Helium

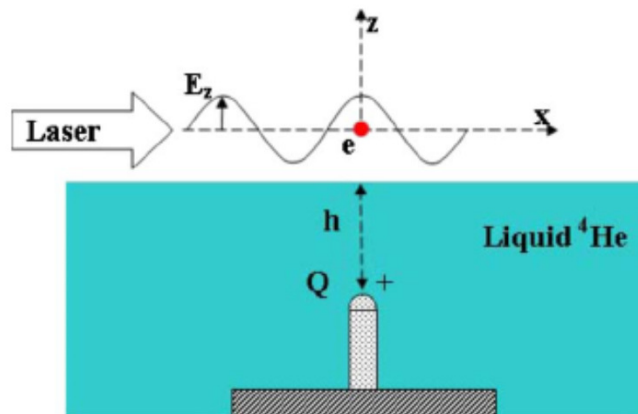


FIG. 1. (Color online) A sketch of an electron confined by a microelectrode  $Q$  submerged by the depth  $h$  beneath the helium surface and driven by a classical laser field propagating along the  $x$  direction.

$$\omega_l = \omega_0 + K\nu.$$

$$\hat{H}_{\text{eff}}^0 = \hbar\Omega e^{i\phi_l} \hat{\sigma}_+ + \text{H.c.} \quad \text{for } K=0,$$

$$\hat{H}_{\text{eff}}^r = i\eta\hbar\Omega e^{i\phi_l} \hat{\sigma}_+ \hat{a} + \text{H.c.} \quad \text{for } K=-1$$

$$\hat{H}_{\text{eff}}^b = i\eta\hbar\Omega e^{i\phi_l} \hat{\sigma}_+ \hat{a}^\dagger + \text{H.c.} \quad \text{for } K=1,$$

$$\hat{H} = \hat{H}_0 + ez\mathcal{E}(x,t)$$

$$\hat{H}_0 = \hbar\nu \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{\hbar\omega_0}{2} \hat{\sigma}_z$$

$$\mathcal{E}(x,t) = E_z \hat{z} \cos(k_l x - \omega_l t + \phi_l)$$

$$\eta = (\omega_0 + K\nu) \sqrt{\hbar / (2m_e \nu)} / c$$

# Evolution-time dependent quantum manipulations

## Evolution-time sensitive quantum gate with a single EH

(i) For  $K \leq 0$ ,

$$\begin{cases} |m\rangle|g\rangle \rightarrow |m\rangle|g\rangle, & m < k \\ |m\rangle|g\rangle \rightarrow \cos(\Omega_{m-k,k}t)|m\rangle|g\rangle + i^{k-1}e^{i\phi_l} \sin(\Omega_{m-k,k}t)|m-k\rangle|e\rangle, & m \geq k \\ |m\rangle|e\rangle \rightarrow \cos(\Omega_{m,k}t)|m\rangle|e\rangle - (-i)^{k-1}e^{-i\phi_l} \sin(\Omega_{m,k}t)|m+k\rangle|g\rangle, \end{cases}$$

(ii) For  $K \geq 0$ ,

$$\begin{cases} |m\rangle|g\rangle \rightarrow \cos(\Omega_{m,k}t)|m\rangle|g\rangle + i^{k-1}e^{i\phi_l} \sin(\Omega_{m,k}t)|m+k\rangle|e\rangle \\ |m\rangle|e\rangle \rightarrow |m\rangle|e\rangle, & m < k \\ |m\rangle|e\rangle \rightarrow \cos(\Omega_{m-k,k}t)|m\rangle|e\rangle - (-i)^{k-1}e^{-i\phi_l} \sin(\Omega_{m-k,k}t)|m-k\rangle|g\rangle, & m \geq k, \end{cases}$$

$$\Omega_{m,k} = \Omega \eta^k \sqrt{(m+k)! / m!} \quad k = |K|$$

Quantum gate with a single EH could be implemented using the ion-trap technique:  
C. Monroe et al, PRL, 75, 4714(1995)

**M. Zhang, H. Y. Jia, and L. F. Wei, Phys. Rev. A, 80 055801 (2009).**

# Evolution-time dependent quantum manipulations

## Quantum manipulations of EHs by controllable spin-orbital interactions

### Why spins?

PHYSICAL REVIEW A 74, 052338 (2006)

#### Spin-based quantum computing using electrons on liquid helium

S. A. Lyon

*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

(Received 17 September 2006; published 30 November 2006)

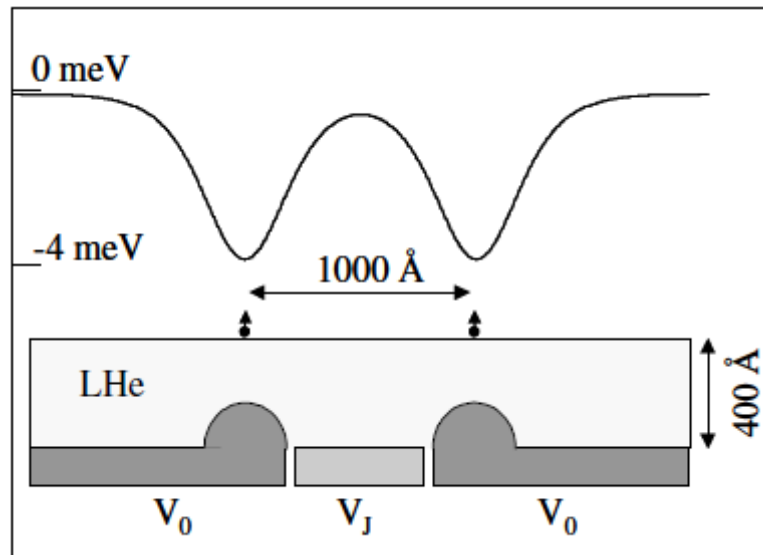
Numerous physical systems have been proposed for constructing quantum computers, but formidable obstacles stand in the way of making even modest systems with a few hundred quantum bits (qubits). Several approaches utilize the spin of an electron as the qubit. Here it is suggested that the spin of electrons floating on the surface of liquid helium will make excellent qubits. These electrons can be electrostatically held and manipulated much like electrons in semiconductor heterostructures, but being in a vacuum the spins on helium suffer much less decoherence. In particular, the spin-orbit interaction is reduced so that moving the qubits with voltages applied to gates has little effect on their coherence. Remaining sources of decoherence are considered, and it is found that coherence times for electron spins on helium can be expected to exceed 100 s. It is shown how to obtain a controlled-NOT operation between two qubits using the magnetic dipole-dipole interaction.



# Evolution-time dependent quantum manipulations

## Quantum manipulations of EHs by controllable spin-orbital interactions

### Why spins?



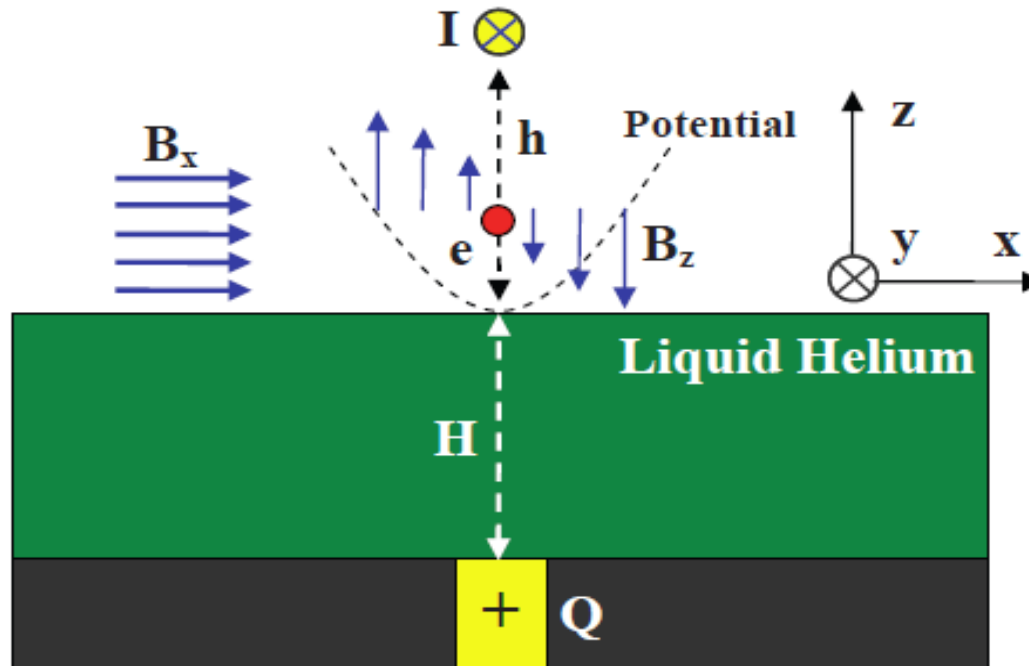
Interspin couplings are implemented by the magnetic dipole-dipole interaction between the spins

Alternatively, the electric dipole-dipole interaction could also be utilized to realize the spin-spin couplings! --Our motivation.

# Evolution-time dependent quantum manipulations

## Quantum manipulations of EHs by controllable spin-orbital interactions

### Spin-orbit coupling with a single trapped electron

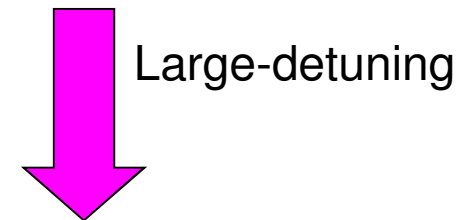
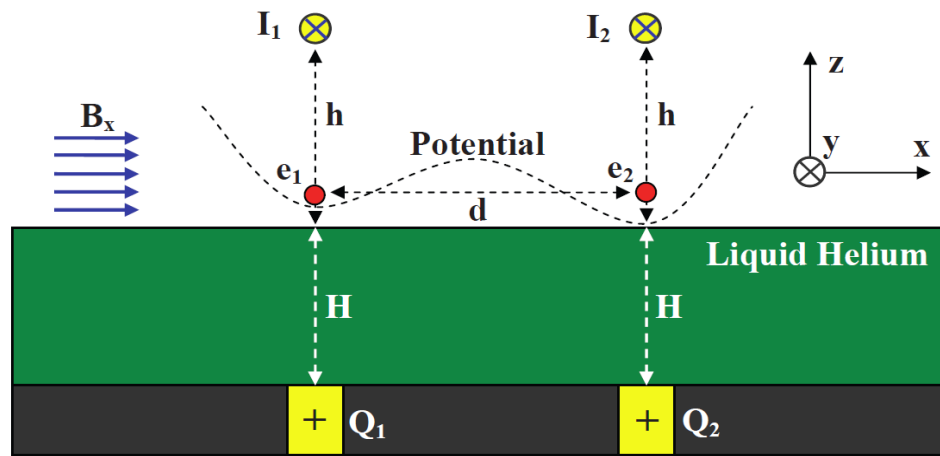


$$\hat{H}_e = \hbar\Omega \left( e^{it\delta} \hat{a} \hat{\sigma}_+ + e^{-it\delta} \hat{a}^\dagger \hat{\sigma}_- \right) \quad \Omega = \frac{g\mu_B\mu_0 I}{4\pi\hbar^2 \sqrt{2\hbar m_e \nu_x}}$$

# Evolution-time dependent quantum manipulations

## Quantum manipulations of EHs by controllable spin-orbital interactions

### Spin-orbit JC coupling between the distant electrons



$$\hat{H}_{\text{JC}} = \frac{\hbar\Omega^2}{\Delta}(\hat{\sigma}_+\hat{b} + \hat{\sigma}_-\hat{b}^\dagger)$$

Orbital Coulomb interaction:

$$V(x, y, z) \approx \frac{e^2}{4\pi\epsilon_0 d^3}(2x_1x_2 + z_1z_2 + y_1y_2)$$

$$\hat{H}_e = \hbar\Omega (e^{it\delta}\hat{a}\hat{\sigma}_+ + e^{-it\delta}\hat{a}^\dagger\hat{\sigma}_-)$$

$$\hat{H}_{ee} = \hat{H}_e + \hbar\tilde{\Omega} (e^{i\Delta t}\hat{a}\hat{b}^\dagger + e^{-i\Delta t}\hat{a}^\dagger\hat{b})$$

$$\Omega^2/\Delta = 2.5 \text{ MHz for } d = 10 \mu\text{m}$$

# Evolution-time insensitive quantum manipulations: adiabatic population passages for single-qubit gate

Consider the reverse field and the Stark shift field, the Hamiltonian of the system is

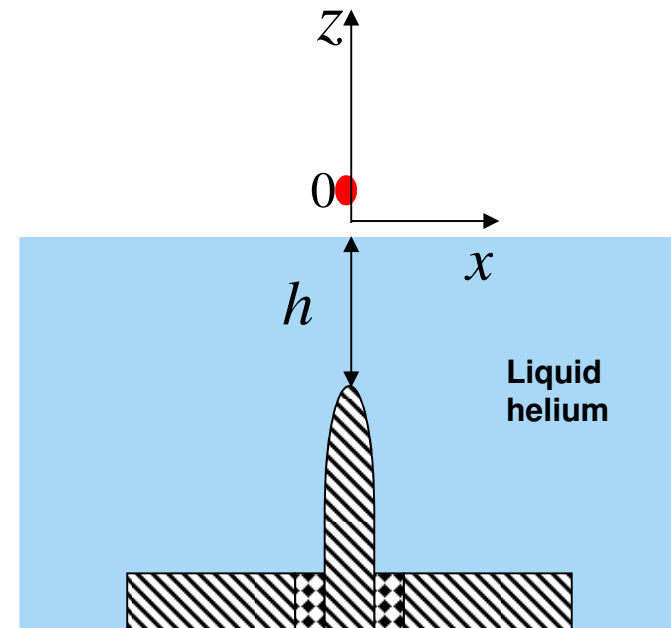
$$H_{\text{int}}(t) = \frac{1}{2} \begin{pmatrix} 0 & \Omega(t) \\ \Omega(t) & 2\Delta(t) \end{pmatrix}$$

$$|\lambda_+(t)\rangle = \sin \vartheta(t)|0\rangle + \cos \vartheta(t)|1\rangle$$

$$|\lambda_-(t)\rangle = \cos \vartheta(t)|0\rangle - \sin \vartheta(t)|1\rangle$$

$$\mu_{\pm} = [\Delta(t) \pm \sqrt{\Delta^2(t) + \Omega^2(t)}] / 2$$

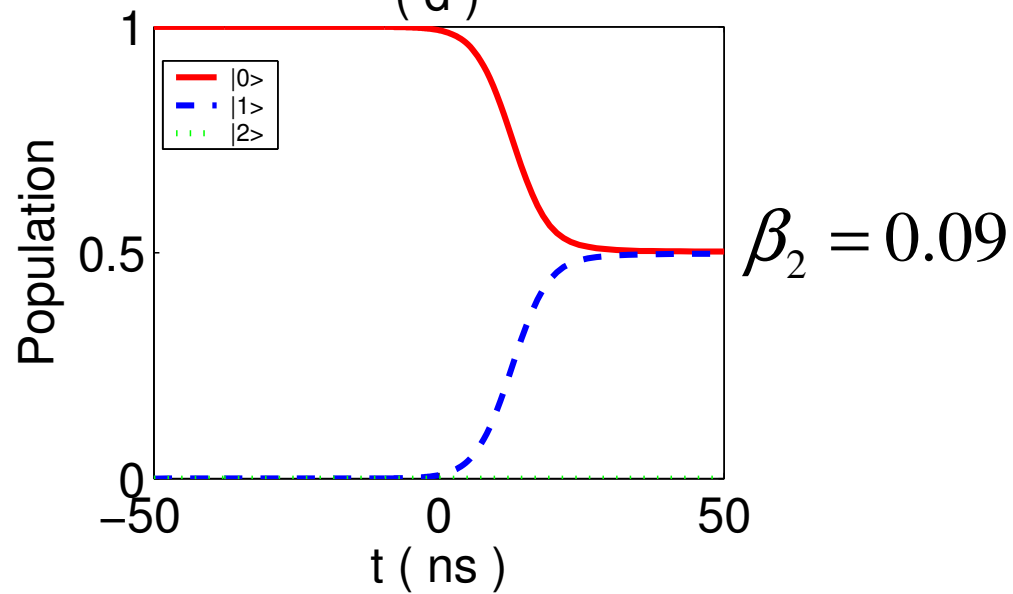
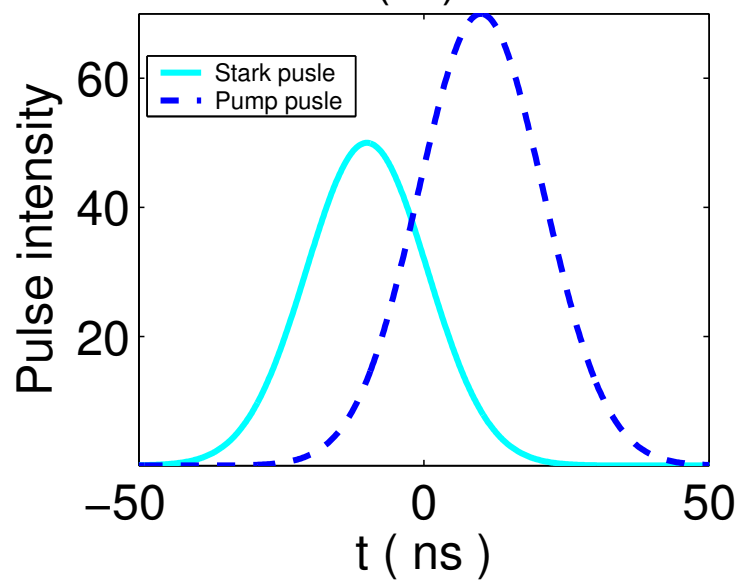
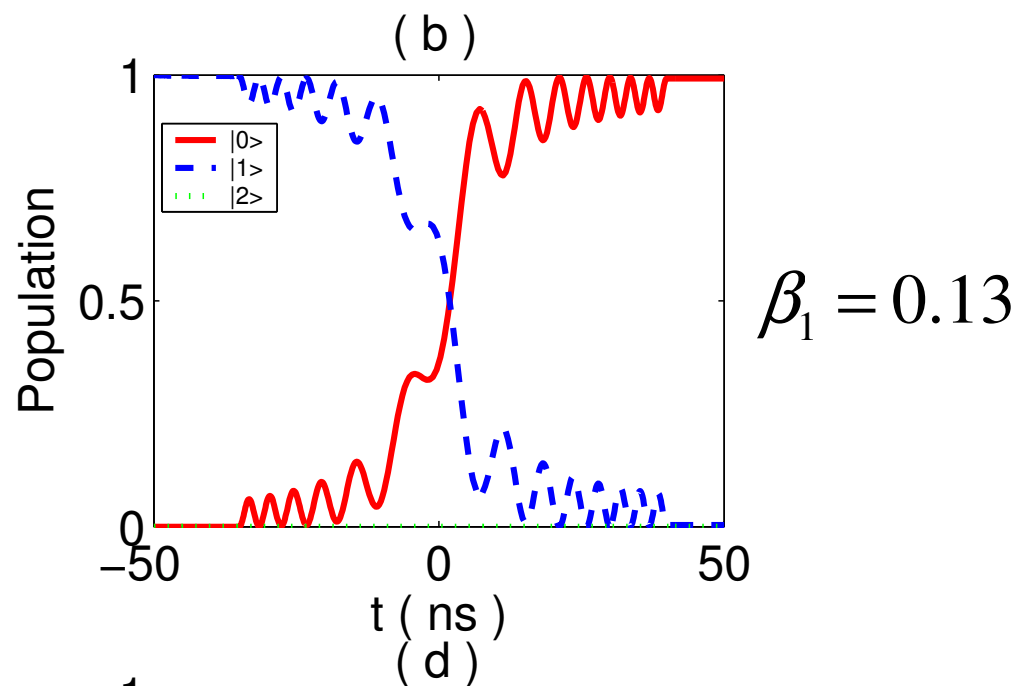
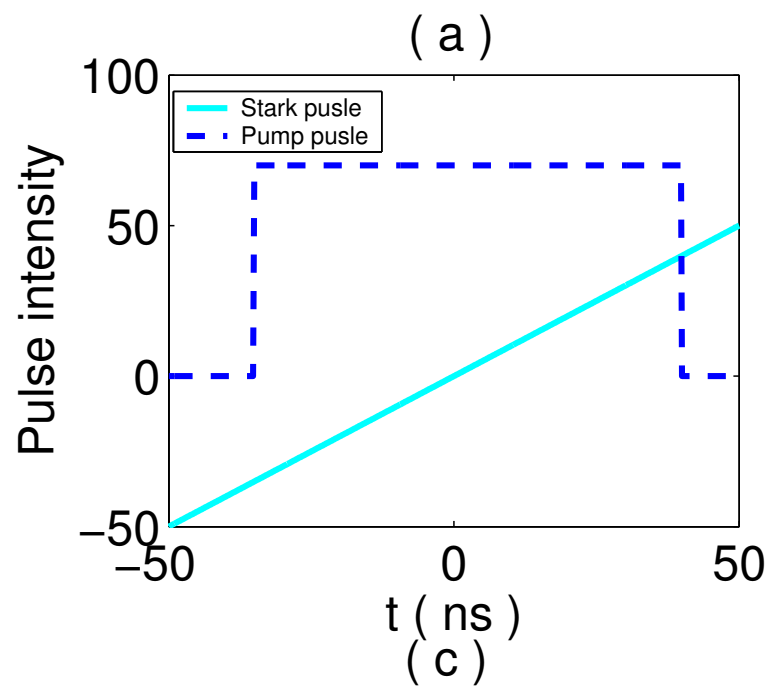
$$\tan 2\theta = \frac{\Omega(t)}{\Delta(t)}$$



Adiabatic parameter

$$\beta = \frac{1}{2} \frac{\left| \Omega(t) \frac{d\Delta(t)}{dt} - \Delta(t) \frac{d\Omega(t)}{dt} \right|}{[\Delta^2(t) + \Omega^2(t)]^{3/2}} \ll 1$$

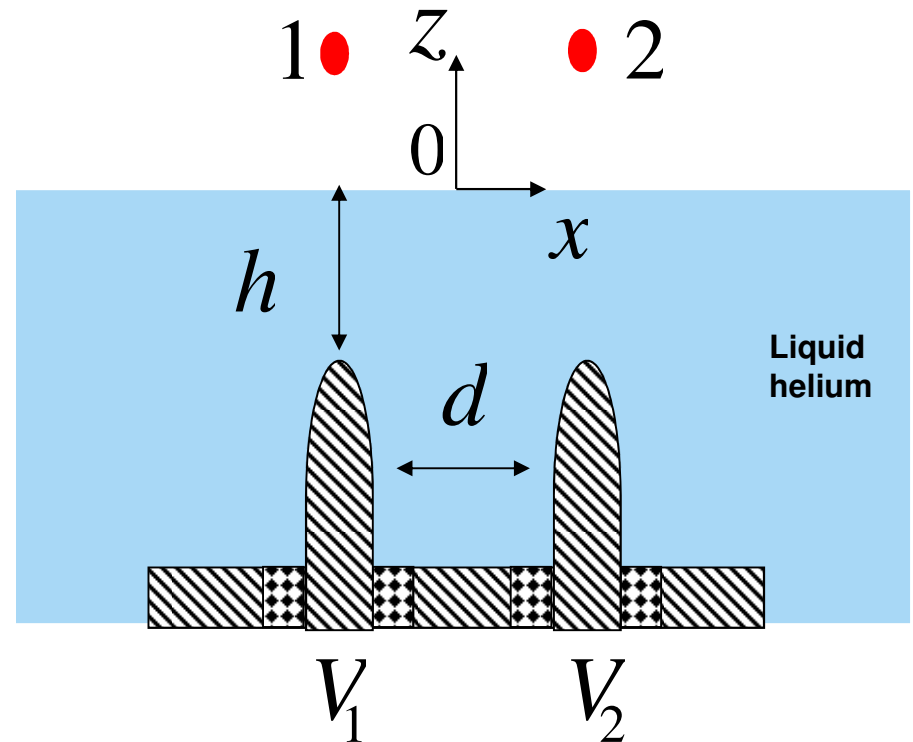
# Numerical simulations



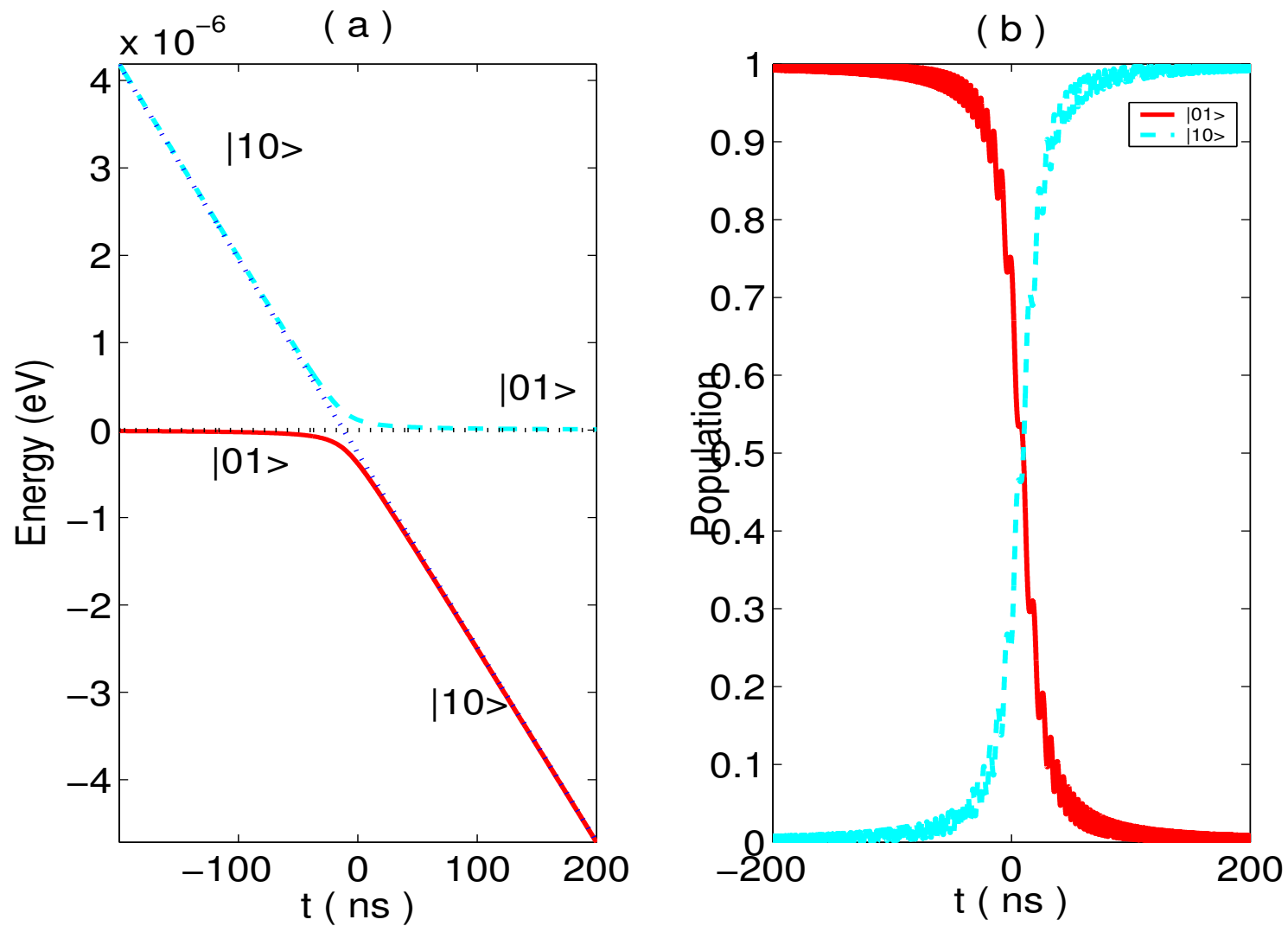
# Two-qubit gate by adiabatic population passages

In order to implement the general i-swap gate, we fix the field applied to the first qubit and change that applied to the second one, then the total Hamiltonian is

$$\hat{H}_{\text{Int}}(t) = \begin{pmatrix} \Delta_1 & 0 & 0 & 0 \\ 0 & \Delta_2 & -2\alpha z_1^{01} z_2^{10} e^{-it\omega} & 0 \\ 0 & -2\alpha z_1^{10} z_2^{01} e^{it\omega} & \Delta_3 & 0 \\ 0 & 0 & 0 & \Delta_4 \end{pmatrix}$$



# Numerical simulations



$$\beta = 0.1$$

# Evolution-time insensitive quantum manipulations: time-evolution operator

---

- We consider a two level system described by the Hamiltonian

$$\hat{H}(t) = A(t)\hat{\sigma}_z + B(t)\hat{\sigma}_x$$

$$\hat{U}(t) = e^{i\alpha(t)\hat{\sigma}_x} e^{i\beta(t)\hat{\sigma}_y} e^{i\gamma(t)\hat{\sigma}_z}$$

- The corresponding time evolution operator determined by

$$\dot{\alpha} = -A(t) \cos[2\alpha(t)] \tan[2\beta(t)] - B(t)$$

$$\dot{\beta} = A(t) \sin[2\alpha(t)]$$

$$\dot{\gamma} = -A(t) \frac{\cos[2\alpha(t)]}{\cos[2\beta(t)]}$$

$$\alpha(0) = \beta(0) = \gamma(0) = 0$$

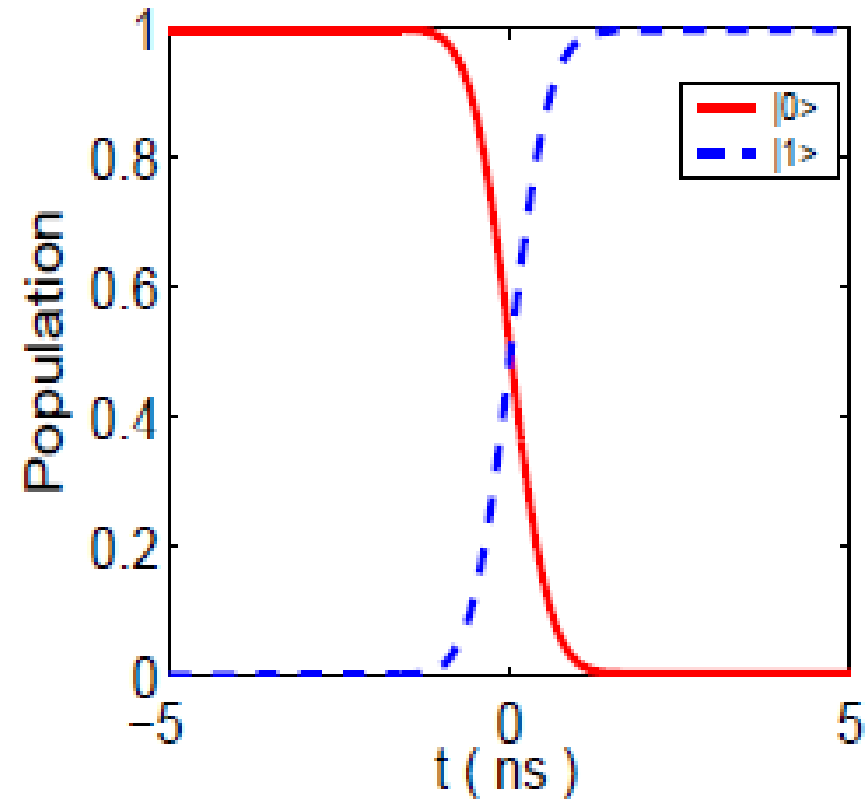


# Single-qubit quantum gate achieved non-adiabatically

$$\hat{H}_i = \frac{eE_{dc}(z_{11} - z_{00})}{2} \hat{\sigma}_z + \frac{e\xi(t)z_{01}}{2} \hat{\sigma}_x$$

$$\xi(t) = 270e^{-\left(\frac{t}{1 \times 10^{-9}}\right)^2}$$

$$E_{dc} = 10e^{-\left(\frac{t}{1 \times 10^{-10}}\right)^2}$$

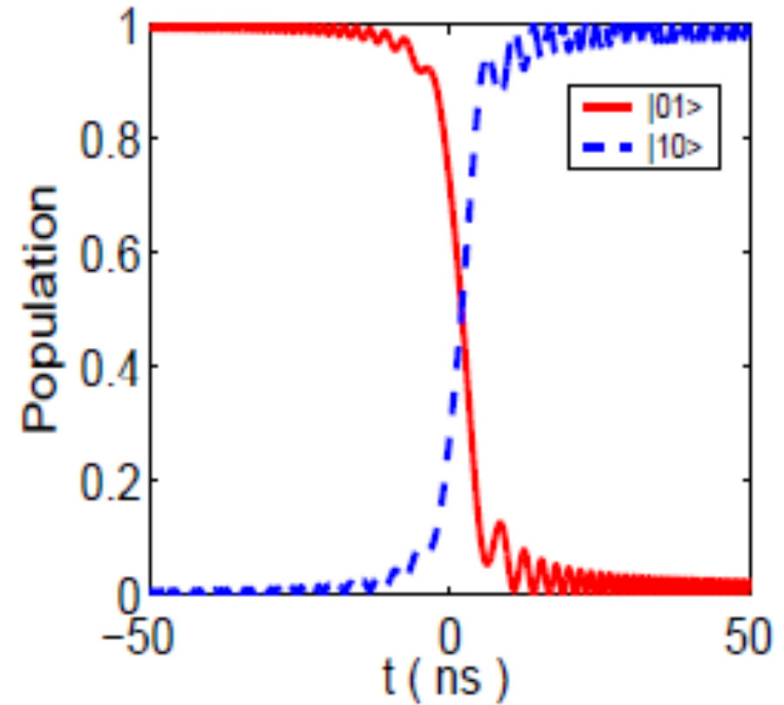


$$\beta_1 = 0.7$$

# Single-qubit quantum gate achieved non-adiabatically

The dynamics of the driven two-qubit system can be described by

$$\hat{H}_2 = \left[ \frac{\Delta'_2 - \Delta'_3}{2} - \frac{1}{2}\hbar\omega \right] (|01\rangle\langle 01| - |10\rangle\langle 10|) - 2\alpha(z_1^{01}z_2^{10}|01\rangle\langle 10| + |10\rangle\langle 01|).$$



$$\beta_2 = 0.5$$

# Application to quantum devices

## Double-frequency device

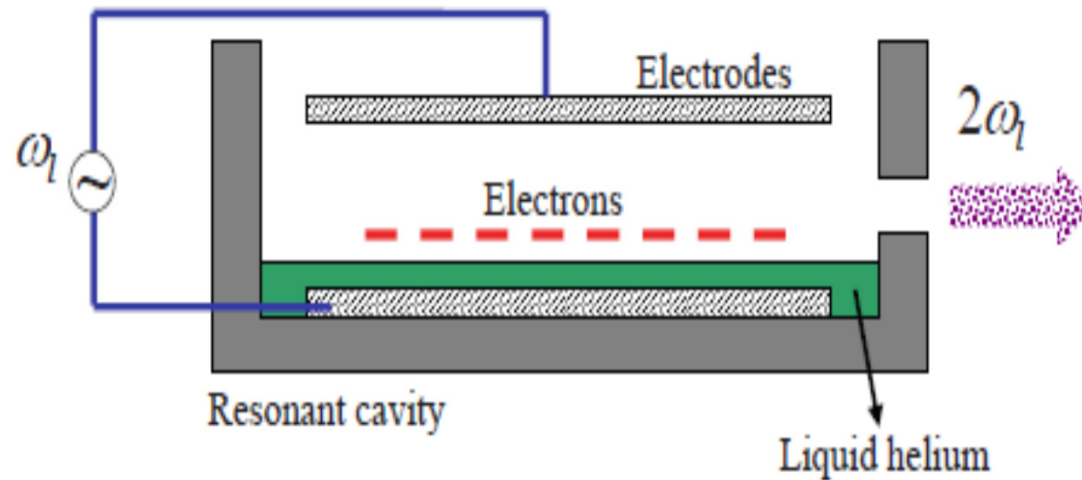
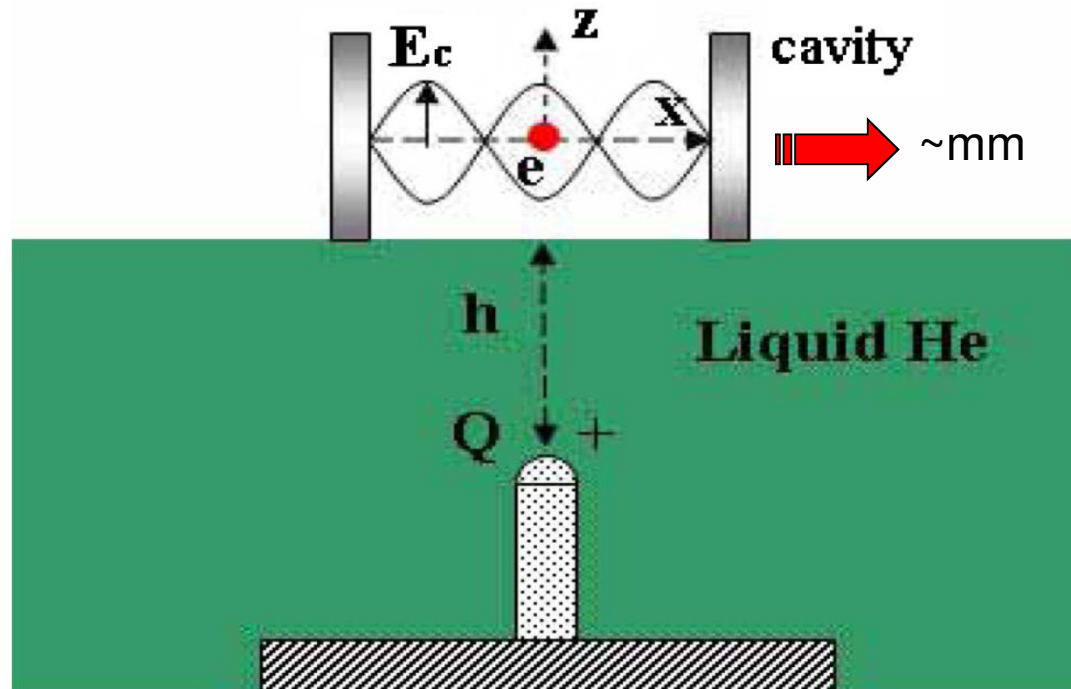


FIG. 3: A sketch of the experimental setup to obtain the frequency-doubling radiations.

M. Zhang et al., [arXiv:1004.2966v1](https://arxiv.org/abs/1004.2966v1)

# Application to quantum devices

## T(G)Hz radiation sources



M. Zhang et al., **Optics Letters**, 35 1686(2010)

# Future Works: with our experimental systems

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**Cryofree dilution refrigerator:  $<15\text{mK}$**

With electrons on Helium:

1. Quantum devices with EHs based on single-quantum manipulations
2. Investigate quantum phase transitions with EHs.

## Conclusions and Discussions

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- EEs could be manipulated coherently
- Quantum gates could be deterministically implemented by evolution-time insensitive population passages
- Quantum manipulations of EEs could be utilized to design novel quantum devices
- Readout of single-quantum state is a challenge for the current technique.

**Thank you  
and  
Welcome to Chengdu**

