Silicon Nitride NEMS at Low Temperatures: Mechanical and Thermal Properties

NPCQS 2012, 24th of April

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welcome to the European Microkelvin Collaboration

Outline

- Introduction to Nanoelectromechanical systems (NEMS)

- Motivation

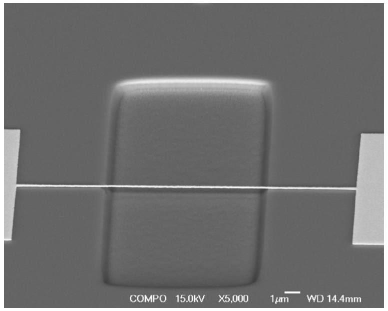
- Nanomechanics:
  - Nonlinear dynamics of a nanomechanical resonator: **<u>Mode coupling</u>**

- Low Temperature properties *(work in progress):*
  - Mechanical energy dissipation: **<u>Q-factor</u>**
  - Heat transport: **<u>Thermal conductivity</u>**

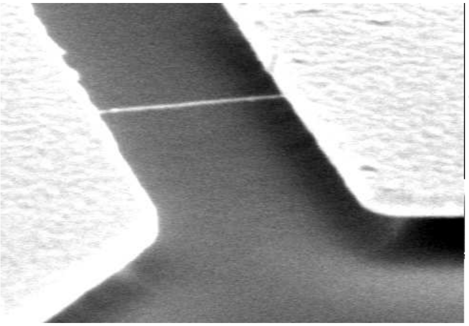
Introduction

- Mechanical vibrating system with cross-sections < 1 μm
  - → SHO model
  - → extremely low masses $< 10^{-15}$ kg
  - → wide resonant frequency range MHz - GHz
  - → materials Si, Si_3N_4, GaAs, diamond, CNTs, graphene, Al, Au…

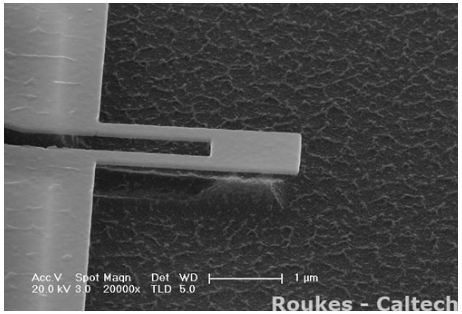
- Common geometries: doubly-clamped beams, cantilevers, membranes...



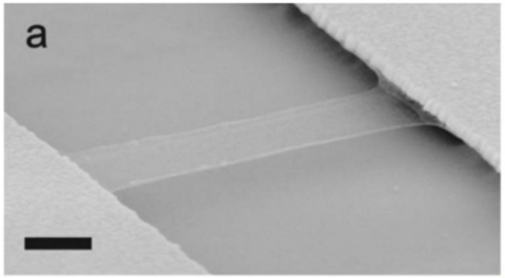
Gold doubly-clamped beam
A Venkatesan *et al*,
Phys. Rev. B **81** 073410 (2010)



CNT
B. Witkamp thesis, *TU Delft*
(2009)



Silicon carbide cantilever
M Li *et al*,
Nature Nanotechnology
Vol 2 (2007)

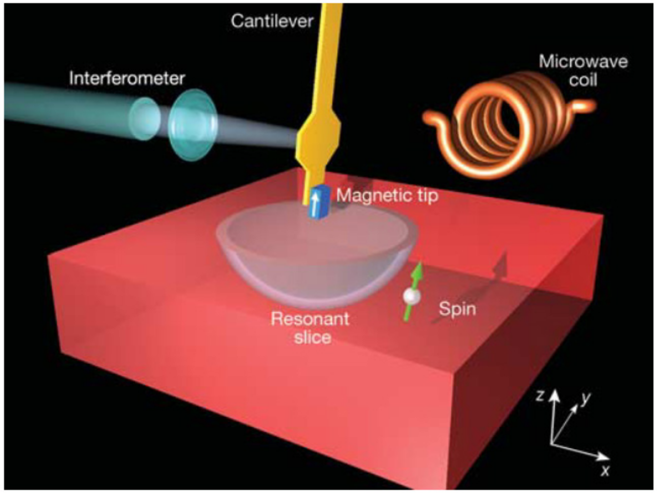


Graphene
A Eichler *et al*,
Nature Nano **Vol 6** (2011)

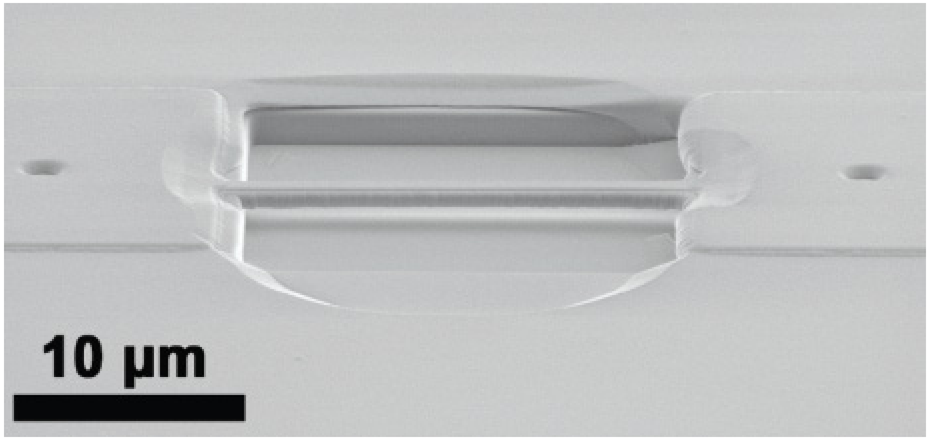
Motivation (I)

- Applications of NEMS

  → Excellent probes for ultrasensitive detection of force, mass, charge etc...



Single spin detection by MRFM
D Rugar *et al, Nature* **Vol 430** (2004)



Fabrication of a Nanomechanical Mass Sensor Containing a Nanofluidic Channel
R. A. Barton *et al, Nano Letters* **Vol 10** (2010)

Need to understand device properties such as dissipation, thermal, nonlinear (dynamic range) to improve sensitivities…

Motivation (II)

MACROSCOPIC WORLD

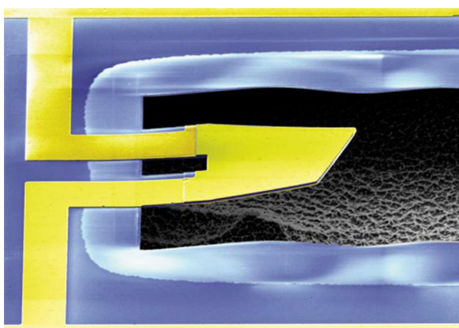
CLASSICAL

MESOSCOPIC WORLD

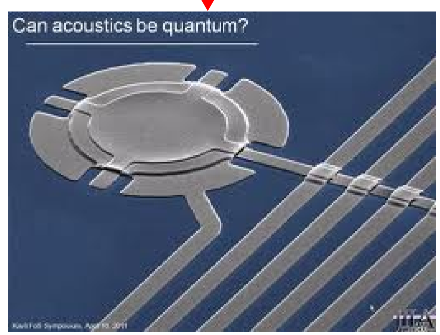
CLASSICAL ⇔ QM

ATOMIC WORLD

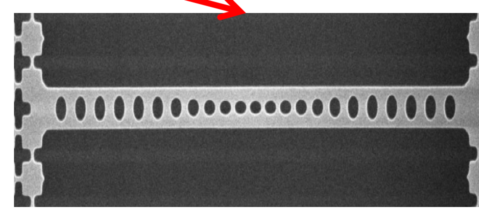
QM



Quantum ground state and single-phonon control of a mechanical resonator
A D O'Connell *et al, Nature* **Vol 464** (2010)



Sideband cooling of micromechanical motion to the quantum ground state
J. D. Teufel *et al, Nature* **Vol 475** (2011)



Observation of Quantum Motion of a Nanomechanical Resonator
A. H. Safavi-Naeini *et al, PRL* **Vol 108** (2012)

FOR QM REQUIRE $\underline{\hbar\omega > k_B T}$

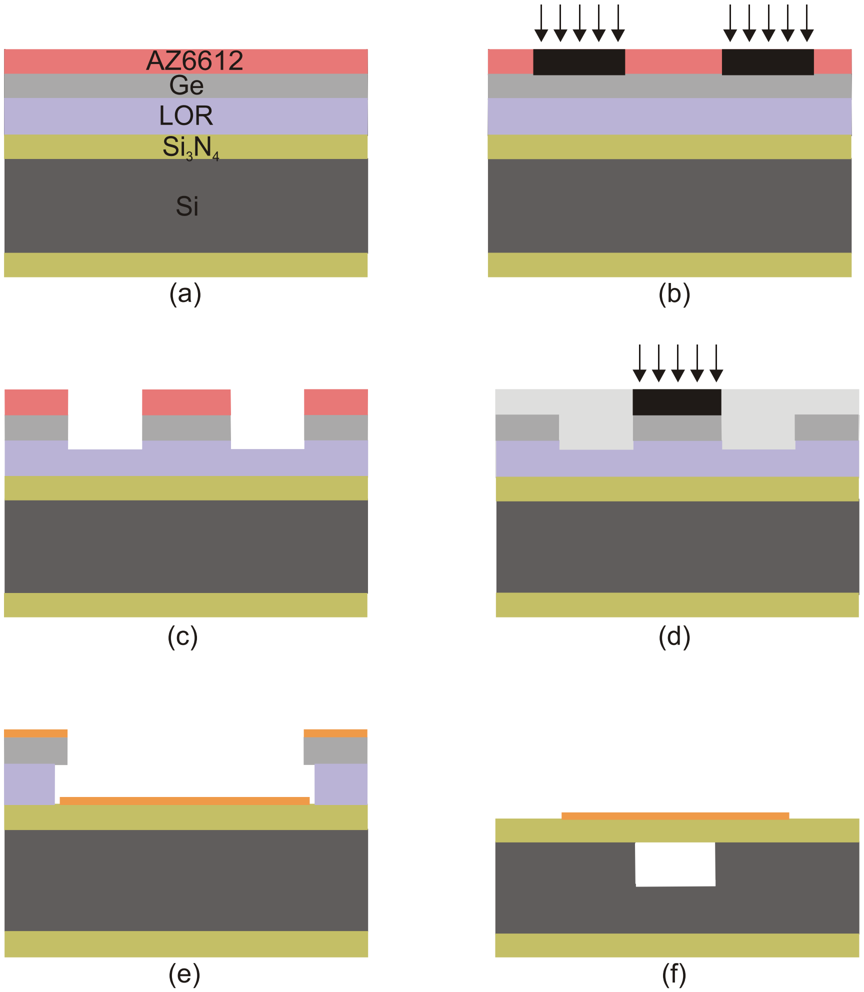
Nanomechanics : Mode Coupling

(a) Device fabrication
(b) Measurement technique
(c) Theory
(d) Results + Conclusions

Mode coupling + dissipation measurements

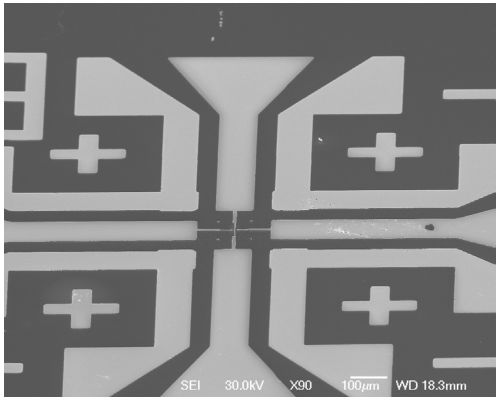
Device fabrication

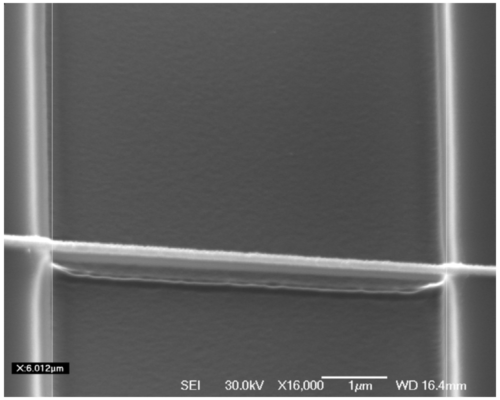
Gold-coated silicon nitride beams



Substrate: Si + Si_3N_4 (LPCVD)

HIGHLY STRESSED σ = 1 GPa !!!

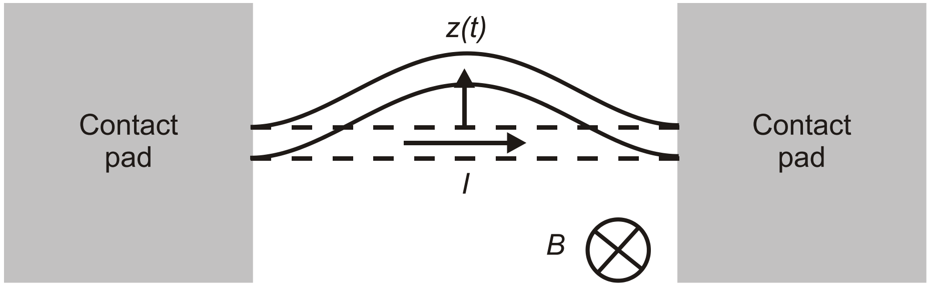




Measurement technique

Magnetomotive

Lorentz force generated perpendicular to both current *I* and magnetic field *B*.



Advantages: Simple + use of cryogenic vacuum
Disadvantages: Use of large magnetic fields affects measurement.

$$Q_L^{-1} = Q_0^{-1} + \beta B^2$$

Q-factor is loaded by external circuitry (Z_{ext}) and R_e of sample itself

$$\beta = \frac{l^2}{\omega_0 m} \frac{\Re(Z_{ext})}{\left|Z_{ext}{}^2\right|}$$

Ref : Cleland & Roukes, *Sens & Actuators* **72** , 256 (1999)

Silicon nitride: Low temperature properties

$$f_{0n} = \frac{\pi}{8}(2n+1)^2 \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}\sqrt{1+\frac{0.97 T_0 L^2}{(n+1)^2 \pi^2 EI}}$$

Length (µm)	Gold thick. (nm)	Mode (n)	f_0 (MHz)	Q_0
25.5	80	1	5.45	10^6
25.5	80	3	16.84	5×10^5
6.4	80	1	31.36	1×10^5
4.4	80	1	55.88	6×10^4
25.5	40	1	7.50	2×10^6
25.5	40	3	22.85	1×10^6
25.5	40	5	39.28	7×10^5

DISSIPATION

MODE COUPLING

Mode coupling: Theory

EOM

$$\underbrace{EI\frac{\partial^4 z}{\partial x^4}}_{\text{bending}} - \underbrace{\left[T_0 + \frac{EA}{2L}\int_0^L \left(\frac{\partial z}{\partial x}\right)^2 dx\right]\frac{\partial^2 z}{\partial x^2}}_{\text{tension}} + \underbrace{\gamma\frac{\partial z}{\partial t}}_{\text{damping}} + \underbrace{\rho A\frac{\partial^2 z}{\partial t^2}}_{\text{inertial}} = F_L$$

External force

<u>Only one mode driven at a time:</u>

Mode displacement $$z_n(x,t) = u_n(t)g_n(x)$$

Mode profile $$g_n(x) = c_{1n}\sinh\left(\frac{k_{1n}x}{L}\right) + c_{2n}\cosh\left(\frac{k_{1n}x}{L}\right) + c_{3n}\sin\left(\frac{k_{2n}x}{L}\right) + c_{4n}\cos\left(\frac{k_{2n}x}{L}\right)$$

Coefficients c_1... can be found applying boundary conditions for a doubly-clamped beam

$$g_n(0) = g_n(L) = \frac{\partial g}{\partial x}(0) = \frac{\partial g}{\partial x}(L) = 0$$

Ref: K. J. Lulla et al, arXiv:1204.4487

Can solve the EOM to obtain equation which is only time dependent

$$\ddot{u}_n + \frac{\omega_{0n}}{Q_n}\dot{u}_n + \omega_{0n}^2 u_n + \lambda I_{nn}^2 u_n^3 = f_n \cos(\omega_n t)$$

Nonlinear coefficient λI_{nn} where:

$$\lambda = \frac{Ew^2}{2\rho L^4} \qquad I_{nn} = \int_0^L g_n'(x) g_n'(x) dx$$

Assume a harmonic solution to solve EOM

$$u_n = a_n \cos(\omega_n t) + b_n \sin(\omega_n t)$$

The amplitudes a_n and b_n can be obtained through Fourier analysis

Ref: K. J. Lulla et al, arXiv:1204.4487

$$r_n = \sqrt{a^2 + b^2} = \frac{f_n}{\sqrt{\underbrace{\left(\omega_{0n}^2 + \frac{3}{4}\lambda I_{nn}^2 r_n^2 - \omega_n^2\right)^2}_{} + \left(\frac{\omega_n \omega_{0n}}{Q_n}\right)^2}}$$

Expanding to lowest order gives

$$\boxed{f_n^{(1)} = f_{0n} + \frac{3}{32\pi^2 f_{0n}} \lambda I_{nn}^2 r_n^2}$$

Key result: Duffing nonlinearity… 'frequency pulling'

Ref: K. J. Lulla et al, arXiv:1204.4487

For mode coupling experiments two modes (n and m) of the same beam driven simultaneously therefore

$$F_L = f_n \cos(\omega_n t) + f_m \cos(\omega_m t)$$

Once again solving the EOM gives a dispersion equation very similar to the single mode case

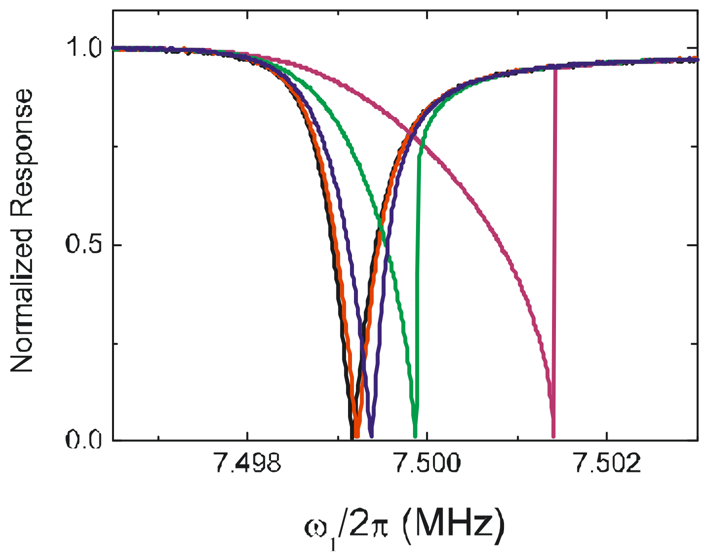
$$f_n^{(2)} = f_n^{(1)} + \frac{1}{8\pi^2 f_n^{(1)}} \lambda\left(\frac{1}{2} I_{nn} I_{mm} + I_{nm}\right) r_m^2$$

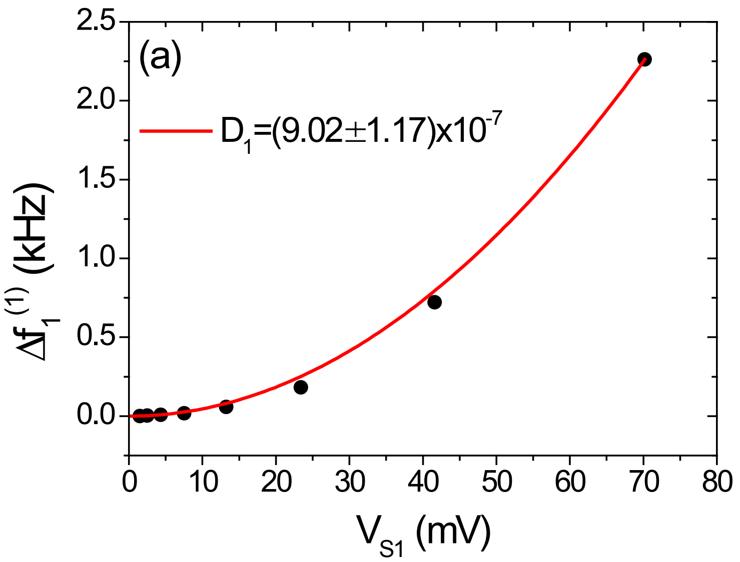
Frequency pulling effect, in a similar fashion to duffing type nonliearity (same source)

Ref: K. J. Lulla et al, arXiv:1204.4487

Results: Single mode

Experiments performed @ 100 mK, 3 T



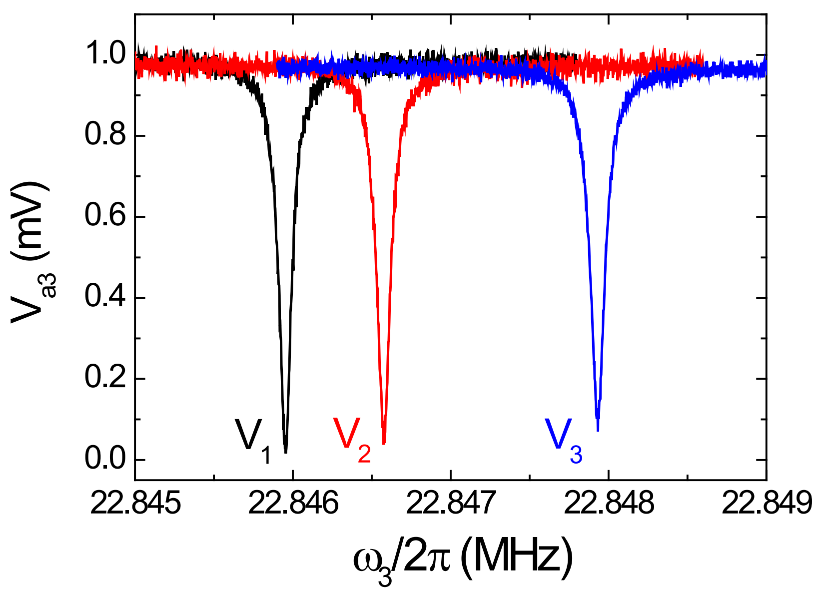


$V_{Sn} = A_n D_n r_n$ → Measured voltage related to the actual amplitude of the NEMS via constants *A* (related to material Known) and *D* (Gain factor in the transmission lines Unknown)

Ref: K. J. Lulla et al, arXiv:1204.4487

Results: mode coupling

Measured the frequency shift in mode *n* as a function of the amplitude of mode *m*.



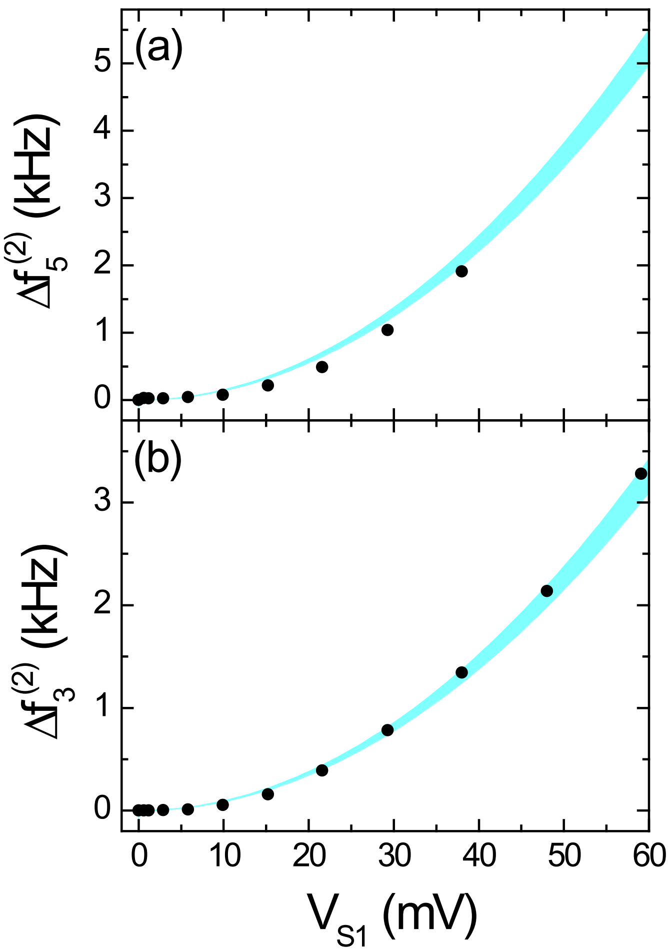
Modes 1 & 3 driven simultaneously:
→ Mode 3 drive kept constant
→ Mode 1 drive increased gradually

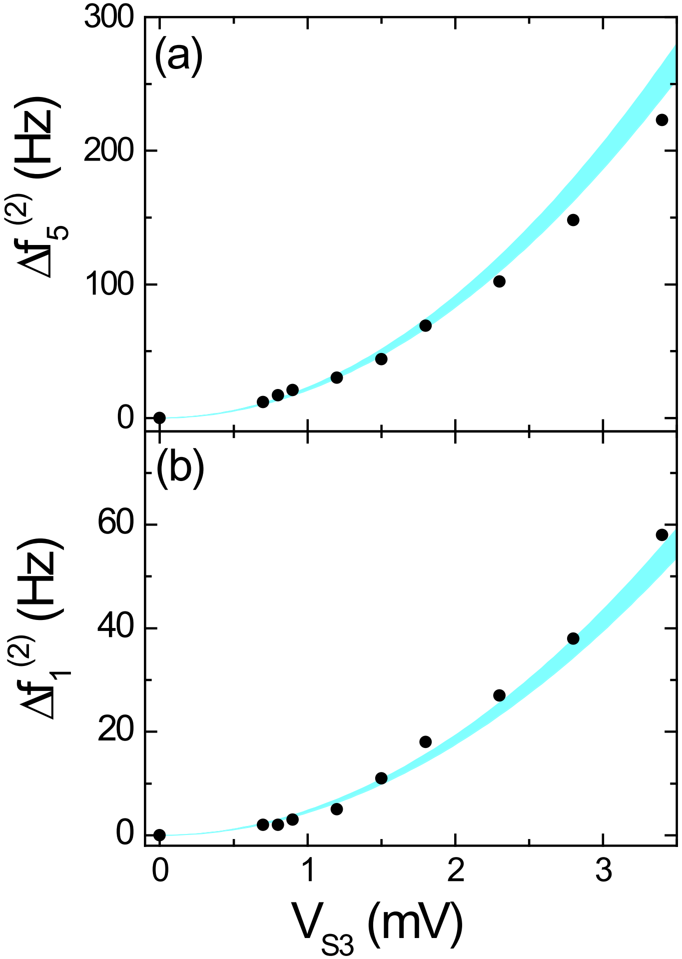
f_3 changes with V_{S1} however V_3 does not vary much, neither does its shape, ie. No energy transfer between modes.

Sensitivity: 7 Hz / nm

Ref: K. J. Lulla et al, arXiv:1204.4487

Results: mode coupling





Ref: K. J. Lulla et al, arXiv:1204.4487

Conclusions: mode coupling

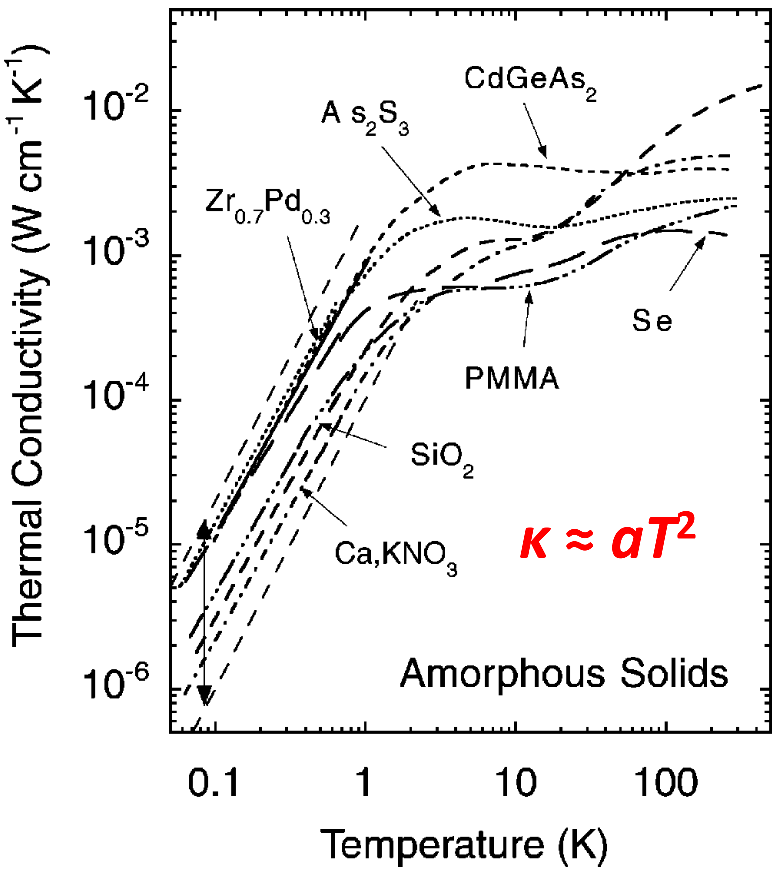
- Good agreement between measurements and theory, coupling between modes is due to the tension induced from the stretching of the beam.
- This inter-modal interaction is very important when describing the mechanical motion of highly-stressed nanomechanical resonators
- What happens at larger drives, is energy transfer between the different modes possible?
- Improve sensitivity by measuring resonators with even larger quality factors?

Ref: K. J. Lulla et al, arXiv:1204.4487

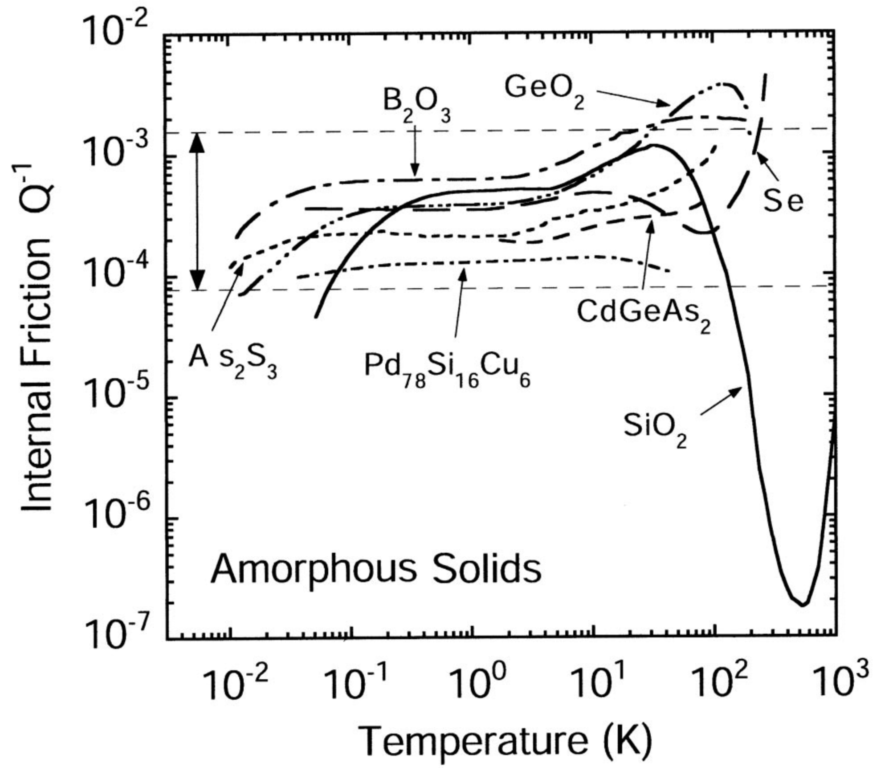
Dissipative and thermal properties at low temperatures

(a) Previous work (experimental + theoretical)
(b) Dissipation in silicon nitride
(c) Thermal conductivity in silicon nitride (measurement technique + results)
(d) Discussion

Previous work: Bulk systems



$\kappa \approx aT^2$



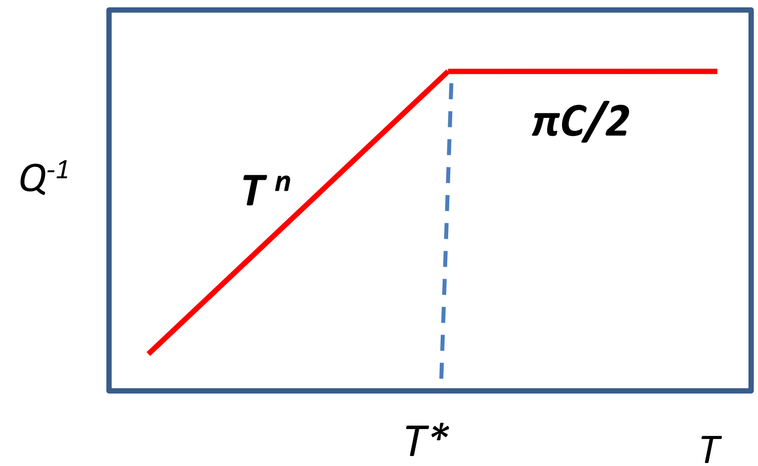
Universal behaviour→ Glassy range

Ref: R. O. Pohl et al, Low temperature thermal conductivity and acoustic attenuation in amorphous solids. *Rev. Mod. Phys.* **74** (2002)

Standard Tunnelling Model

Two-level systems (TLS) with broad distribution of energies *E* and relaxation times τ → **Anharmonic oscillations**

TLS ←→ electron/phonon interaction



Thermal conductivity (phonons):

$$\kappa(T) = \frac{\rho k_b^3}{6\pi\hbar^2}\frac{v}{P\gamma}T^2$$

Interaction	Q_0^{-1}
Resonant	Negligible when $\hbar\omega \ll k_B T$
Relaxation	aCT^n/ω ($\omega\tau_{min} \gg 1$) $\pi C/2$ ($\omega\tau_{min} \ll 1$)

n = 3 (phonons), 1 (electrons) ***C*** – Coupling strength

Ref: S Hunklinger and A K Raychaudhuri. *Progress in low temperature physics*, volume **IX**. North-Holland, 1986; W A Phillips. *Tunnelling states in amorphous solids*. J. Low Temp. Phys., **7**, 1987.

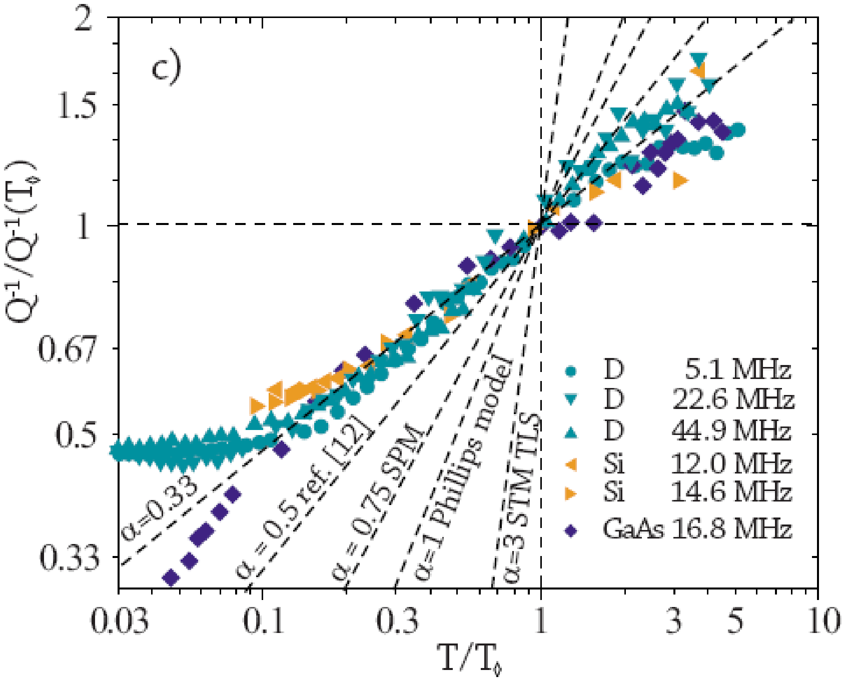
Previous work: Mesoscopic systems

Thermal conductance of silicon nanowires follows T^3 dependence as predicted by the Debye model for perfect crystals.

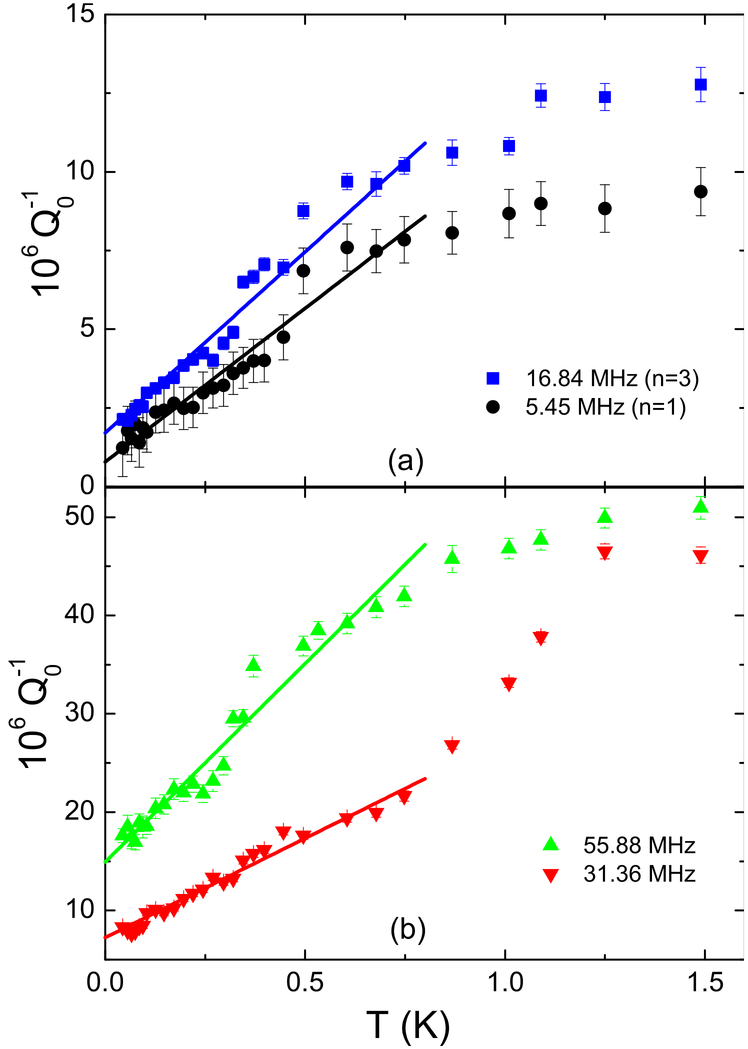
→ O. Bougeois et al., *JAP* **101**, 014104 (2007)

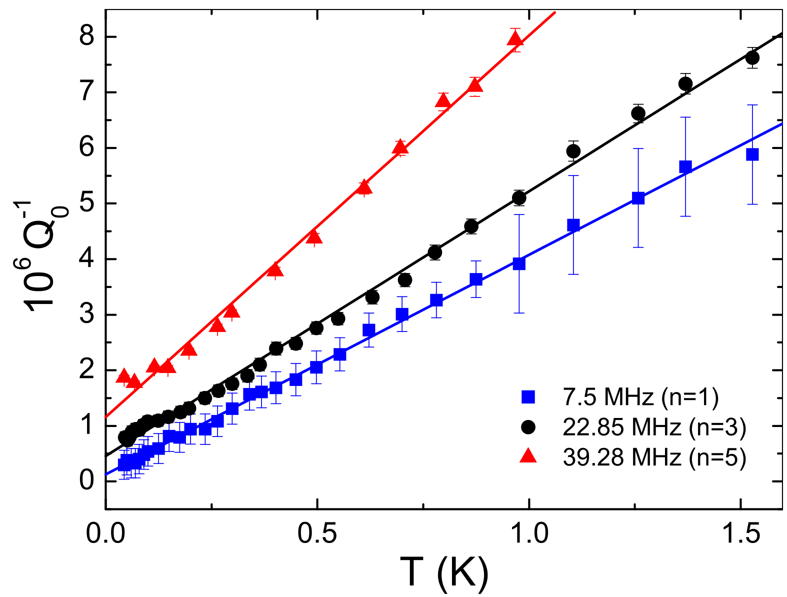
Dissipation measurements in various materials seems to show the same behaviour ... Universal signature in the dissipation in NEMS ?

→ M. Imboden and P. Mohanty, *PRB* **79**, 125424 (2009)



Dissipation vs Temperature



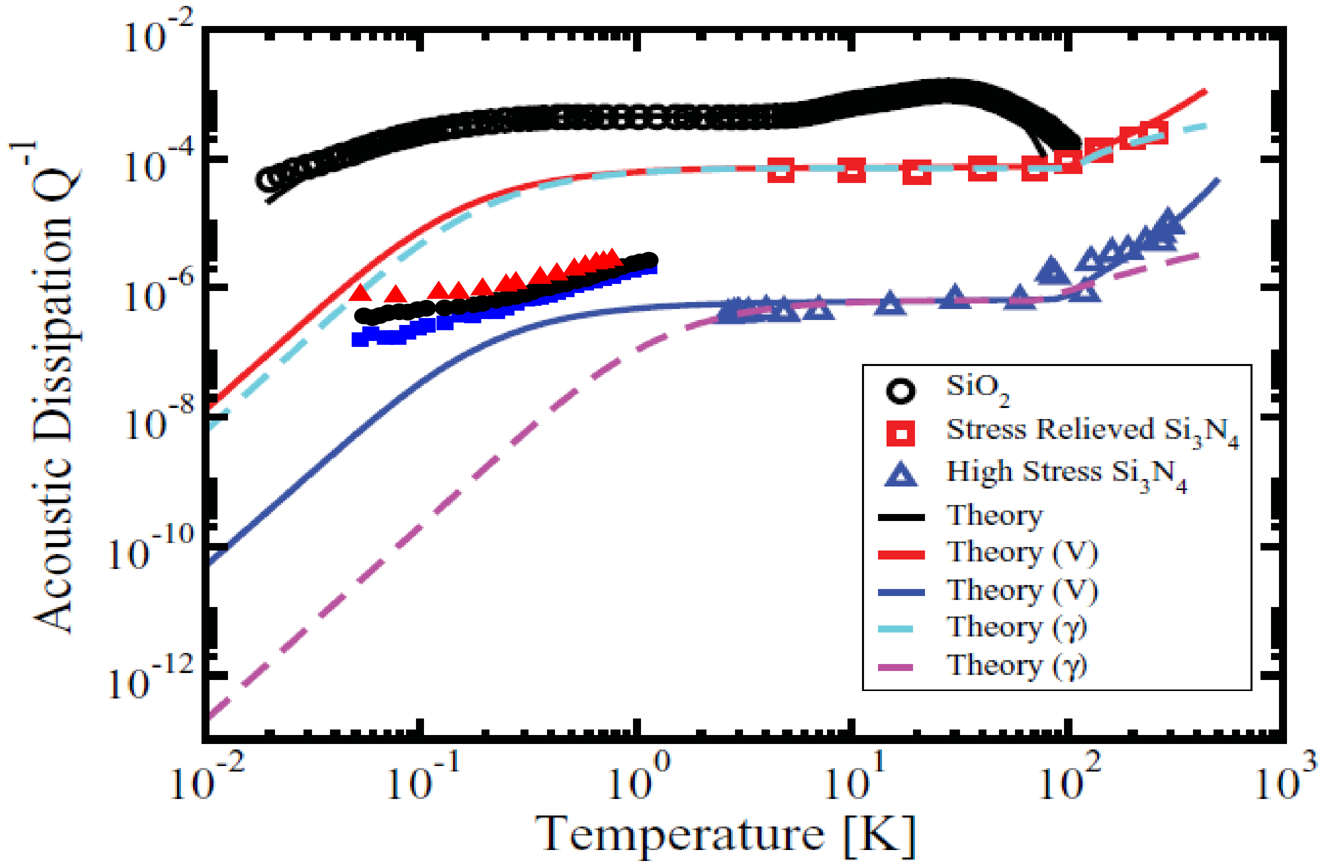


Straight lines correspond to the power law T^n where n=1

SiN + Al: $Q_0^{-1} \sim T$ (T < 600 mK)

Ref: T. Rocheleau et al., *Nature* **463** , 7 (2010)

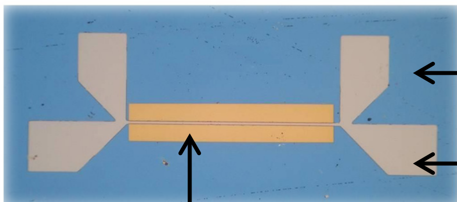
Dissipation: Comparison to theory



Ref: J. Wu and C. C. Yu, *How stress can reduce dissipation in glasses*, *PRB,* vol. 84, 174109 (2011).

Thermal measurements: Devices

Membranes

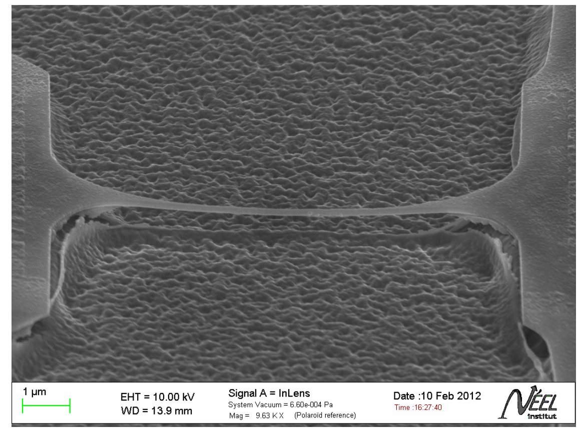


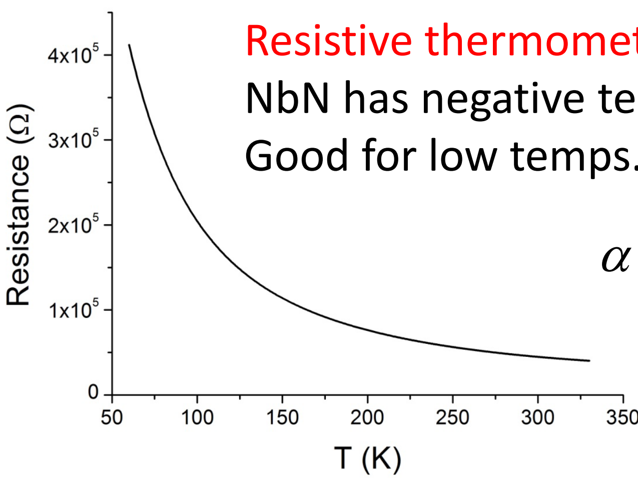
Silicon nitride

Thermometer (NbN)

Membrane thickness=100 nm

Doubly clamped beams



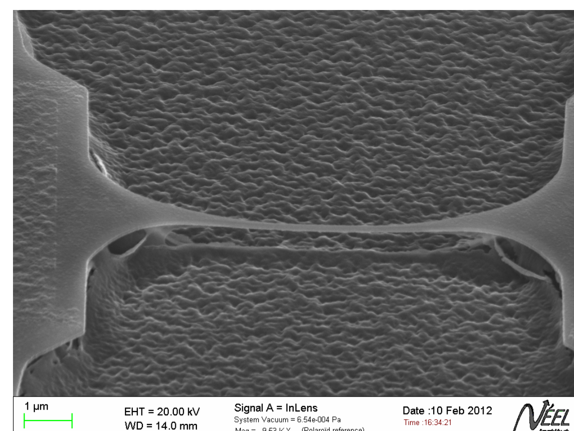


Resistive thermometry:

NbN has negative temp. Coeff.

Good for low temps.

$$\alpha = -\frac{1}{R}\frac{dR}{dT}$$



Thermal Measuremens: technique

The 3ω method

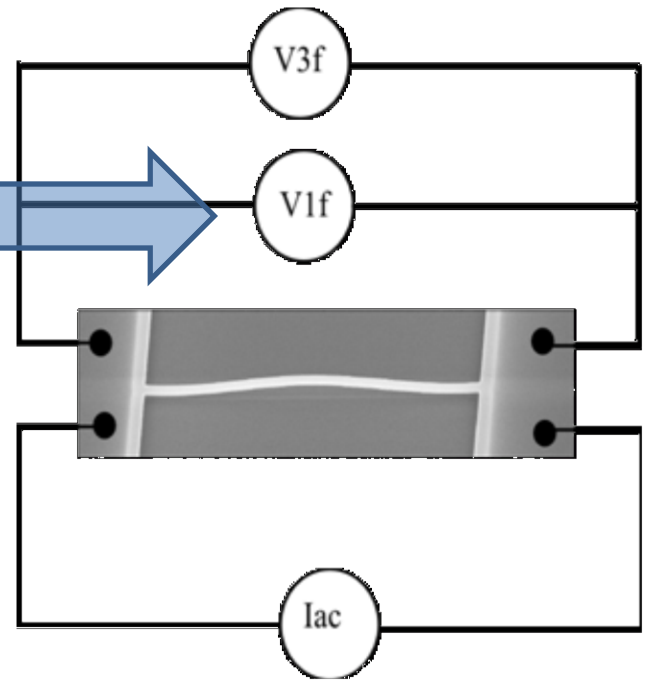
- Ac current passed through NbN transducer : $I_{ac} = A\sin(\omega t)$

- Generate oscillating *T* proportional to power dissipated : $I_{ac}^2 \rightarrow 2\omega$

- Resistance proportional to temperature $\rightarrow 2\omega$

- $V_{3\omega} = I_{ac}R \rightarrow 3\omega$ (at low freq → *K*)

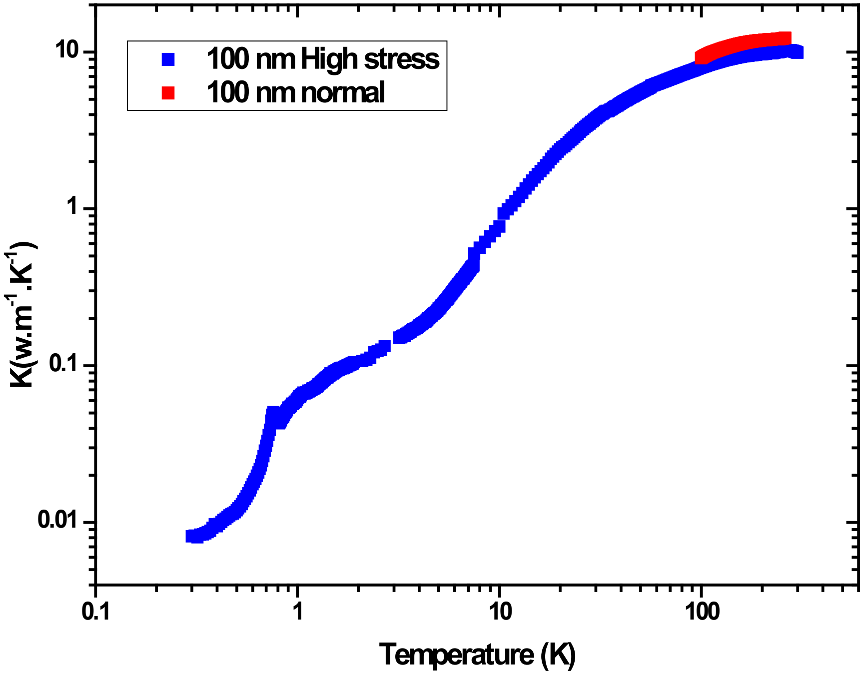
Disadvantage:
Large signal at ω however it can be filtered out using a Wheatstone bridge for example



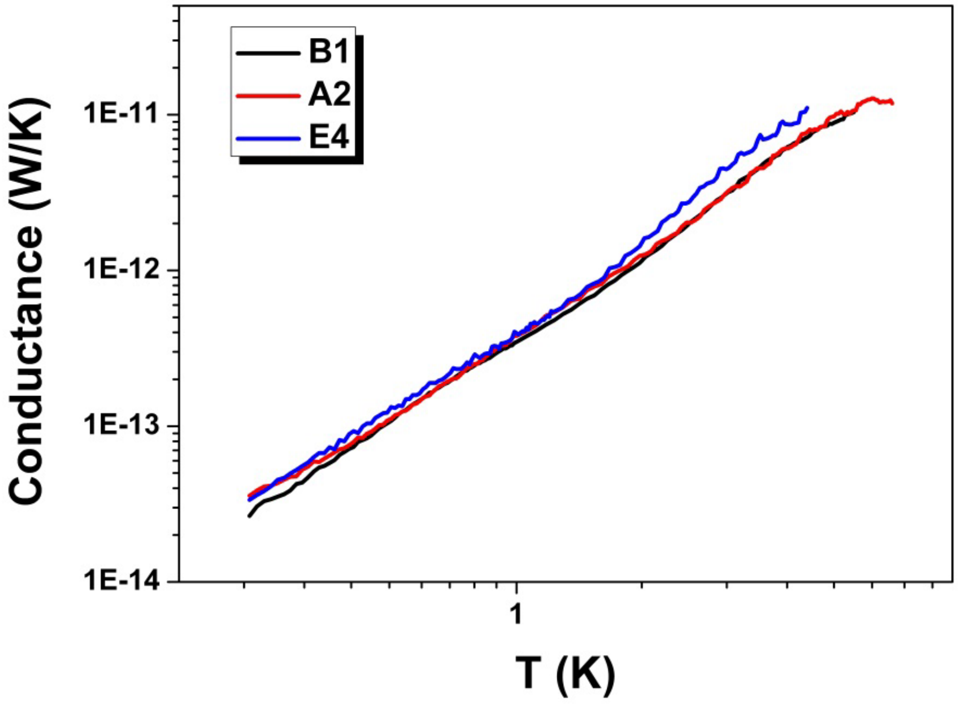
Ref: N. O. Birge and S. R. Nagel, Rev. Sci. Instrum. **58,** 1464 (1987)
L. Lu, W. Yi and D. L. Zhang, Rev. Sci. Instrum. **72**, 2996 (2001)

Thermal measurements: Results

Membranes



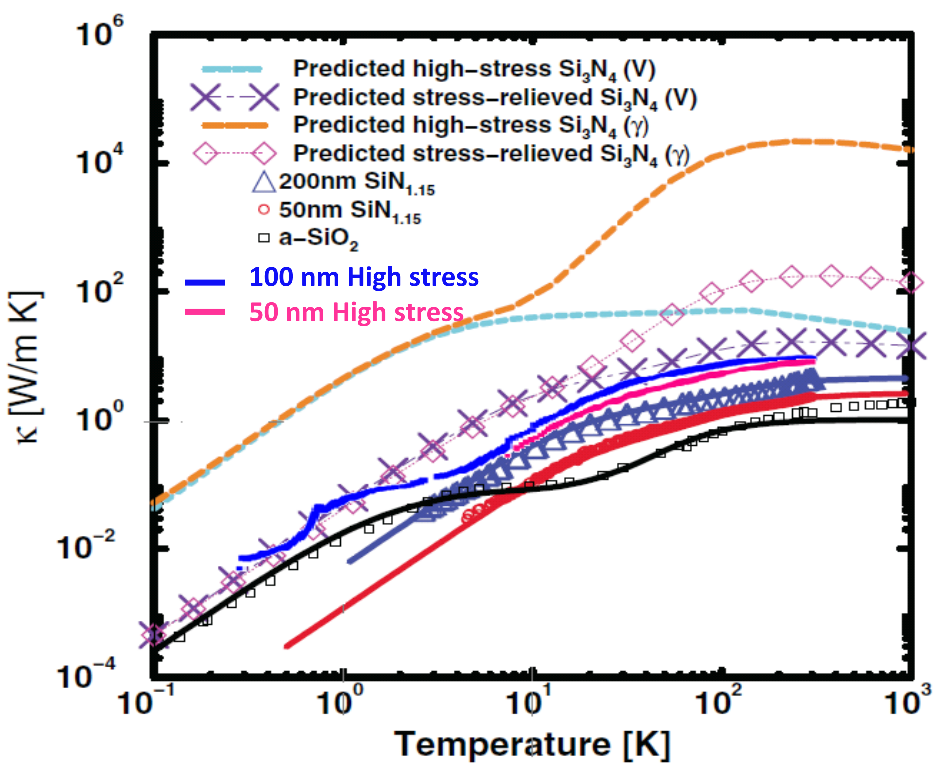
Beams



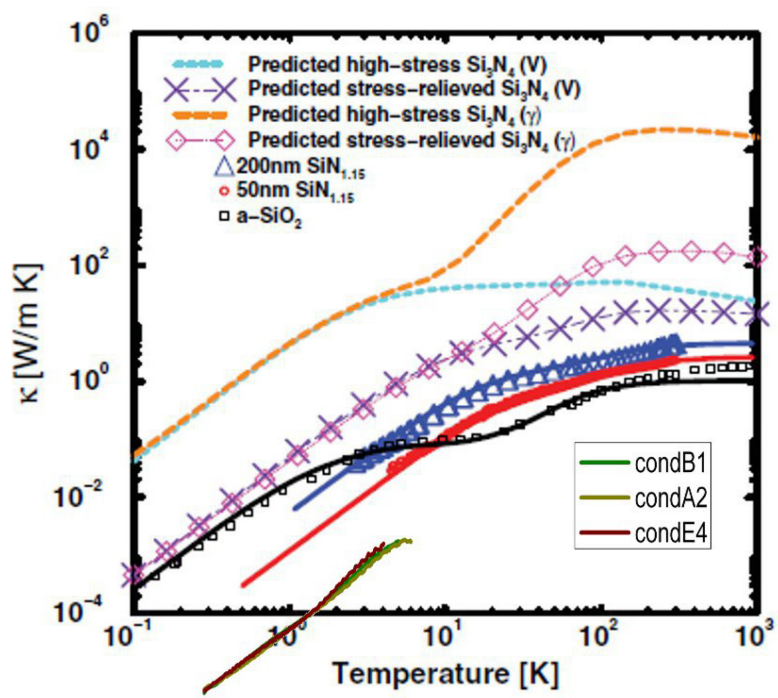
- For both structures **K ≈ aT^2**

Thermal conductivity: Comparison to theory

Membranes



Beams



Ref: J. Wu and C. C. Yu, *How stress can reduce dissipation in glasses,*
PRB, vol. 84, 174109 (2011).

Conclusions

DISSIPATION

- n = 1, STM indicates TLS-electron interaction (electrons in the gold layer??)
  → Previous measurements on gold resonators revealed a $T^{0.5}$

- Some features are reminiscent of TLS dominated dissipation however no quantitative agreement with theory

- The finite value of $Q_0^{-1}(0)$ = clamping loss?

THERMAL CONDUCTIVITY

- Preliminary results show that the thermal conductance varies as T^2 however it depends on size + geometry.

- Characteristic physical length scales play a key role
  1. Phonon mean free path (phonon transport ballistic or not?)
  2. Dominant phonon wavelength (a few hundreds of nm below 1K)

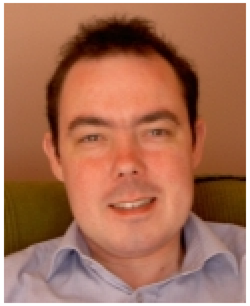
Summary + Future work

- Highly stressed silicon nitride leads to higher resonance frequencies and much larger Q-factors than other NEMS of similar dimensions.
- **Mode coupling:** We understand! (At least in the regime we have studied)
- **Thermal measurements + dissipation :** We do not understand!
- Keep measuring !!
- Measure low stress nitride samples to quantify the effect of the stress
- Come up with a theory which unifies TLS, dimensionality and stress

Acknowledgements

Nottingham

Prof. J. Owers-Bradley



Dr. A. Armour

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Grenoble

Dr. E. Collin

Dr. O. Bourgeois

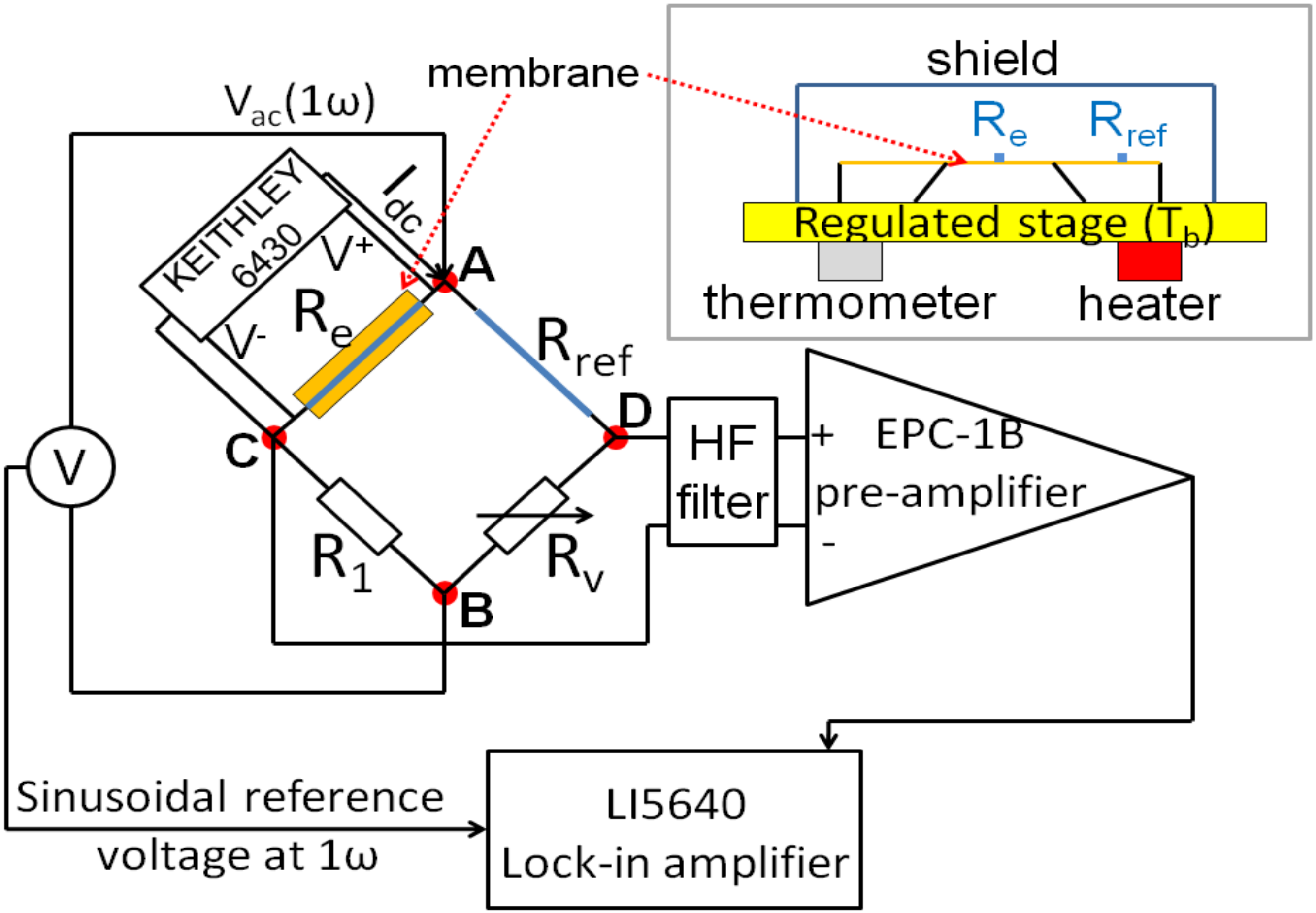
M. Defoort

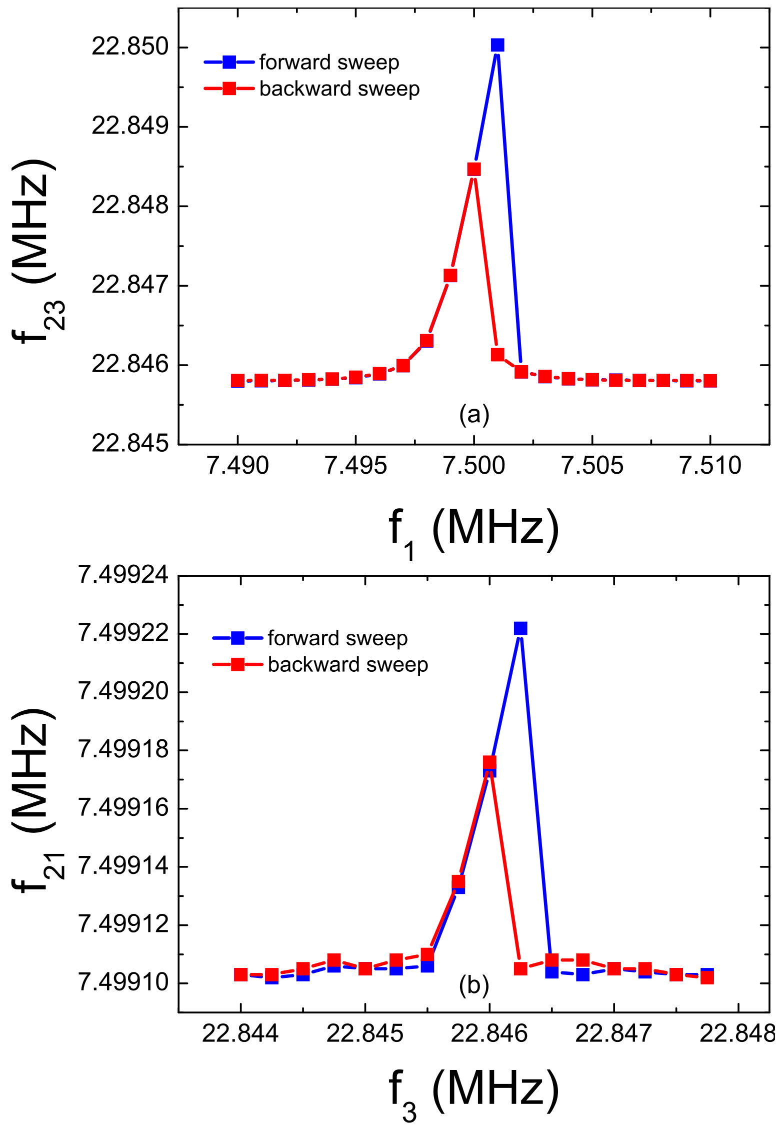
C. Blanc

H. Ftouni

Thank you !

3-ω method + wheatstone bridge set-up





forward sweep
backward sweep
(a)
22.850
22.849
22.848
22.847
22.846
22.845
7.490
7.495
7.500
7.505
7.510
f_{23} (MHz)
f_1 (MHz)
forward sweep
backward sweep
(b)
7.49924
7.49922
7.49920
7.49918
7.49916
7.49914
7.49912
7.49910
22.844
22.845
22.846
22.847
22.848
f_{21} (MHz)
f_3 (MHz)