OIST OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY

Lecture 4: Implementations and the Future

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QU²

Implementations

Aim: to implement quantum information processing

→ build machines that can carry out quantum communication and quantum computing

Challenges: systems are very small, imperfections, decoherence, limited experience in quantum engineering techniques

→ we are getting better at controlling single quantum particles, decoherence free subspaces, quantum error correction

Many candidates for physical implementations of quantum information processing have been identified and more are added regularly.

cold atoms and ions
NMR
quantum dots

SQUIDS
floating electrons
CQED
...

Goals

Goals:

- ➡ encode qubits in physical system
- ➡ process these qubits
- ➡ have a universal set of gates for quantum computation
- ➡ quit readout (measurement)
- ➡ store qubits
- ➡ minimize decoherence
- ➡ correct errors

Problems:

- ➡ qubit may be in Hilbert space of more than two dimensions (truncate)
- ➡ coupling to environment can lead to decoherence (isolate)
- ➡ imperfect gates don’t deliver precisely the desired transformation
- ➡ preparation has to be reliable, no defects
- ➡ readout has to be technically feasible

Criteria

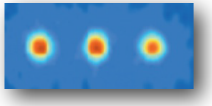
DiVincenzo Criteria (for a good implementation of a quantum computer)

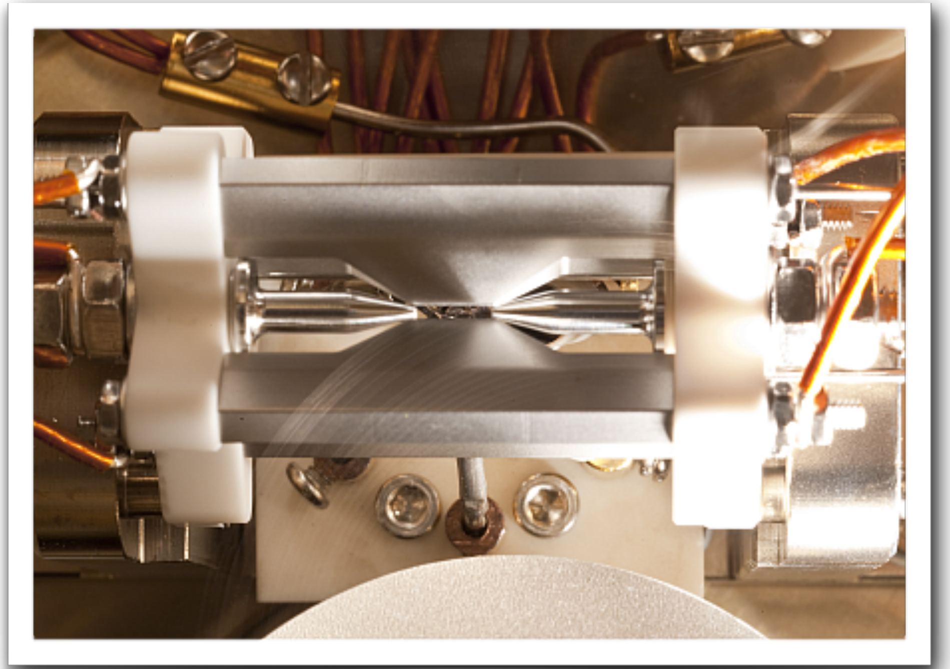
1. Scalability
2. Ability to initialise
3. Long decoherence times
4. Universal set of quantum gates
5. Qubit measurement capability

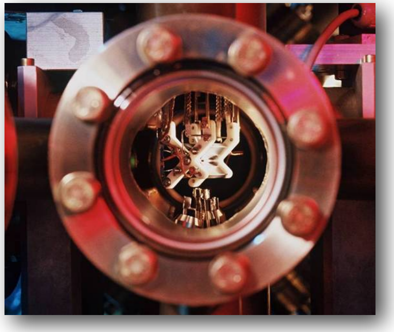
Additional Criteria

6. Interconvertibility between physical qubits
7. Faithfully transmit flying qubits

Trapped Ions







Trapped Ions

- trapped ions were a system identified early for realising QIP
  - well isolated (vacuum), initialised in ground state

- quantum information is stored in internal energy levels
  - long lifetimes, spectroscopic technologies advanced

- ion to ion coupling is obtained via collective harmonic motion of the crystal, which is quantised
  - free lunch for qubit-qubit coupling

4 out of 5 criteria ✓

Quick Reminder about notation:

- angular momentum of ion is the vector sum of spin and orbital angular momentum

$$\vec{J} = \vec{s} + \vec{L}$$

- spectroscopic notation: $^{2s+1}\{L\}_J$ with

$$L = 0 \rightarrow S$$
$$L = 1 \rightarrow P$$
$$L = 2 \rightarrow D$$

Atom Field Interactions

Absorption
(coherent)

Stimulated Emission
(coherent)

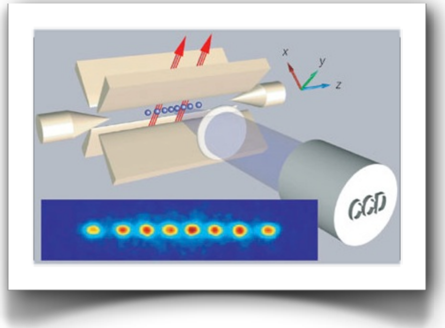
emitted photon is copy of the *trigger* photon

Spontaneous Emission
(incoherent)

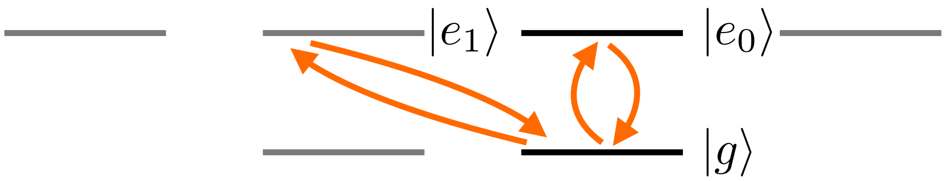
emitted photon is random in direction and phase

Cirac Zoller Proposal

- take *N* ions in a linear trap, each interacting with a single laser beam
- Iions are confined by a harmonic potential in each directions, with the frequency in the longitudinal direction much less than in the transversal ones → linear configuration



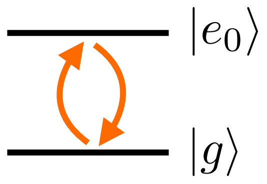
- excitation of dipole forbidden transition ${}^2S_{\frac{1}{2}} \to {}^2D_{\frac{5}{2}}$ → long lifetimes!
- typical level scheme:



- choose transition to $|e_0\rangle$ or $|e_1\rangle$ using different polarisation

Single Qubit Rotation

- tune laser field and polarisation to transition $|g\rangle \rightarrow |e_0\rangle$



interaction strength

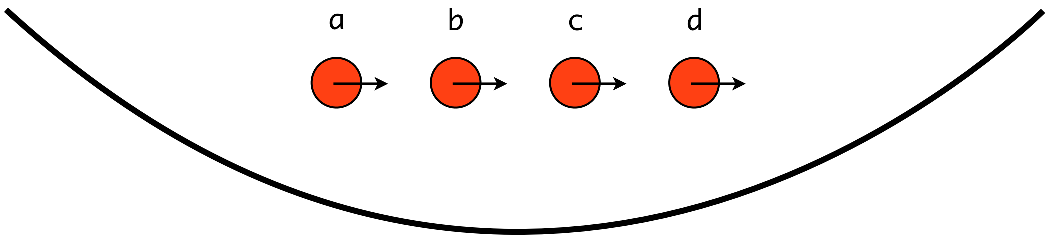
laser phase

$$\hat{H}_n = \frac{\Omega}{2}\left[|e_0\rangle_n\langle g|e^{-i\phi} + h.c.\right]$$

for the nth ion

- for evolution time $t = \frac{k\pi}{\Omega}$ (pi-pulse)
  - $|g\rangle_n \rightarrow \cos(k\pi/2)|g\rangle_n - ie^{i\phi}\sin(k\pi/2)|e_0\rangle_n$
  - $|e_0\rangle_n \rightarrow \cos(k\pi/2)|e_0\rangle_n - ie^{-i\phi}\sin(k\pi/2)|g\rangle_n$

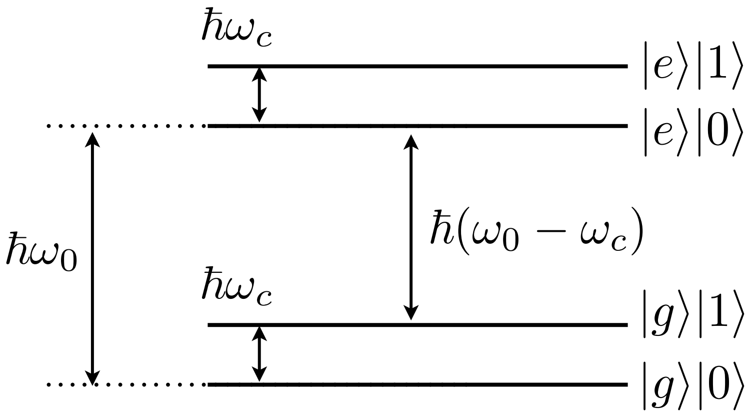
Interaction Hamiltonian



$$|\psi\rangle = |g_a, g_b, e_c, g_d, e_\nu\rangle$$

- → cool ions down, so that vibrational modes are quantised
- → centre of mass mode has common amplitude and phase

Energy levels for a single atom



- → note that a red-detuned laser with $\omega_L = \omega_0 - \omega_c$ acting on $|e\rangle|0\rangle$ will put the whole chain into the first excited mode!

Unitary Evolution

- laser acting on the n^{th} atom is detuned by centre-of-mass motion excitation energy
- change of electronic state is accompanied by creation/annihilation of one phonon

$$\hat{H}_{nq} = \frac{\eta}{N}\frac{\Omega}{2}\left[|e_q\rangle_n\langle g|\hat{a}e^{-i\phi} + h.c.\right]$$

$$\eta = \sqrt{\frac{\hbar^2 k^2 \cos^2\theta}{2m\nu_x}}$$

- apply laser for time $t = \dfrac{k\pi}{\Omega\eta/N}$ gives the evolution operator

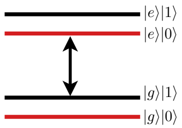
$$U_n^{k,q}(\phi) = \exp\left[-ik\frac{\pi}{2}\left(|e_q\rangle_n\langle g|\hat{a}e^{-i\phi} + h.c.\right)\right]$$

which has no effect on $|g\rangle_n|0\rangle$ and only couples

$$|g\rangle_n|1\rangle \rightarrow \cos(k\pi/2)|h\rangle_n|1\rangle - ie^{i\phi}\sin(k\pi/2)|e_q\rangle_n|0\rangle$$

$$|e_q\rangle_n|0\rangle \rightarrow \cos(k\pi/2)|e_q\rangle_n|0\rangle - ie^{-i\phi}\sin(k\pi/2)|g_n\rangle_n|1\rangle$$

Two Qubit Gate



Basis States:

$$|0\rangle|0\rangle \quad |0\rangle|1\rangle \quad |1\rangle|0\rangle \quad |1\rangle|1\rangle$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$|g_a\rangle|g_b\rangle|0\rangle \quad |g_a\rangle|e_b\rangle|0\rangle \quad |e_a\rangle|g_b\rangle|0\rangle \quad |e_a\rangle|e_b\rangle|0\rangle$$

1. apply laser pulse of frequency $\hbar(\omega_0 - \omega_c)$

$\hat{U}_a$:

$$|g_a\rangle|g_b\rangle|0\rangle \rightarrow |g_a\rangle|g_b\rangle|0\rangle$$

$$|g_a\rangle|e_b\rangle|0\rangle \rightarrow |g_a\rangle|e_b\rangle|0\rangle$$

→ no effect

$$|e_a\rangle|g_b\rangle|0\rangle \rightarrow -i|g_a\rangle|g_b\rangle|1\rangle$$

$$|e_a\rangle|e_b\rangle|0\rangle \rightarrow -i|g_a\rangle|e_b\rangle|1\rangle$$

→ phase can be adjusted by length of laser pulse $\longrightarrow e^{i\omega t}$

Two Qubit Gate

2. apply laser pulse to change phase of atom *b* if $|g_b\rangle|1\rangle$

(use additional level outside scheme)

$$\hat{B}_b : \quad |g_a\rangle|g_b\rangle|0\rangle \longrightarrow |g_a\rangle|g_b\rangle|0\rangle$$

$$|g_a\rangle|e_b\rangle|0\rangle \longrightarrow |g_a\rangle|e_b\rangle|0\rangle$$

$$-i|g_a\rangle|g_b\rangle|1\rangle \longrightarrow +i|g_a\rangle|g_b\rangle|1\rangle$$

$$-i|g_a\rangle|e_b\rangle|1\rangle \longrightarrow -i|g_a\rangle|e_b\rangle|1\rangle$$

3. apply first gate again

$$\hat{U}_a : \quad |g_a\rangle|g_b\rangle|0\rangle \longrightarrow |g_a\rangle|g_b\rangle|0\rangle$$

$$|g_a\rangle|e_b\rangle|0\rangle \longrightarrow |g_a\rangle|e_b\rangle|0\rangle$$

$$+i|g_a\rangle|g_b\rangle|1\rangle \longrightarrow |e_a\rangle|g_b\rangle|0\rangle$$

$$-i|g_a\rangle|e_b\rangle|1\rangle \longrightarrow -|e_a\rangle|e_b\rangle|0\rangle$$

Two Qubit Gate

full effect of the gate

$$\hat{W} = \hat{U}_a \hat{V}_b \hat{U}_a :$$

$$|g_a\rangle|g_b\rangle|0\rangle \rightarrow |g_a\rangle|g_b\rangle|0\rangle$$

$$|g_a\rangle|e_b\rangle|0\rangle \rightarrow |g_a\rangle|e_b\rangle|0\rangle$$

$$|e_a\rangle|g_b\rangle|0\rangle \rightarrow |e_a\rangle|g_b\rangle|0\rangle$$

$$|e_a\rangle|e_b\rangle|0\rangle \rightarrow -|e_a\rangle|e_b\rangle|0\rangle$$

← phase change!

to make a CNOT we consider

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|g_b\rangle \pm |e_b\rangle\right) \rightarrow$$

$$\hat{W}|g_a\rangle|\pm\rangle = |g_a\rangle|\pm\rangle$$

$$\hat{W}|e_a\rangle|\pm\rangle = |e_a\rangle|\mp\rangle$$

← inverts!

CNOT Gate

→ **Hadamard** $\hat{H}|e\rangle = |-\rangle$ $\hat{H}|g\rangle = |+\rangle$

→ $\hat{C} = \hat{H}\hat{W}\hat{H}$

$$\hat{C}|g_a\rangle|g_b\rangle = |g_a\rangle|g_b\rangle$$

$$\hat{C}|g_a\rangle|e_b\rangle = |g_a\rangle|e_b\rangle$$

$$\hat{C}|e_a\rangle|g_b\rangle = |e_a\rangle|e_b\rangle$$

$$\hat{C}|e_a\rangle|e_b\rangle = |e_a\rangle|g_b\rangle$$

→ depending on the state of ion *a*, the state of ion *b* gets inverted (negated)

$$\hat{C} = CNOT$$

Other ideas for implementation

System	Quantum Bit
SQUIDs	currents running in two directions
Superconductors	number of electrons on a superconducting island
NMR	spin in inhomogeneous environments
Quantum Dots	electronic states of dot (artificial atoms)
Electrons on liquid He	electron spins
Cavity QED	number of photons in a cavity
Neutral atoms	internal states
Linear Optics	position of the photon

→ about 40 qubits needed to rival current classical machines

Future

Any large scale overhaul of information technology in the 21st century has to include Quantum Information!

→ bigger and better devices

→ new algorithms

→ commercialisation

→ deeper understanding of concepts like entanglement

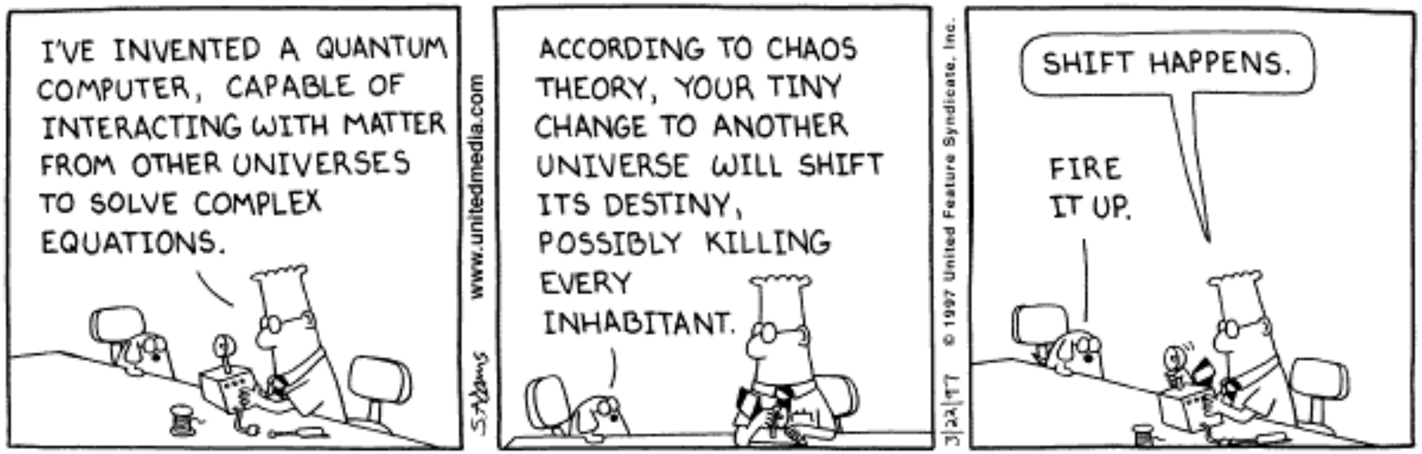
And finally....

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b')$$

All expectation values are bound by ± 1 , but QM predicts $S_{\max} = 2\sqrt{2}$

→ why is correlation strength limited?

Title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