Lecture 3A: Grover's Search Algorithm

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What is a search algorithm?

Searching is an important task that modern computers do.

Classically, the only way to do a search is to systematically examine all the possibilities until you find the solution.

If the search space has N entries, then the time taken to complete a search is $O(N)$ (on average, $N/2$). No knowon classical algorithm can do better than this.

Grover's algorithm works in time $O(\sqrt{N})$

Setting up the problem

1. We wish to search through a list of N elements, y_x
2. Each element has an index x in the range 0 → N – 1
3. Assume $N = 2^n$, so we can store x in n qubits
4. The search problem has M solutions, where 1 ≤ M ≤ N
5. Rather than dealing with the list itself, we focus on the index of the list, x
6. Key idea: given some value of x , we can tell whether y_x solves the search problem. We assume that we can construct some device for the task and call that as ORACLE

Algorithm

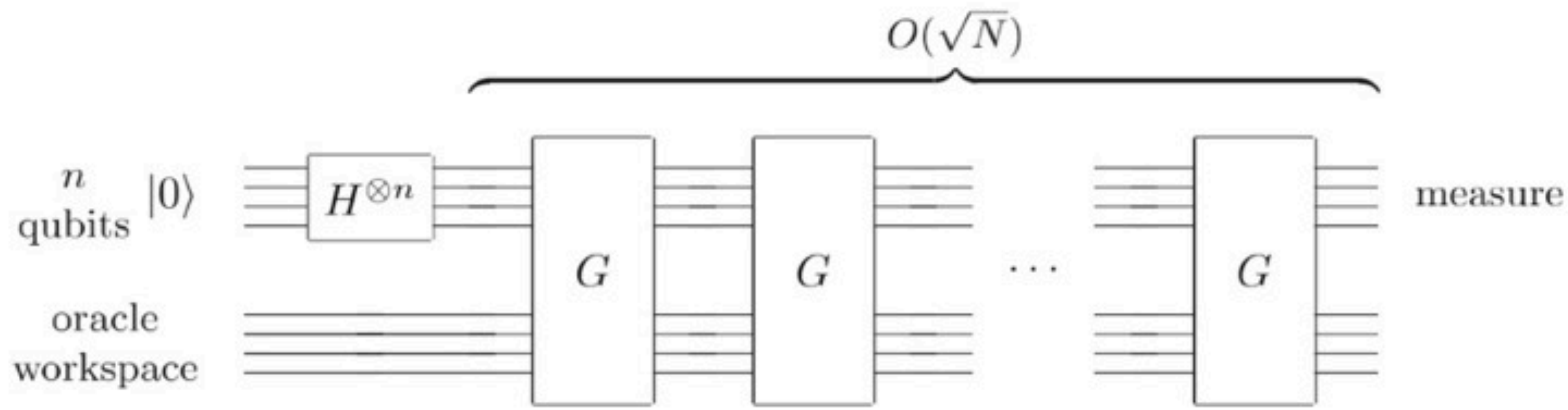

Figure 6.1. Schematic circuit for the quantum search algorithm. The oracle may employ work qubits for its implementation, but the analysis of the quantum search algorithm involves only the n qubit register.

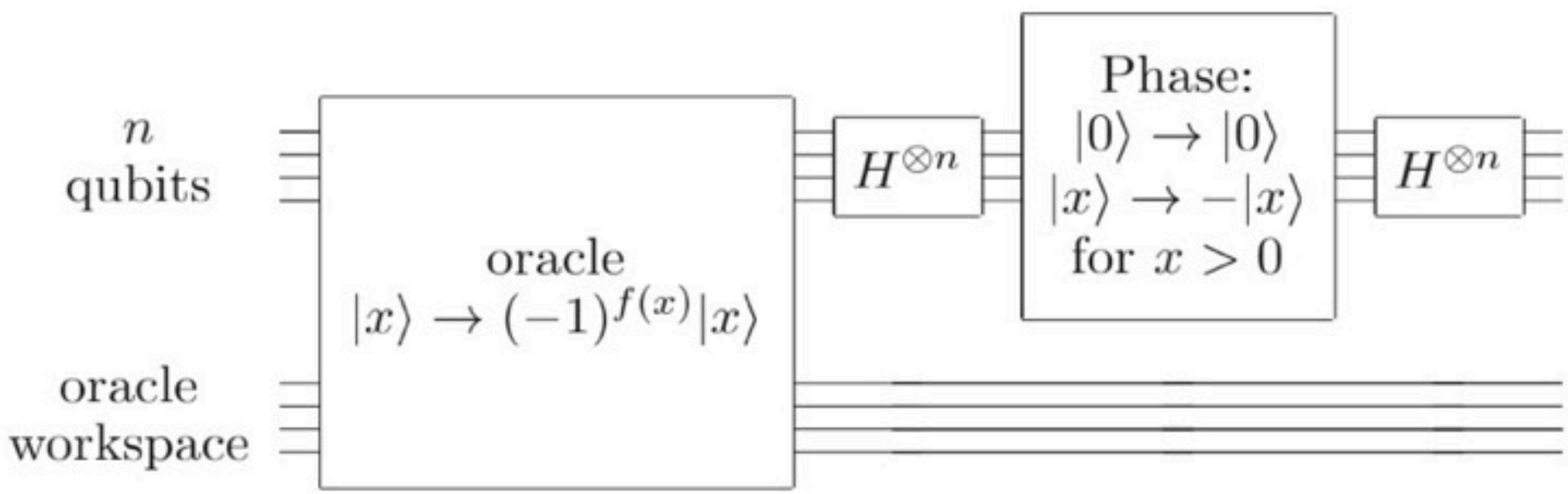

Figure 6.2. Circuit for the Grover iteration, G.

Algorithm

Algorithm: Quantum search

Inputs: (1) a black box oracle O which performs the transformation $O|x\rangle|q\rangle = |x\rangle|q \oplus f(x)\rangle$, where $f(x) = 0$ for all $0 \le x < 2^n$ except x_0, for which $f(x_0) = 1$; (2) $n + 1$ qubits in the state $|0\rangle$.

Outputs: x_0.

Runtime: $O(\sqrt{2^n})$ operations. Succeeds with probability $O(1)$.

Procedure:

1. $|0\rangle^{\otimes n}|0\rangle$	initial state
2. $\rightarrow \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$	apply $H^{\otimes n}$ to the first n qubits, and HX to the last qubit
3. $\rightarrow \left[(2|\psi\rangle\langle\psi| - I)O\right]^R \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$ $\approx |x_0\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$	apply the Grover iteration $R \approx \lceil \pi\sqrt{2^n}/4 \rceil$ times.
4. $\rightarrow x_0$	measure the first n qubits