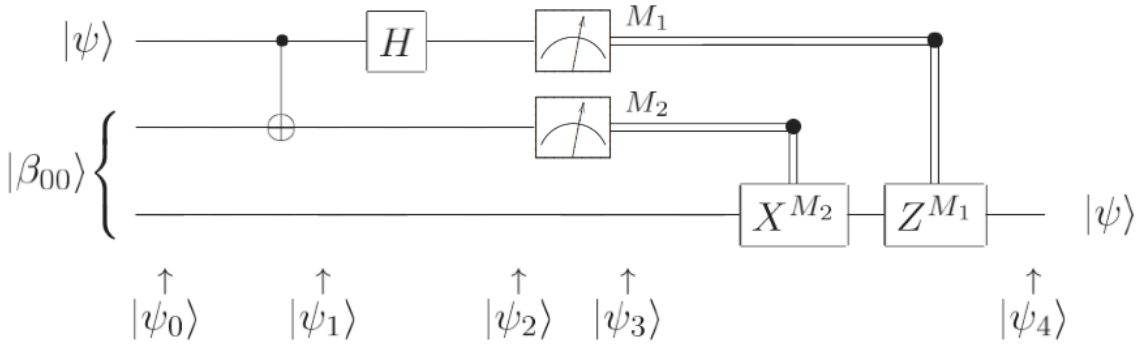
Tutorial 2 Problems: Correlated Quantum Systems

1) Prove the Bell state: $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ cannot be written in the factorized form $|\phi\rangle_1 |\psi\rangle_2$.

2) Show the following quantum circuit realizes quantum teleportation for an arbitrary state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.



Assume Alice and Bob share the entangled state $|\beta_{00}\rangle = |\Phi^+\rangle$. The CNOT gate and Hadamard gates are as follows:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Identify what single qubit operations Bob must apply to recover the initial state. [Hint, use Dirac notation.]

3) Show the anti-correlated Bell-state $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ will violate the CHSH inequality when the measurements taken are

$$A_1 = \sigma_z, \quad A_2 = \sigma_x$$
$$B_1 = \frac{-\sigma_z - \sigma_x}{\sqrt{2}}, \quad B_2 = \frac{\sigma_z - \sigma_x}{\sqrt{2}}$$