Tutorial 1 Problems: Fundamental Concepts for QI

★★ **Exercise 2.2: (Matrix representations: example)** Suppose V is a vector space with basis vectors $|0\rangle$ and $|1\rangle$, and A is a linear operator from V to V such that $A|0\rangle = |1\rangle$ and $A|1\rangle = |0\rangle$. Give a matrix representation for A, with respect to the input basis $|0\rangle, |1\rangle$, and the output basis $|0\rangle, |1\rangle$. Find input and output bases which give rise to a different matrix representation of A.

★ **Exercise 2.7:** Verify that $|w\rangle \equiv (1, 1)$ and $|v\rangle \equiv (1, -1)$ are orthogonal. What are the normalized forms of these vectors?

★★ **Exercise 2.11: (Eigendecomposition of the Pauli matrices)** Find the eigenvectors, eigenvalues, and diagonal representations of the Pauli matrices X, Y, and Z.

★ **Exercise 2.16:** Show that any projector P satisfies the equation $P^2 = P$.

★ **Exercise 2.19: (Pauli matrices: Hermitian and unitary)** Show that the Pauli matrices are Hermitian and unitary.

★★★ **Exercise 2.22:** Prove that two eigenvectors of a Hermitian operator with different eigenvalues are necessarily orthogonal.

★ **Exercise 2.26:** Let $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Write out $|\psi\rangle^{\otimes 2}$ and $|\psi\rangle^{\otimes 3}$ explicitly, both in terms of tensor products like $|0\rangle|1\rangle$, and using the Kronecker product.

★ **Exercise 2.27:** Calculate the matrix representation of the tensor products of the Pauli operators (a) X and Z; (b) I and X; (c) X and I. Is the tensor product commutative?

★★ **Exercise 2.30:** Show that the tensor product of two Hermitian operators is Hermitian.

★★ **Exercise 2.37: (Cyclic property of the trace)** If A and B are two linear operators show that

$$\mathrm{tr}(AB) = \mathrm{tr}(BA). \tag{2.62}$$

★ **Exercise 2.40: (Commutation relations for the Pauli matrices)** Verify the commutation relations

$$[X, Y] = 2iZ; \quad [Y, Z] = 2iX; \quad [Z, X] = 2iY. \tag{2.73}$$

★ **Exercise 2.59:** Suppose we have qubit in the state $|0\rangle$, and we measure the observable X. What is the average value of X? What is the standard deviation of X?