OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY
沖縄科学技術大学院大学

Origami and Deployable Mechanisms

OIST Workshop, 28 – 31 May 2019

Location: OIST Conference Center, OIST, Tancha, Onna-son, Okinawa, Japan

Book of Abstracts

Organizers:
Eliot Fried (OIST)
Johannes Schönke (OIST)
Gianni Royer-Carfagni (University of Parma)

mathematics
mechanics
materials

This event is organized by the Mathematics, Mechanics, and Materials Unit (https://groups.oist.jp/mmmu)

Description of the Workshop

Connecting Kinematic and Curved Origami with Classical and Deployable Mechanisms

Deployable structures are exemplified by everyday items like foldable chairs, lifesaving arterial stents, solar panels for powering spacecrafts, and cargo-sorting DNA robots. The underlying theory and design of these objects is a very active field of research that builds on findings for classical mechanisms. Origami, the art of paper folding, has at least two subcategories which are closely related to deployable and classical mechanisms. One subcategory is kinematic origami, which involves the creation of folded paper structures that possess internal degrees of freedom that allow for changes in shape. Although the many commonalities between these creations and deployable mechanisms are obvious, members of the mechanics community have only recently realized the huge potential for applying kinematic origami to innovative packaging, robotics, mobile architecture, nanotechnology, and medical devices. The other subcategory is curved origami, which involves the creation of rigid structures by folding paper along curved creases. In this case, there are close links to the theories of nonlinearly elasticity of plates and shells, again with numerous applications, interior design, portable architecture, lighting, and stowage included. To understand and advance these modern research disciplines, it is crucial to forge connections with the classical field of mechanics, leading to new discoveries and applications.

Abstracts

In the following, all abstracts are given in the order of the workshop program.

Morphology of Modular Kinetic 4R Structures

Valentina Beatini

Dept. Arch. and Built Environment
Northumbria University, Newcastle upon Tyne, UK
valentina.beatini@northumbria.ac.uk

Origami inspired structures are structural surfaces whose modular geometry can be designed so to reach target design performances, structural but also thermal, acoustical and so on. Being kinetic, they foster to imagine those performances as changeable in response to external stimuli, and yet a full control of their motion and stability is needed. Through a comparison with planar 4R-linkages, this study analyzes the morphological possibilities of modular origami inspired 4R-linkages, with focus on transformable and retractable 1-DOF mechanisms.

Streoplexy

Maurizio Brocato

University Paris-Est, Laboratoire GSA – Géométrie Structure Architecture,
École nationale supérieure d'architecture Paris-Malaquais,
14 rue Bonaparte, 75006 Paris, France.
maurizio.brocato@paris-malaquais.archi.fr

The title of this talk is the ambitious proposal of a neologism, modifying "stereotomy" to suggest that the purpose of our study is not cutting stones but weaving them.

A number of structures made of ashlar are presented. These constructions are characterised by a non-traditional use of the material, obtained thanks to particular shapes of the blocks, that interlock each others, and, in some cases, by means of a new way to include a steel reinforcement.

If the geometrical design of such systems has required the development of some original methods, it is interesting to note that their mechanics introduces a new way to think stone structures, where the blocks, due to their interlocking, withstand flexure. Hence these examples demonstrate that it is possible to use stone as any other building material and open the way to rewrite stone structures beyond the usual reverse funicular scheme.

Studying flat vaults, it appears that a certain analogy can be set with foldable sheets, which can be interesting for further developments of these systems.

Utilizing Torsional Deflections in Crease Design for Thick Origami

Guimin Chen

School of Electro-Mechanical Engineering
Xidian University, Xi'an 710071, P.R. China
guimin.chen@gmail.com

Lamina emergent torsional (LET) joints are widely used in origami-adapted foldable devices to accommodate thickness. This talk will discuss some useful equations for designing torsional segments in LET joints. Based on these equations, the best aspect ratio of torsional segments is also suggested. To further reduce parasitic motions of LET joint, a new type of LET joint called the membrane-enhanced LET joint is presented. The use of membrane-enhanced LET joints are demonstrated in designs such as a kaleidocycle (a 6R Bricard linkage), a degree-4 origami vertex, and an Origami water bomb.

Programmable Origami Metamaterials

Yan Chen

Motion Structure Laboratory, School of Mechanical Engineering
Tianjin University, Tianjin 300354, China
yan_chen@tju.edu.cn

Origami-inspired mechanical metamaterials exhibit exotic behaviours and properties that are programmable based on their geometric design. The underlying units of these metamaterials are foldable crease patterns, which can be either rigid or non-rigid. The mechanism of local deformation associated with rigid patterns can be analytically determined. However, the global material deformation for non-rigid patterns is difficult to characterize theoretically. Yet, without the rigidity constraints, the non-rigid origami has a much larger collection of patterns with wider choice of design parameters, and therefore offers greater application potentials. A non-rigid square-twist origami pattern were proposed with theoretically predictable and programmable global mechanical behaviour achieved using its rigid counterpart. We identified two distinct deformation paths of the rigid pattern and analytically obtained the energy level for each path. Also, we demonstrated experimentally that the non-rigid structure and its rigid origami counterpart bifurcated during deformation to follow the lower-energy path under all conditions tested, which offers an in-depth understanding of the accurate folding process. The mechanical properties of the overall structure can be programmed by the geometric design parameters of the origami pattern, and by controlling the material properties of the creases and central square facets. We envisage that this work could inspire a new class of non-rigid metamaterials with programmable behaviour, serving for different desired application requests in mechanical field including energy absorption, vibration isolation, and even extending to the emerging optical, acoustic and thermal ones.

General Variational Framework for Finding the Equilibrium Shape of a Paper Sheet Subjected to Edge Loads

Yi-chao Chen [1], Roger Fosdick [2], Eliot Fried [3]

[1]University of Houston, Texas, USA
ychen11@Central.UH.EDU

[2]University of Minnesota, Minnesota, USA

[3]Okinawa Institute of Science and Technology (OIST), Okinawa, Japan
eliot.fried@oist.jp

The equilibrium shape of a paper sheet must satisfy the constraint of unstretchability, which is the very reason that a dome-like origami can be formed only through folding. In this talk, we study the equilibrium shape of a paper sheet, without folding, when subjected to edge loads. The constraint of unstretchability enables us to represent a possible shape of the paper sheet through a space curve, and thus to reduce the bending energy functional of the deformed paper sheet from a surface integral to a line integral. The equilibrium shape corresponds to a stationary point of the total energy functional, consisting of the bending energy of the paper sheet and the potential energy of the edge loads. The equilibrium equations of the paper sheet, obtained from the Euler-Lagrange equation of the energy functional, are a system of ordinary differential equations that offers great simplicity in finding the equilibrium shape of the paper sheet.

Configuration Spaces of Origami Mechanisms, Kaleidoscopes, and Molecular Crystals

Gregory S. Chirikjian

National University of Singapore and Johns Hopkins University
gregc@jhu.edu

This talk discusses the configuration spaces of collections of rigid bodies that move in unison with discrete symmetries between them. This scenario arises in origami mechanisms, kaleidoscopes, and molecular crystals. Whereas the configuration space of a single rigid body is the group of orientation-preserving Euclidean motions, the configuration space of a collection of rigid bodies that move in lock step constrained by a discrete symmetry group is actually a coset space (or quotient space) of the configuration space of a single body by the discrete symmetry group. This configuration space, called a 'motion space' has many applications ranging from those listed above to the analysis of symmetry breaking in liquid crystals and the phase problem in protein crystallography. This talk discusses this mathematical object, with an eye towards the range of applications to which the analysis can be applied, including origami mechanisms. Of particular importance is the characterization those coordinated motions which place symmetry mates in collision, since only the complement of this space is physically viable. See, for example, the following references for background.

References

[1] Chirikjian, GS. Mathematical aspects of molecular replacement. I. Algebraic properties of motion spaces. *Acta Crystallographica Section A: Foundations of Crystallography* v67, no. 5 (2011): 435-446.

[2] Chirikjian, GS. Kinematics Meets Crystallography: The Concept of a Motion Space. *Journal of Computing and Information Science in Engineering* 15, no. 1 (2015): 011012.

[3] Chirikjian, GS., and B Shiffman. Collision-free configuration-spaces in macromolecular crystals. *Robotica* 34, no. 8 (2016): 1679-1704.

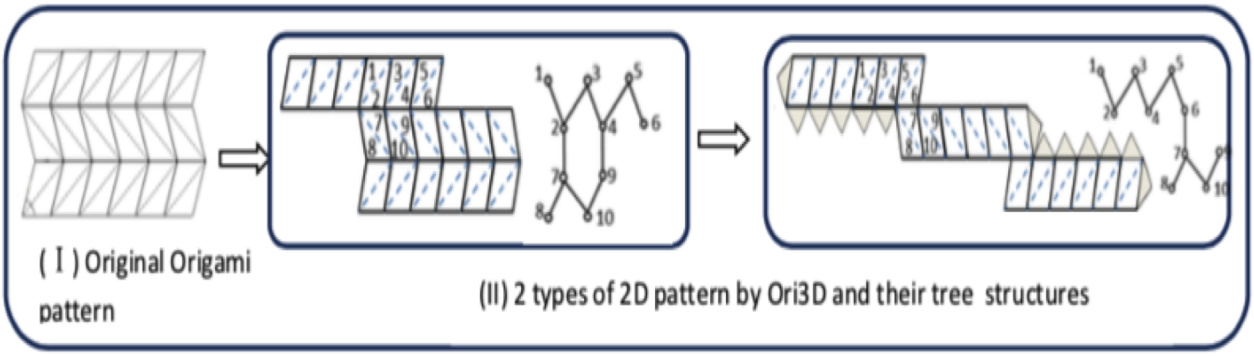
[4] Chirikjian GS, Sajjadi S, Shiffman B, Zucker SM. Mathematical aspects of molecular replacement. IV. Measure-theoretic decompositions of motion spaces. *Acta Crystallographica Section A: Foundations and Advances* 2017 Sep 1;73(5):387-402.

Development of Two Pattern Generating Methodologies for Origami Performing Robots

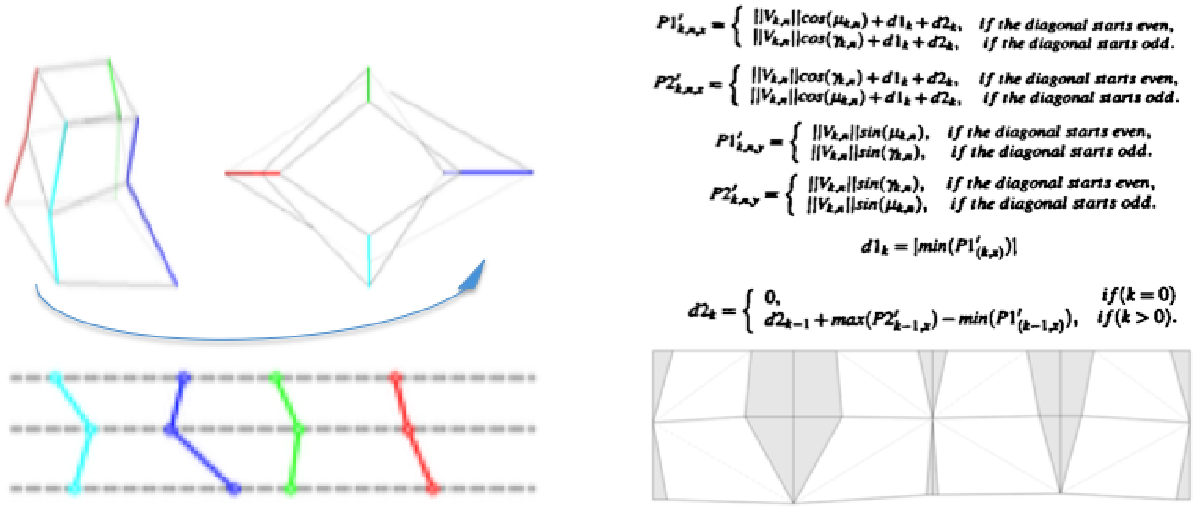
Luis Diago

Meiji Institute for Advanced Study of Mathematical Sciences
Meiji University, Tokyo, Japan
luis_diago@meiji.ac.jp

Current pattern generating methodologies (e.g. TreeMaker, Origamizer, ORI-REVO) generate complicated patterns that can only be folded by human hands. Figure 1 shows two pattern design methodologies that will be discussed focussing on origami performing robots [1, 2].



a) Opening the tree structure of traditional origami patterns [1]



b) Star-based polygonal decomposition of a 3D structure [2]

Figure 1: Two pattern design methodologies

References

[1] - Hoan, Hagiwara, YuBo, Thao, Aparatus, method and program to create an origami pattern. Japanese Patent No. 2016-170 597(23/09/2016)

[2] - J. A. Romero, L. A. Diago, C. Nara, J. Shinoda, and I. Hagiwara, Crease Pattern Simplification for Automatic Folding, In Proceedings of 7OSME, Vol.4 pp.1313–1327.

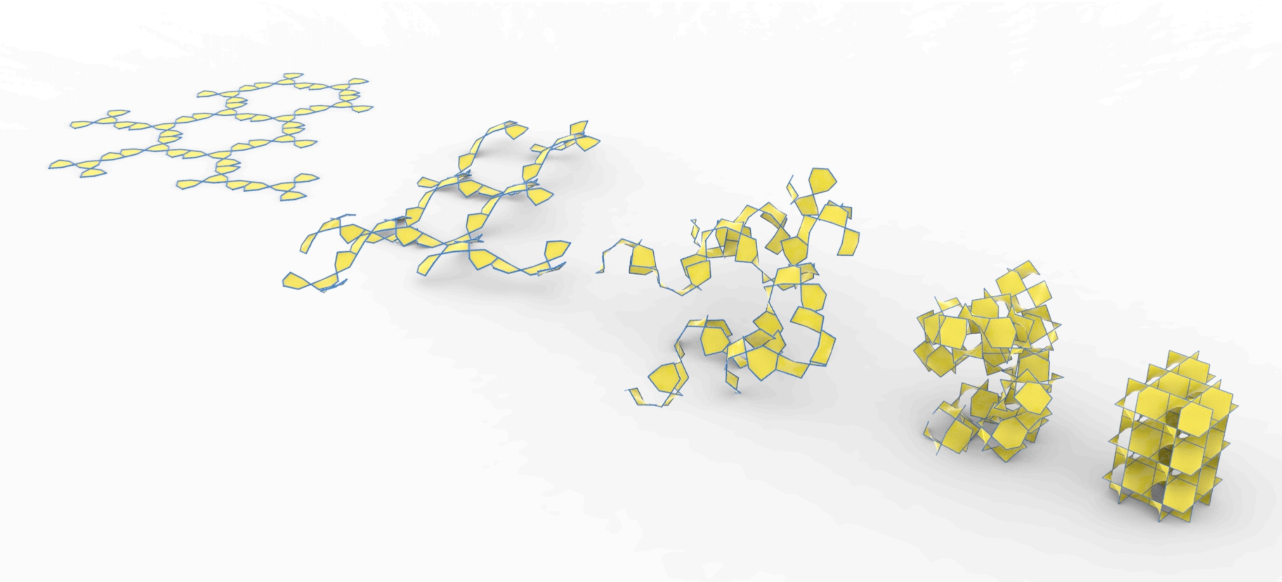
(SHORT PRESENTATION)

Bridging the Euclidean with the Hyperbolic: Origami-Inspired Self-Folding of Minimal Surface Structures

Sebastien Callens, Nazlı Tümer, Amir Zadpoor

Delft University of Technology, The Netherlands
S.J.P.Callens@tudelft.nl

Origami-inspired folding is increasingly used as a platform for the development of deployable structures and mechanisms as well as a novel 2D-to-3D fabrication paradigm. An inherent limitation of origami, however, is its inability to achieve intrinsic curvature in the folded structures. Folding a flat sheet is essentially an isometric deformation of that sheet, which maintains the intrinsic flatness or developability of the sheet (zero Gaussian curvature). Although some methods exist to locally introduce intrinsic curvature, e.g. by tucking away material or by strategically cutting and pasting the sheet, smooth intrinsically curved structures cannot be created with traditional origami. Here, we present a novel folding approach to transform a flat material into curved, hyperbolic morphologies based on triply periodic minimal surfaces (TPMS), which are attractive geometries for several applications. By exploiting the inherent hyperbolic symmetries of TPMS, we design a net of foldable patches that can smoothly transition between a flat state and the final minimal surface. The foldable patches are physically constructed by attaching a pre-stretched membrane to a rigid, foldable frame: the pre-stretch drives self-folding and causes the membrane to adopt a curved minimal surface shape. Our approach opens up new avenues in 2D-to-3D fabrication of complex cellular structures, such as surface-functionalized tissue scaffolds and 3D metamaterials.



Corresponding publication:

Callens, Sebastien J.P., Nazlı Tumer, and Amir A. Zadpoor. Hyperbolic origami-inspired folding of triply periodic minimal surface structures. Applied Materials Today 15 (2019): 453–461

(SHORT PRESENTATION)

New Types of Rigid-Foldable Quadrilateral Creased Papers

Zeyuan He

Engineering Department, University of Cambridge
zh299@cam.ac.uk

Consider a surface in $\mathbb{R}^3$ that is homomorphic to a disk. A quadrilateral creased paper is the union of this surface (the paper) and a quadrilateral mesh embedded on this surface, which is not necessarily developable. Based on a nearly-complete classification of rigid-foldable Kokotsakis quadrilaterals from Ivan Izmestiev, here we will present some large rigid-foldable quadrilateral creased papers with the following additional requirements: (1) There is at least one rigid folding motion for which no folding angle remains constant. (2) The quadrilateral creased paper is infinitely extendable in both longitudinal and transverse directions. (3) The sector angles, which define the crease directions, can be solved quadrilateral by quadrilateral. All the quadrilateral creased papers described in this paper only have one degree of freedom in each branch of their rigid folding motion.

References

[1] Izmestiev, I. (2017). Classification of flexible Kokotsakis polyhedra with quadrangular base. *International Mathematics Research Notices*, 2017(3), 715-808.

[2] Zeyuan He and Simon D. Guest. (2018). On Rigid Origami II: Quadrilateral Creased Papers. *arXiv preprint* arXiv:1804.06483.

Fold Printing and Fold Mapping

Matthew Gardiner

Ars Electronica Futurelab, Ars Electronica Strasse 1. Linz, Austria

https://matthewgardiner.net, hello@matthewgardiner.net

The art of oribotics, a field concerning the art and science of robotics and origami, involves a process of imagining and creating geometry for static and kinetic origami structures to fit a desired form. It is a two-fold difficult task. Firstly, tools for calculating the geometry are few, and those existing lack key aesthetic and functional criteria specific to my artistic practice. Secondly, material issues, such as durability and complex foldability, compound the issues for fabrication. Existing methods, even those applying digital fabrication, pose complex folding problems that confound origami experts.

Our research conducted case studies into the practices of leading origamists working with software, fabrication and materials to analyse and summarise processes. Analysis of these led to the synthesis and identification of differentiating criteria that inspired the invention of two key methods: **Fold-Mapping** and **Fold-Printing**. Fold-Mapping abstracts naturally occurring origami patterns into fold-molecules for tessellation across target geometric surfaces. It allows an artist to prioritise the sculptural shape of the result while seeking a kinetic solution through experimentation with different fold-molecules. A developability algorithm then flattens the crease pattern into geometry for fabrication. Fold-Printing allows the fabrication of Fold-Mapping results. It includes results of high-order complex-foldability by 3D printing whereby polymers are deposited onto textiles forming a durable polymer plate-structure separated by perfect textile hinges.

For proof of concept, the methods are presented as an evaluated set of successful traits and developed solutions. These produce developable crease patterns from target geometries and afford the fabrication and foldability of complex origami (known as ORI*) objects. Fold-Printing allows near impossible-to-fold patterns to become foldable objects. The methods do not succeed in all circumstances, and that success is additionally dependent on the author's experience of origami structures. The aesthetic qualities and material properties of the artistic results have distinct qualities that qualify them to meet the particular criteria required for ORI* objects.

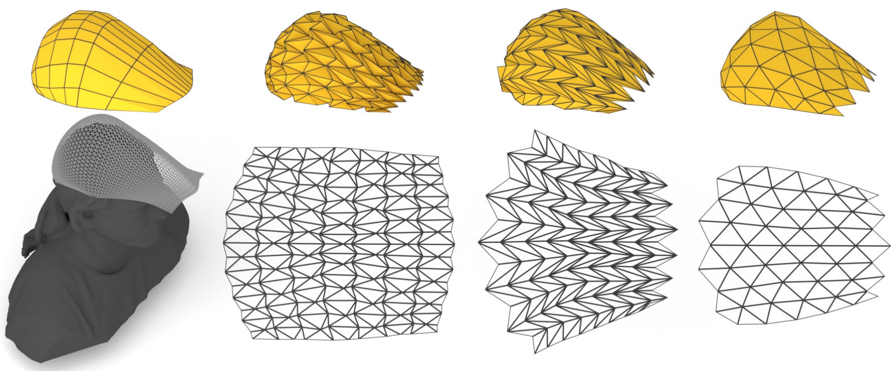


Figure 1: **Fold Mapping** used to create parametric designs for origami headwear from various *natural folding* patterns

Figure 2: **Fold Printing** Polymers are 3D printed onto textiles affording complex origami geometries in flexible, durable prototypes.

Origami as a Means to Explore Responsive Architecture

Arzu Gönenç Sorguç

Department of Architecture, Faculty of Architecture
Middle East Technical University, 06800 Ankara, Turkey
arzug@metu.edu.tr

Architecture and built environment has been undergoing an increasingly rapid transformation not only because of the advancements in technology, but also because of increasing environmental problems. Today terms like responsiveness, performance, smart and etc. are typical adjectives describing buildings which are becoming machine-like and more complex than before.

In this vein, motion in various level is integrated into buildings having Euclidian and/or non-Euclidian forms of new topologies which deliberates themselves in new tectonics. Kinetic systems which are embedded in the facades of buildings, new movable/portable building components, and new curved forms forcing the limits of the material and fabrication technologies compel architects to re-explore design tools and media once again. These new tectonics and the need for more responsive structures turning static buildings into dynamic systems require a seamless integration of form and mechanism.

Origami which provides an excellent means to understand non-deformed spatial transformations, structural stability inherited in the form, regular and non-regular tessellations as well as motion and thus mechanisms that can be derived based on their fold-unfold states has always potentials to be a means for architectural design.

In this presentation first existing buildings with kinetic structures and their projection into origami and then their most possible mechanisms based on the motion are to be presented. Then how curved folding and tessellated kinetic facades out of curved folding components and their examples are to be illustrated both in digital and physical medium in a mutual way in design process. Finally curved folding in 4d printing is to be discussed together with other possible use in architectural design.

Origami-Based Compliant Mechanisms

Larry Howell

Department of Mechanical Engineering, Brigham Young University
lhowell@byu.edu

The application of origami principles to materials other than paper presents interesting engineering challenges. This presentation will describe research addressing several of the most pressing of needs, including thickness accommodation methods, surrogate folds, and rigid foldability. Mathematical models and compliant mechanism theory are presented as tools that can help address these challenges and facilitate the development of origami-based engineering systems. The concepts will be demonstrated in a range of applications, including space mechanisms, consumer products, and medical devices. The connection between curved origami and current research on mechanisms on developable surfaces will also be discussed.

Origami × Mechanical Engineering: Introduction to Origami-Based Vibration Isolator

Sachiko Ishida
Department of Mechanical Engineering, Meiji University, Tokyo, Japan
sishida@meiji.ac.jp

In this talk, origami-based vibration isolator is introduced as an application of mathematical origami in mechanical engineering. Mathematical origami that is generated by mathematical interpretation and computation of paper folding provides important ideas for the design of new structures and mechanisms in engineering. The vibration isolator (Fig. 1(a)) is designed based on an origami foldable cylinder inspired from the twist buckling pattern (Fig. 1(b)). The origami foldable cylinder has negative spring stiffness in a given displacement range during folding/deploying motions (Fig. 2(a)). By adding a coil spring that has positive spring stiffness to the cylinder, the total spring stiffness is cancelled out and it works as zero-stiffness spring that does not transmit any excitation to the other side of the cylinder around the region where the stiffness is ideally zero (Fig. 2(b)). To support sufficient load in the application, the origami foldable cylinder is remodeled as a truss structure using versatile mechanical elements including coil springs, shafts, and universal joints, but the above-mentioned spring characteristics are still maintained. The performance of the prototype vibration isolator is evaluated through excitation experiments (Fig. 3(a)) via the use of harmonic oscillations. The results indicate that the isolator with the current specification is able to suppress the transmission of vibrations (Fig. 3(b)).

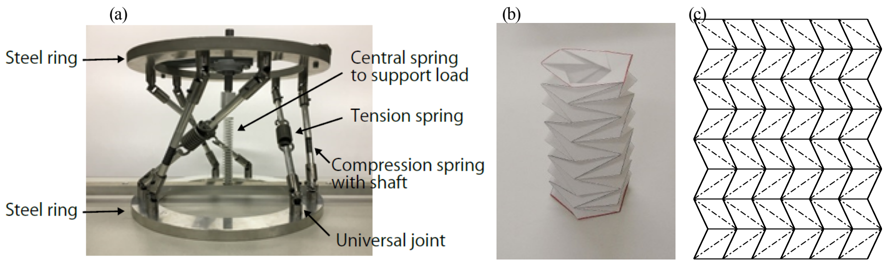


Figure 1: Vibration isolator and its origami model [1]; (a) prototype vibration isolator; (b) original origami model; (c) folding pattern.

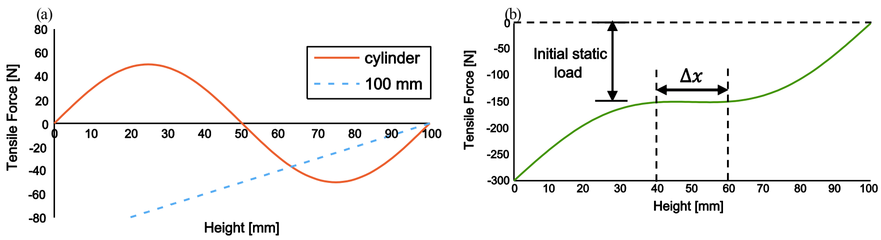


Figure 2: Schematic loadheight diagrams [2]; (a) foldable cylinder and a linear spring; (b) foldable cylinder with a linear spring.

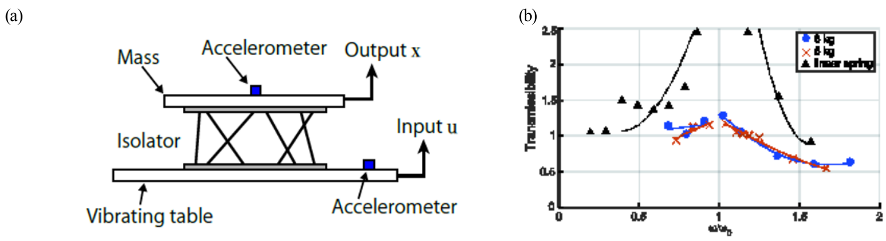


Figure 3: Experimental setup and results of excitation tests [2].

References

[1] Ishida, S., Suzuki, K., and Shimosaka, H., 2017, Design and Experimental Analysis of Origami-inspired Vibration Isolator with Quasi- zero-stiffness Characteristic, *ASME Journal of Vibration and Acoustics*, 139(5), p. 051004.

[2] Inamoto, K., and Ishida, S., 2019, Improved Feasible Load Range and Its Effect on the Frequency Response of Origami-Inspired Vibration Isolators with Quasi-Zero-Stiffness, *ASME Journal of Vibration and Acoustics*, 141(2), p. 021004

Origami Through Thick and Thin: Rigidly Foldable Mechanisms

Robert J. Lang

Lang Origami
Altadena, California, USA
robert@langorigami.com

Origami, the art of paper-folding, provides a rich vein of material for the realization of mechanisms, whether used to fabricate a static structure from sheet-like material, or to realize deployable/morphable structures. Traditional origami uses paper, which permits many types of manipulation: folding, of course, but also curved bending, rolling creases through the paper, and dynamic changes in the crease pattern, to name a few. With most engineering materials, however—metal, plastic, composites—the range of possible manipulations is far more limited. These limitations have led to a focus on those origami patterns that allow motion only along fixed folds with no bending of panels or dynamic alteration of the crease pattern: so-called *rigidly foldable* origami. With that focus has come an interest in methods of synthesizing rigidly foldable patterns.

A closely related issue is that while traditional origami patterns may usually be designed by neglecting the paper thickness—the *zero-thickness approximation* (ZTA)—in engineering based on origami structures, the thickness of the materials is typically non-negligible. While many origami patterns may be found that are rigidly foldable, a naïve application of a ZTA fold pattern to thick materials can result in breakage—of the kinematics, of the mechanism, or both.

And so there is interest in the coupled topics of rigidly foldable origami mechanisms, thick-panel origami mechanisms, and their applications to technological problems. The last decade has seen vibrant growth in the field of research on thick/rigidly foldable origami mechanisms. In this presentation, I will review some recent work on methods of synthesizing both thin- and thick-origami mechanisms, including design methods for rigidly foldable quadrilateral meshes and techniques for transforming arbitrary zero-thickness foldable mechanisms into thick-panel equivalents that preserve the kinematics of the underlying pattern.

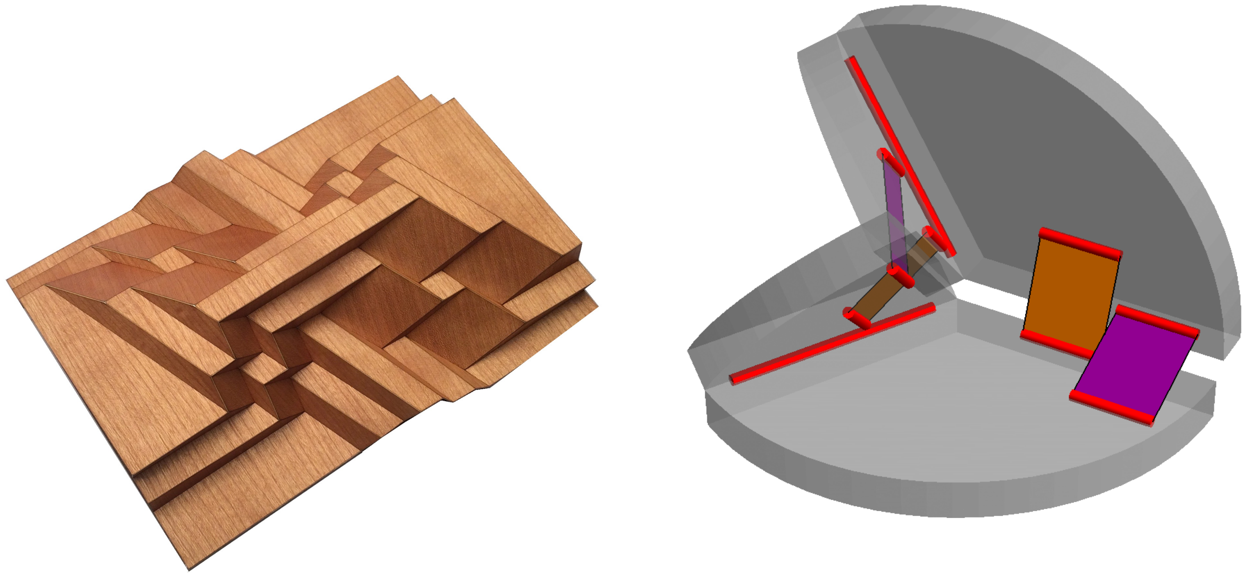


Figure 1: Left: a rigidly foldable mechanism fabricated from wood veneer laminate. Right: a thick rigidly foldable vertex with four-bar linkage hinges.

Kirigami Derived from the Type III Bricard Linkages

Shengnan Lu

Institute of Robotics, Beihang University (BUAA)
lvshengnan5@gmail.com

Kirigami is similar to origami in that it is originally a form of paper art. The major difference is: origami deals only with folding of the paper; kirigami includes both folding and cutting.

Among Bricard's overconstrained 6R linkages, the third type has two collapsed configurations, where all joint axes are coplanar. The existence of two configurations where all joints are coplanar allows us to use the distribution of rotational axes of the Bricard linkage as the crease pattern of novel kirigami.

This talk will focus on the parameterization, kinematics, and design of a type of 1-dof kirigami derived from the type III Bricard linkage. Using the two coplanar states of the constituent Bricard units, the kirigami is able to cover a large surface with a specific outline when deployed, and can be folded compactly into a stack of much smaller planar shapes.

Folding Product Design Using the Principles of Origami

Brian Ignaut

Degrees of Freedom LLC
https://www.instagram.com/degreesoffreedomco
brian@enjoydof.com

The growing field of origami engineering has been celebrated as an unexpected intersection of art and science. While our technical community continues to find new ways to use a familiar art form to generate compelling research, our research is also generating new ways to create similarly compelling art. The rules and framework which define rigid foldability offer a new kinetic palette for folding product design. This palette can be used to bring unexpected folding behavior to familiar objects, creating products which are tactile, functional, and beautiful. Degrees of Freedom was founded in 2018 with the goal of making beautiful mechanical products for people to enjoy, and is using origami engineering as its chosen medium.

To date, three folding products have been completed and commercialized by Degrees of Freedom. These designs use a variety of four or six-bar mechanisms to create at least one but often two single degree of freedom folding configurations. Selection and implementation of folding behavior is guided by the goal of enhancing, not compromising, the product's purpose, whether functional or aesthetic. This can be achieved through the application of simple design principles, like the use of folds in all operating states to maintain a reduced form factor in storage applications, or the use of developable mechanisms to maintain aesthetics. Folding may also be used to improve products even before they reach the customer, as intended with the Designed-Offset (DOF) joint which offers an avenue to manufacturing cost reduction for 3D folding structures.

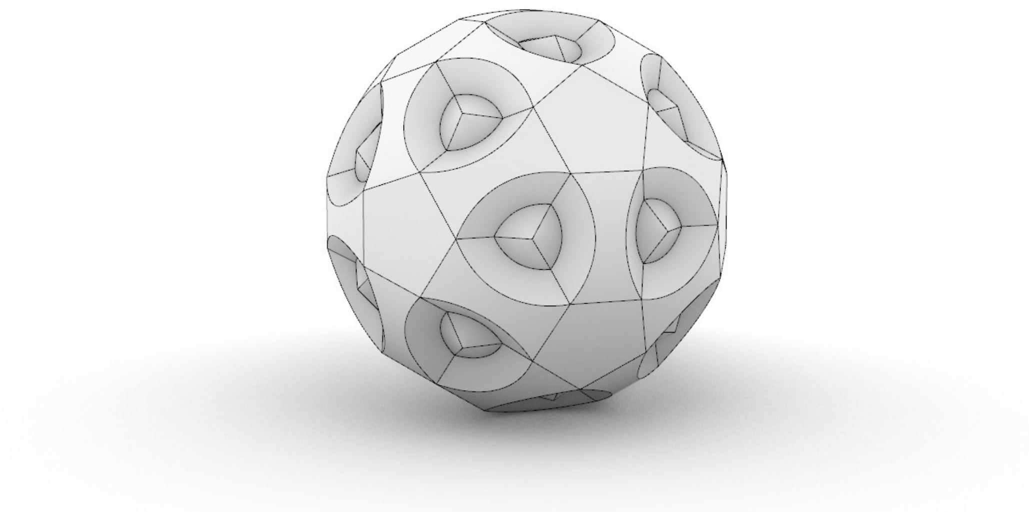
As the addition of functional requirements to a system is unlikely or unable to improve the efficiency of meeting any one its requirements, the addition of folding behavior to products presents special challenges to the designer both creatively and technically. Though the resulting inefficiencies may be unacceptable in engineering, art is a field where these imperfections can be tolerated or even celebrated, creating a space where technical origami can evolve and expand in new ways. Degrees of Freedom attempts to operate in this space, bringing enjoyment, inspiration, and advancement to its audience in the process.

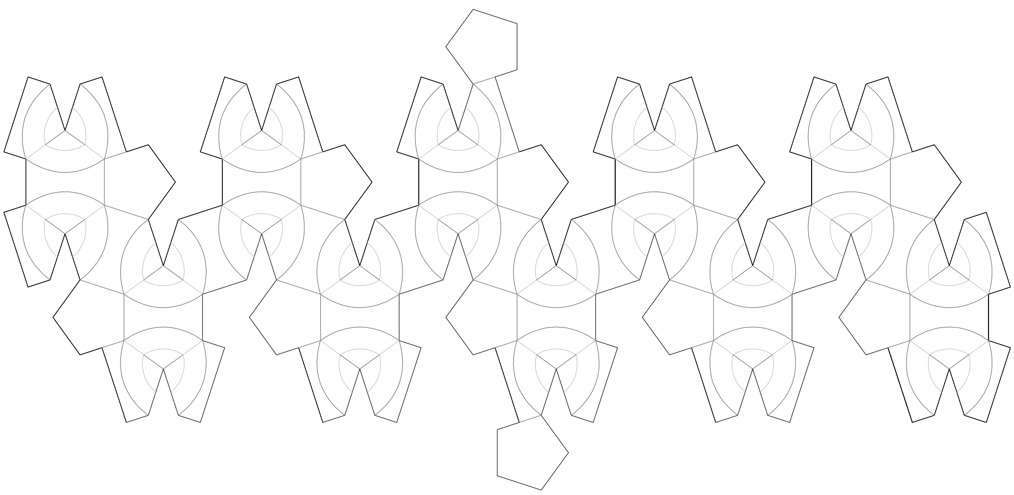
On Mathematical Folding of Curved Crease Origami: Sliding Developables and Parametrizations of Folds into Cylinders and Cones

Klara Mundilova

Institute of Discrete Mathematics and Geometry, Technical University Vienna, Austria
klara.mundilova@tuwien.ac.at

Motivated by the art of curved crease origami we study mathematical models for folding ideal paper along curved creases. We approach the problem of finding mathematical descriptions in terms of developable surfaces of shapes which can be folded from real paper but where their rigorous description is often unknown. For that we investigate a particular one-parameter family of surfaces isometric to a given planar surface patch. We parametrize the crease curves which fold those surfaces into cylinders and cones. Finally, we apply our methods to explore curved crease origami designs, such as tessellations of the plane and a right circular cylinder and models of spherical regular polyhedra.





Making Waves...
Analogue Kinetic Sculptures That Seek to Combine the Sensuousness of Nature with the Logic of Math

Reuben Margolin

Kinetic Artist
reubenmargolin@yahoo.com

Artist Statement:
For the past fifteen years I have been working on a series of monumental mechanical mobiles inspired by water that combine the logic of mathematics with the sensuousness of nature. So far I've completed over thirty suspended installations that undulate in different patterns. They are made of wood, aluminum, and copper, and are driven by overhead structures containing electric motors, cams, levers, helices, thousands of pulleys, and miles of cable or string. Although their modular fluidity is reminiscent of digital graphics, their motion is instead produced by layering the effects of simple mechanical components. Peoples delight in both the dynamic overhead structures as well as the suspended wave has been immensely gratifying.

While working on these installations I remain grounded in many spheres. As an artist I have a sense that art should be beautiful, well proportioned, and emotionally moving. The mathematician in me delights in taking equations off the page and giving them form. The modern part of me strives to create innovative patterns, while the old-fashioned part of me delights in the simplicity of analog machinery. The engineer in me knows that both longevity and elegance follow from streamlined design, in which every element visually describes the forces it must address, as well as in precise, high-quality craftsmanship. The tinkerer in me has garnered experiences with a wide range of materials, while the naturalist in me loves organic forms. In these wave installations I have found the multiple parts of myself coming together.

Bio:
Reuben Margolin was raised in Berkeley, California. A love of math and physics propelled him to Harvard, where he changed paths and got a degree in English. He then went on to study traditional painting in Italy and Russia. In 1999 he became obsessed with the movement of a little green caterpillar, and set out to make wave-like sculptures. In 2004 he moved to his current studio in Emeryville and began making a series of large-scale undulating installations that attempt to combine the logic of mathematics with the sensuousness of nature. He has since made over 30 of these mechanical mobiles and shown them internationally. He also makes pedal-powered rickshaws and has collaborated on a couple large-scale pedal-powered vehicles.

Mechanisms and Self-Stresses in the Design of Rigid Origami Structures

Andrea Micheletti

University of Rome Tor Vergata
micheletti@ing.uniroma2.it

Rigid origami structures are intended here as assemblies of rigid polygonal panels hinged together at their edges according to given origami patterns, typically with applications as deployable and morphing structures. Their kinematics is usually described by following two equivalent modeling approaches: (1) by considering corresponding multi-connected chain of rigid bodies, or (2) by looking at corresponding bar-and-joint frameworks, with bars along the edges and joints at the vertices of the origami. The first part of this talk is dedicated to the problem of finding a deployment path between a 'closed' and an 'open' configuration of a rigid origami structure. Clearly, in statics, the two approaches are not equivalent to each other. In particular, a rigid origami structure modeled as in (1) often possesses a large number of self-stress states, making the design against external loads more complicated. In this respect, we distinguish door hinges from sliding hinges, a sliding hinge differs from a door hinge in that it permits the relative translation between two panels along the hinge axis. In the second part of this talk we present the problem of finding a statically determined assignment of door and sliding hinges for Miura-ori-like assemblies. The talk is based on an ongoing joint work with Alessandro Tiero.

Simulation of Curved Folding

Jun Mitani

University of Tsukuba
mitani@cs.tsukuba.ac.jp

Several methods have been proposed for designing shapes obtained by folding curves by calculation based on geometric constraints. The degree of freedom in design is not high intrinsically because of the severe constraints. However, if we actually fold a sheet of paper, we can often fold the crease pattern, a network of folding lines, that cannot be folded theoretically. And then we can often obtain shapes that do not meet the geometrical constraints. This is considered that the physical properties of the material of paper allow a small strain near the folds. This fact greatly enhances the freedom of curved origami design. Because some conventional origami simulators simulate folding motion of origami under isometric constraint conditions, they cannot cope with shape changes with strain. On the other hand, the Origami Simulator, developed by Ghassaei et al., can handle strain since the sheet of paper is modeled as a spring-network. The shape of origami is represented as a triangle mesh, and each side of a triangle is represented with a spring having a damper that keeps the length constant. However, as a disadvantage, the folding line needs to be a line segment, and curves cannot be handled. To address this problem, we approximated curves by polylines, and added a feature on the Origami Simulator to triangulate the area surrounded by folding lines so that the triangle edges locate along the rulings of developable surface patches. Although there are still limitations, the folded shapes of complicated patterns with curved folding lines, that cannot be folded theoretically, were generated with our approach (Figure 1). The validity of the method was confirmed by comparing the 3D models obtained by the simulation and physically folded paper.

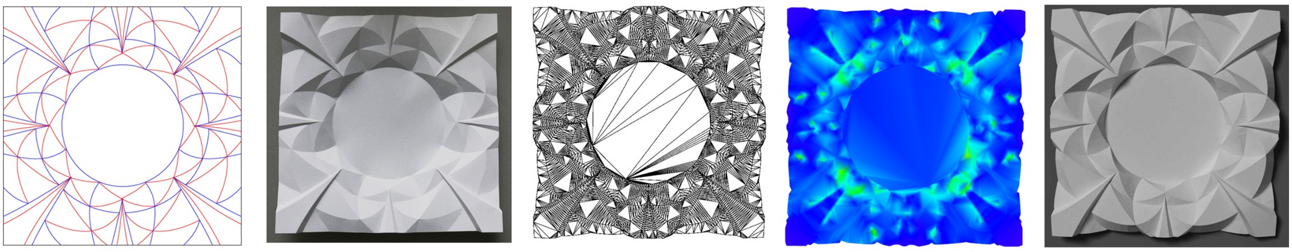


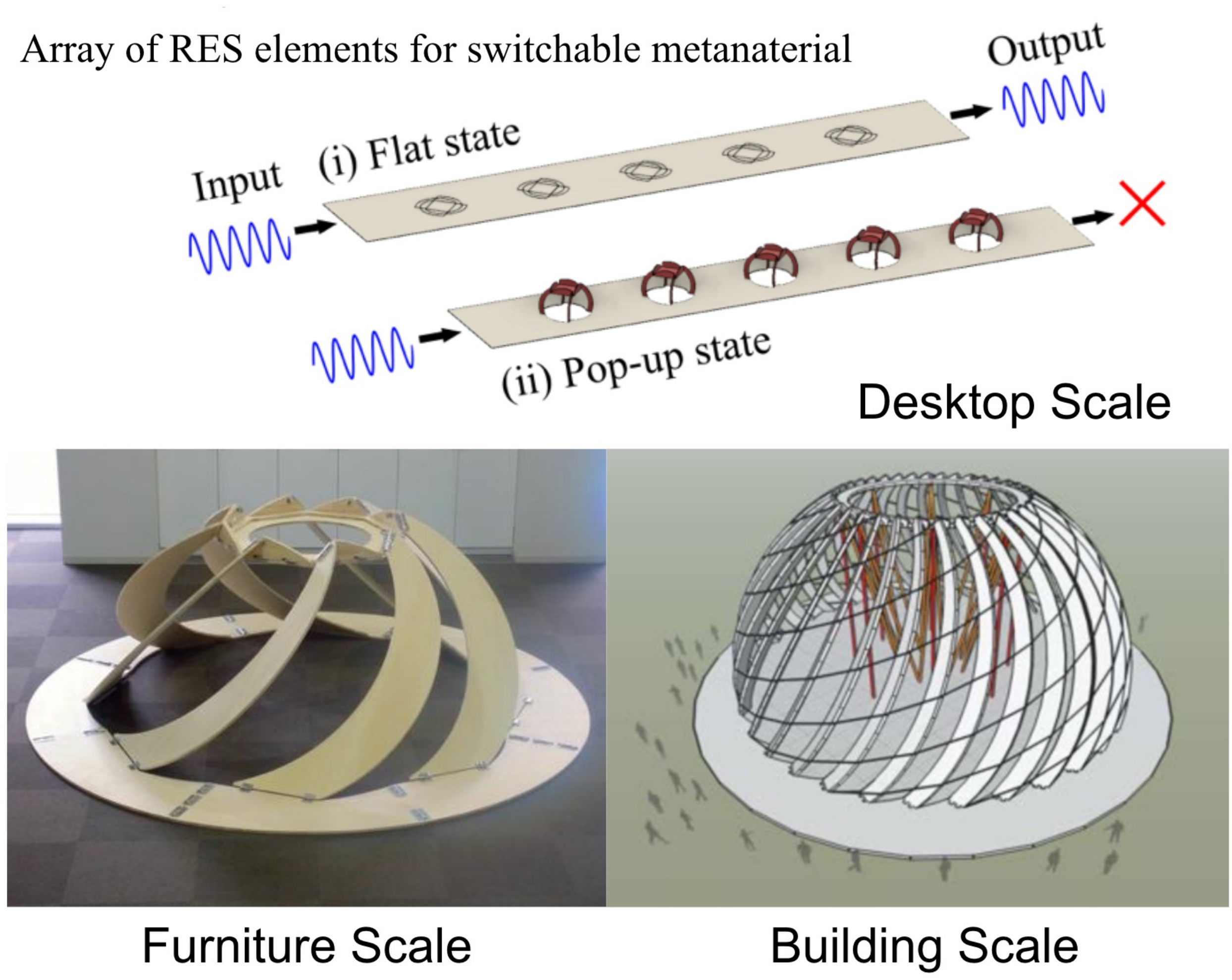
Figure 1: (from left to right) Crease pattern, photo, triangle mesh after simulation, strain map, and rendered CG image.

Forms and Functions of RES (Rotational Erection System)

Yoshinobu Miyamoto

Aichi Institute of Technology
miyamoto@aitech.ac.jp

We explore potential applications of RES (Rotational Erection System) on various scales. Firstly we review the recent studies in metamaterials with RES-like structure in nanometer scales. Secondly, we explain our preliminary experiment of a switchable mechanical device in centimeter scale with multiple RES elements for vibration property control. Thirdly we show the example designs in furniture scale and building applications. Finally, we compare two types of RES, one with bistability and the other with rigid foldability in terms of compatibility to the scales.



A Geometric Approach to the Computational Determination of the Finite Mobility of Linkages, Reconfiguration and Singularity Analysis

Andreas Müller

Institute of Robotics, Johannes Kepler University Linz
a.mueller@jku.at

The finite and differential mobility is the main characteristic of a linkage, i.e. a system of rigid bodies interconnected by lower pair joints. Of particular interest is the local aspect, i.e. the finite and differential mobility in a given (assembly) configuration. Various approaches have been proposed in the past, of which the higher-order local analysis could be deemed the most generally applicable method. Over the last decade a comprehensive computational framework for such a local analysis has been developed. This framework makes use of screw and Lie group theory that give rise to algebraic formulations for all necessary objects within the algorithms. The local analysis allows to establish the finite local mobility and the differential mobility at nearby regular configurations, to identify kinematic singularities and singularities of the configuration variety, and to determine finite singular motions. Such a local analysis eventually allows to investigate the reconfiguration of linkages, a phenomenon prevailing in origami structures.

This presentation summarizes modeling steps using screws, and the main relations for the computational local analysis. Special topics are also discussed such as singularities where submanifolds of the configuration space intersect tangentially and singularities where no tangent space can be defined. Finally the local approach is put into perspective together with global formulations and methods from computational geometry.

Continuous Flattening of Polyhedra Using the Kite Property

Chie Nara
Organization for the Strategic Coordination of Research and Intellectual Properties
Meiji University, Tokyo, Japan
cnara@jeans.ocn.ne.jp

We use the terminology *polyhedron* for a closed polyhedral surface which is permitted to touch itself but not self-intersect. A polyhedron can always be folded by creases like a piece of paper and a *flat folding* of a polyhedron is a folding by creases into a multi-layered planar shape. The topic presented here are related to the problem proposed by E. Demaine et al. ([1], Open Problems 18.1 and 18.3 in [2]): can every flat folded state of a polyhedron be reached by a continuous folding process?

It is shown in [3, 4] if a polyhedron has rigid faces, its volume does not change even if the polyhedron flexes. Hence, to obtain a flat folded state of a polyhedron continuously, the polyhedral surface should change by moving creases (edges). Three methods were proposed by the author et al. for continuous flat folding of polyhedra (see [5, 6, 7]). Among them the method using the kite property is useful in a sense that it requires small portions of the polyhedron's surface for moving creases.

The kite property was found by the author and shown in [5] first, and extended to a general case in [8]. For two points x and y $dist\{x, y\}$ denotes the distance from x to y,

Proposition 0.1. *Let $K = abcd$ be a kite with $|ab| = |ad|$ and $|cb| = |cd|$, and h the center of K (see Fig. 1 (a)). For any pair $\{l, m\}$ with $||ah| - |ch|| \leq l \leq |ac|$ and $0 \leq m \leq |bd|$, there is a unique point q in the line segment dh so that the resulting figure obtained by folding mountain creases on ah, aq, cq and qd and valley creases on ah, hc and qh, satisfies $dist\{a, c\} = l$ and $dist\{b, d\} = m$, as shown in Fig. 1 (b). The resulting fugure is called a folded kite with wing-shape at q and denoted $K(l, m)$. For any two folded kites $K(l_1, m_1)$ and $K(l_2, m_2)$ there is a family $\{K(l_t, m_t) : 1 \leq t \leq 2\}$ such that $K(l_t, m_t)$ converges to $K(l_2, m_2)$ as t increases from one to two.*

In this talk we show some applications of the kite property for the continuous flattening problems of polyhedra, together with industrial applications, e.g., folding helmets and boxes with thickness.

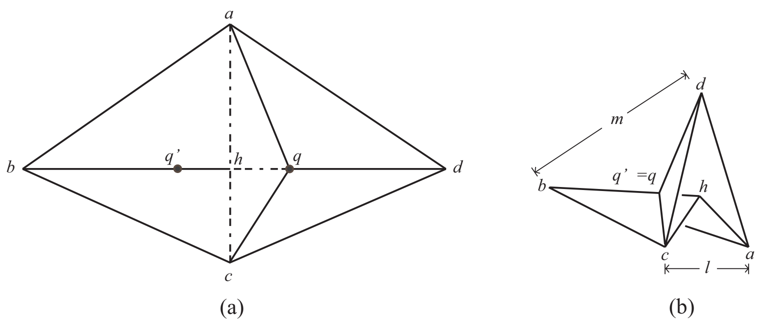


Figure 1: An example of a folded kite with wing-shape.

References

[1] E. D. Demaine, M. D. Demaine and A. Lubiw. Flattening polyhedral. Unpublished manuscript, 2001.

[2] E. D. Demaine and J. O'Rourke, *Geometric Folding Algorithms: Linkages, Origami, Polyhedra*, Cambridge University Press, 2007.

[3] I. Sabitov, The volume of polyhedron as a function of its metric, *Fundam. Prikl. Mat.*, 2, 1235–1246 (1996).

[4] R. Connelly, I. Sabitov, A. Walz, The bellows conjecture. *Beiträge Algebra Geom.*, 38, 1–10 (1997).

[5] J. Itoh, C. Nara, Continuous flattening of Platonic polyhedra, In Proc. Computational Geometry, Graphs, and Applications (CGGA 2010), LNCS, 7033, Springer, 108–121 (2011).

[6] J. Itoh, C. Nara, and C. Vîlcu, Continuous flattening of convex polyhedra. In *Revised Papers, 16th Spanish Meeting on Computational Geometry (EGC 2011)*, LNCS, vol. 7579, pp. 85–97. Springer, 2012.

[7] Z. Abel, E. D. Demaine, M. L. Demaine, J.-i. Itoh, A. Lubiw, C. Nara, J. O'Rourke, Continuously flattening polyhedra using straight skeletons. In *Proc. 30th Annual Symposium on Computational Geometry (SoCG)*, 396–405, 2014.

[8] C. Nara, Continuous flattening of some pyramids, *Elem. Math.* 69, 45–56 (2014).

Lattice Expandable Origami for Architecture and Design: ORIGAMI-SCISSOR Hinged Deployable Structures

Esther Rivas-Adrover

University of Cambridge
era.esther@gmail.com

Throughout the history of deployable structures origami and scissor-hinged have been different types out of many others that exist. Rivas-Adrover has unified both types making an ORIGAMI-SCISSOR hinged deployable structure which is a lattice thick origami made with hinged bars that enable the creases to expand and contract. ORIGAMI-SCISSOR hinged deployable structures can simultaneously or independently fold or unfold as an origami, and expand or contract as a scissor hinged structure; they therefore provide an extra degree of freedom to its thick origami counterpart. A geometry method has been developed to realize this, and a prototype has been built that demonstrates this concept. This novel technology opens new design possibilities that range from engineering, transformable architecture, stage design or industrial design.

Video illustrating the ORIGAMI-SCISSOR hinged pattern:
https://www.youtube.com/watch?v=Jw42lt229f4

Matrix Analysis of Möbius Kaleidocycles

Eliot Fried [1], Gianni Royer-Carfagni [2,3], Johannes Schönke [1]

[1]Okinawa Institute of Science and Technology (OIST), Okinawa, Japan
eliot.fried@oist.jp, johannes.schoenke@oist.jp

[2]Department of Engineering and Architecture, University of Parma, Parma, Italy

[3]Construction Technologies Institute - National Research Council of Italy (ITC-CNR), San Giuliano Milanese, Italy
gianni.royer@unipr.it

As first recognized by Maxwell [1], a spatial system of points connected by lines (the bars of a truss) has particular structural properties when the rank of the kinematic matrix, or equivalently the static matrix, does not equal the number of unknowns or, more generally, when the solution of the corresponding linear system of equations does not exist, or that solution is not unique. Usually, a statically under-determined structure allows for inextentional mechanisms for which the lengths of connecting bars remain unchanged. However, there are special arrangements for which these mechanisms are infinitesimal, namely arrangements in which there are no first order variations of the lengths of the bars for given infinitesimal displacements. Following the seminal work by Pellegrino and Calladine [2], the kinematical characterization of such mechanisms is complemented by a static property: there are states of self-stress that can stiffen the infinitesimal mechanisms to the first order. Fascinating structures like tensegrities are based on this property.

Here, we classify the possible mechanisms (finite or infinitesimal) for a new class of linkages, first presented by Schönke and Fried [3], formed by closed chains of an arbitrary number $N \geq 7$ of identical links whose ends are joined by revolute hinges. The mid-axis of each link is orthogonal to the hinges at its ends and those hinges form a certain acute twist angle. Remarkably, although a closed chain of N links should have $N - 6$ internal degrees of freedom according to the classical mobility rule, whatever the number N of links there is an N-dependent "critical twist angle" for which the linkage exhibits a single degree of freedom. The resulting motion is a continuous eversion much like that exhibited by the rings of six tetrahedra called kaleidocycles [4]. Since the hinge orientation induces a nonorientable topology equivalent to the topology of a threefold Möbius band, linkages of this type have been called Möbius Kaleidocycles.

The kinematic matrix is constructed by using as variables the rotation angles of the revolute hinges, so that the work conjugate entities are the moments that act across the hinges. In general, there are $N-6$ independent "undistortional" movements that leave unchanged the relative positions of the hinges in each link: these movements form a basis for all possible movements of the linkage. At the "critical twist angle" we verify that an additional mechanism is nucleated. Thus, in total there are $N-5$ independent mechanism but, remarkably, only one mechanism remains finite. Correspondingly, the system admits only one state of self-stress, which imparts first order stiffness to all the possible infinitesimal modes of motion except the mode constituting the finite eversion. This state of self-stress can be obtained by manufacturing the hinges in one or more links with a twist angle slightly below the critical one and forcibly closing the chain.

References

[1] Maxwell, J.C., On the calculation of the equilibrium stiffness of frames, *Phil. Mag.* 27, 294299, 1864.

[2] Pellegrino, S., and Calladine, C.R., Matrix analysis of statically and kinematically indeterminate frameworks, *Intern. J. Sol. Struct.* 22(4),409428, 1986.

[3] Schönke, J., and Fried, E., Single degree of freedom everting ring linkages with nonorientable topology, *Proc. Nat. Acad. Scien. of the USA*, 116(1), 90-95, 2019.

[4] Schattschneider. D., "Creating kaleidocycles and more". In Senechal, M., and Fleck, G., eds, *Shaping space. A polyhedral approach*, Chap. 1, 5760. Birkhauser, Basel, 1988.

Geometric Challenges in Structural Origami

Tomohiro Tachi

University of Tokyo
tachi@idea.c.u-tokyo.ac.jp

Dynamic and static structural properties of origami arise from the arrangement of folding patterns and the assembly of folded sheets. Structural origami is a field that is rapidly developing through a collaboration between engineering, design, art, and computation. The talk will share recent exciting developments in this field and highlight some geometric questions that come from the studies.

Structures and Materials Inspired by Origami

Zhong You

Department of Engineering Science, University of Oxford, Parks Road, Oxford, U.K.
zhong.you@eng.ox.ac.uk

Recently origami folding, a traditional art form of paper folding, has been of increasing interest to engineers. It has been found that the formation of origami structures and the shape change capability of some origami objects can be readily parameterised and applied to the development of new structures and devices. Since most of the sheet materials used in engineering applications are relatively rigid in comparison with paper, particular attention has been drawn on to rigid origami, a subset of origami that permit continuous motion between folded states along the pre-determined folding creases without the need for twisting or stretching of the facets. This allows the patterns to be readily manufactured from modern materials such as plastic, metal, or carbon-fibre sheets, producing patterns that are sufficiently strong and durable to be of use in large-scale applications.

In this talk, I shall focus on two types of origami structures, one being the origami morphing structures that are capable of large shape changes, whilst the other the tubular and sandwich structures where origami is used to design such structures to acquire superior mechanical properties. I shall show that not only can origami technique be effectively exploited to develop novel foldable structures, but it can be used to induce a particular failure mode that leads to a higher energy absorption capability. I shall complete my talk with some remarks on future research directions.