A Dimension-reduction Method and Its Applications to some Phase-transition Problems

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Many natural or man-made structures are thin or slender, and to model their deformations one can use reduced lower-dimension equations instead of the original three-dimensional ones. There is a long history in constructing proper 1-D beam equations or 2-D plate/shell equations, however until recently there still lacks a consistent method to do the dimension reduction for a general loading condition. In this talk, I shall present a systematic reduction method based on series expansions (coupled with asymptotic expansions for nonlinear problems). To illustrate the main ideas, starting from the 2-D field equations for the plane-stress problem, I shall give the derivation of a rigorous asymptotic beam theory, together with a pointwise error estimate. Then, the similar procedure is used to study stress-induced phase transitions in some slender/thin shape memory alloy (SMA) structures based on a constitutive model in literature. For a thin SMA layer, the reduced beam-like model consists of three differential equations for austenite, martnesite and phase transition regions, based on which analytical solutions for the outer-loop, upper and lower plateau stresses are obtained and instability phenomena observed in experiments are captured. For a slender SMA cylinder (wire), the reduced equations lead to the generalization of the classical Ericksen's 1-D results. Unlike the 1-D case, for which there infinitely-many equally stable states, the model with 3-D effect reveals there is only one optimal state (by the nergy criterion). Also, the average axial stress is still the 1-D Maxewell stress, although there is a non-uniform distribution. Finally, based on the asymptotic method, we provide an analytical way to quantitatively determine the up-down-up material response and a systematic procedure to calibrate material constants through tension tests of a pre-twisted SMA tube.

References

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