Stability of spin liquid phases of high spin alkaline earth atoms on honeycomb lattice

Edina Szirmai

Workshop on Coherent Control of Complex Quantum Systems
Okinawa, 16. 04. 2014.

In collaboration with:
M. Lewenstein, P. Sinkovicz, G. Szirmai, A. Zamora
Quantum simulations with ultracold atoms

- Quantum simulation of fundamental models (properties, phenomena).
- Novel behavior, completely new phases are expected due to the high spin.
Quantum simulations with ultracold atoms

Quantum simulation of fundamental models (properties, phenomena).

Novel behavior, completely new phases are expected due to the high spin.

In topological phases of spin liquids the quasiparticles have fractional statistics. They are nonlocal and resist well against local perturbations. Promising qubit candidates.

Quantum information
Quantum simulations with ultracold atoms

Quantum simulation of fundamental models (properties, phenomena).

Novel behavior, completely new phases are expected due to the high spin.

Low energy excitations above spin liquids can be described by effective gauge theories. Aim: to study various gauge theories with ultracold atoms.

Quantum information

Simulation of gauge theories
Quantum simulations with ultracold atoms

- Quantum simulation of fundamental models (properties, phenomena).
- Novel behavior, completely new phases are expected due to the high spin.

A possible explanation for the mechanism of high-$T_c$ superconductivity and their strange behavior in the non-superconducting phase based on the strong magnetic fluctuation in doped Mott insulators. These fluctuation can be treated within the spin liquid concept.

- Quantum information
- Simulation of gauge theories
- High-$T_c$ superconductors
Quantum simulations with ultracold atoms

Spin-5/2 fermions
- Spin-5/2 ultracold atom experiments with $^{173}$Yb.
- Various manganise complexes; NaReO$_4$, NH$_4$ReO$_4$ with $^{185}$Re or $^{187}$Re etc.

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Quantum information

Simulation of gauge theories

High-$T_c$ superconductors
Simulation of high spin magnetism

Competing spin liquid states of spin-5/2 fermions in a honeycomb lattice
  - Properties of chiral spin liquid state
  - Stability of the spin liquid states beyond the mean-field approximation
  - Finite temperature behavior
  - Experimentally measurable quantities

Summary
Simulation of high spin magnetism
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Atoms loaded into an optical lattice
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- Periodic potential: standing wave laser light.
Simulation of high spin magnetism

**Atoms loaded into an optical lattice**

- **Periodic potential**: standing wave laser light.
- **Interaction between the neutral atoms**:
  - van der Waals interaction
  - in case of alkaline-earth atoms: spin independent s-wave collisions

![Diagram](image_url)
Simulation of high spin magnetism

Atoms loaded into an optical lattice

- Periodic potential: standing wave laser light.
- Interaction between the neutral atoms:
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Easy to control the model parameters:

- Interaction strength: Feshbach resonance
- Localization: laser intensity
- Lattice geometry
Simulation of high spin magnetism

The Hubbard Hamiltonian with n.n hopping and on-site interaction:

\[ H = -t \sum_{<i,j>} c_{i,\alpha}^\dagger c_{j,\alpha} + U \sum_{i} c_{i,\alpha}^\dagger c_{i,\beta}^\dagger c_{i,\alpha} c_{i,\beta}. \]

spin independent interaction \( \rightarrow \) SU(N) symmetry
Simulation of high spin magnetism

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spin independent interaction \(\rightarrow\) SU(N) symmetry

**Strongly repulsive limit:** \(U/t \rightarrow \infty\)

repulsive interaction, \(f = 1/(2F + 1)\) filling (\# of particles = \# of sites)

Perturbation theory up to leading order with respect to \(t\).

- \(t\) preserves \(S\) and \(m_S\)
- nearest-neighbor hopping
Simulation of high spin magnetism

Effective Hamiltonian (spin-$F$ fermions in the $U/t \to \infty$ limit)

$$H = J \sum_{\langle i,j \rangle} c^\dagger_{i,\alpha} c^\dagger_{j,\beta} c_{j,\alpha} c_{i,\beta}$$

- nearest-neighbor interaction
- the same spin dependence that has the original model

Without long range magnetic order: spin liquid state
Simulation of high spin magnetism

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no particle transport
Simulation of high spin magnetism

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Without long range magnetic order: spin liquid state
Competing spin liquid states of spin-5/2 fermions on honeycomb lattice
Finite temperature field theory

\[ H[c, c^\dagger] = -J \sum_{\langle i,j \rangle} c_{i,\alpha} c_{j,\alpha} c_{j,\beta} c_{i,\beta} \]
Finite temperature field theory

\[ H[c, c^\dagger] = -J \sum_{\langle i,j \rangle} c^\dagger_i,\alpha c_j,\alpha c^\dagger_j,\beta c_i,\beta \]

Partition function: \( Z = \int [dc][dc] \exp(-\int_0^\beta d\tau L[c, \bar{c}]) \)

\( L[c, \bar{c}] = \sum_i \bar{c}_{i,\alpha} \partial\tau c_{i,\alpha} + H \quad \text{and} \quad \#\text{atoms} = \#\text{sites} \)
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Decoupling procedure:

Hubbard-Stratonovich transformation:

\[ L[c, \bar{c}] \rightarrow L[c, \bar{c}; \varphi, \chi] \]

auxiliary fields: \( \varphi_i \) (on-site, real), and \( \chi_{i,j} \) (link, complex)

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Finite temperature field theory

\[ H[c, c^\dagger] = -J \sum_{\langle i,j \rangle} c_i^\dagger c_j, \alpha c_j, \alpha c_i^\dagger c_i, \beta \]

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\[ Z[\varphi, \chi] = \exp(-\int_0^\beta d\tau \sum_{\langle i,j \rangle} \left[ \frac{1}{2} |\chi_{i,j}|^2 + \ln \det G_{i,j}(\tau) \right]) \]

Saddle-point approximation \( \rightarrow \) and beyond...
Ground state spin liquid states

\[ \chi_{i,j} = J \text{tr} \left( G_0 \frac{\partial \Sigma}{\partial \chi_{i,j}} \right) \]

(\(\Sigma\): self-energy)

\[ \Pi_1 = \chi_1 \chi_2 \chi_3 \chi_4 \chi_5 \chi_6 = |\Pi_1|e^{i\phi_1} \]
Ground state spin liquid states

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>a) Chiral spin liquid state</td>
<td>-6.148</td>
</tr>
<tr>
<td>b) Staggered flux state</td>
<td>-6.062</td>
</tr>
<tr>
<td>c) Valence bond crystal</td>
<td>-6</td>
</tr>
</tbody>
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Greek symbols and notation correspond to:

a) Chiral spin liquid state
b) Staggered flux state
c) Valence bond crystal
Preserves the global SU(6) invariance of the Hamiltonian and every lattice symmetry.

$\Phi = \frac{2\pi}{3}$ flux generated per plaquette $\Rightarrow$ spontaneous time reversal symmetry breaking.

Consequences:
- Integer quantum-Hall effect, transverse conductivity $C = 6$.
- Chiral edge states appear.
- Anyon quasiparticles: spinon with $\Phi = \frac{\pi}{3}$ elementary flux.

And the low energy effective theory is a U(1) gauge theory: Chern-Simons theory.
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Ground state — chiral spin liquid

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And
- The low energy effective theory is a U(1) gauge theory: Chern-Simons theory.
- U(1) gauge theory simulator.
Finite temperature behavior

All the spin liquid phases "melt" around the same critical temperature.
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- No new state occurs as lowest free energy SP solution.
- The SP free energies approach each other without crossing.
- The chiral state remains the lowest free energy solution even at $T > 0$. 

\[
\text{Finite temperature behavior}
\]
Stability analysis

Stability matrix: \( C_{\mu,\nu} \sim \frac{\partial^2 (G_0 \Sigma)^2}{\partial \chi_\mu \partial \chi_\nu} \)

chiral spin liquid

quasiplaquette
Stability analysis

Stability matrix: $C_{\mu,\nu} \sim \frac{\partial^2 (G_0 \Sigma)^2}{\partial \chi_{\mu} \partial \chi_{\nu}}$

The quasiplaquette state collapses into the lower free energy chiral spin liquid state.
Experimentally measurable quantities

Structure factor: \( S(r, \tau; r', 0) = \langle S_z(r, \tau)S_z(r', 0) \rangle \)

- **a) chiral spin liquid**
- **b) quasiplaquette**
Structure factor: \( S(r, \tau; r', 0) = \langle S_z(r, \tau) S_z(r', 0) \rangle \)

Unambiguous features → Suitable tool to distinguish the phases.
Experimental measurable quantities

Spectral density: \( \rho_{\text{tot}}(\omega) = \sum_k \text{Im} S(k, \omega) \)
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Unambiguous features $\rightarrow$ Suitable tool to distinguish the phases.
Summary

- Time-reversal symmetry breaking spin liquid state of spin-5/2 fermionic atoms on honeycomb lattice.
- Stable even at finite temperature.
- Experimentally probable.
- \( \rightarrow \) Simulation of a U(1) gauge theory.

G. Szirmai, E. Sz., A. Zamora, M. Lewenstein, PRA 84 011611(R) (2011)