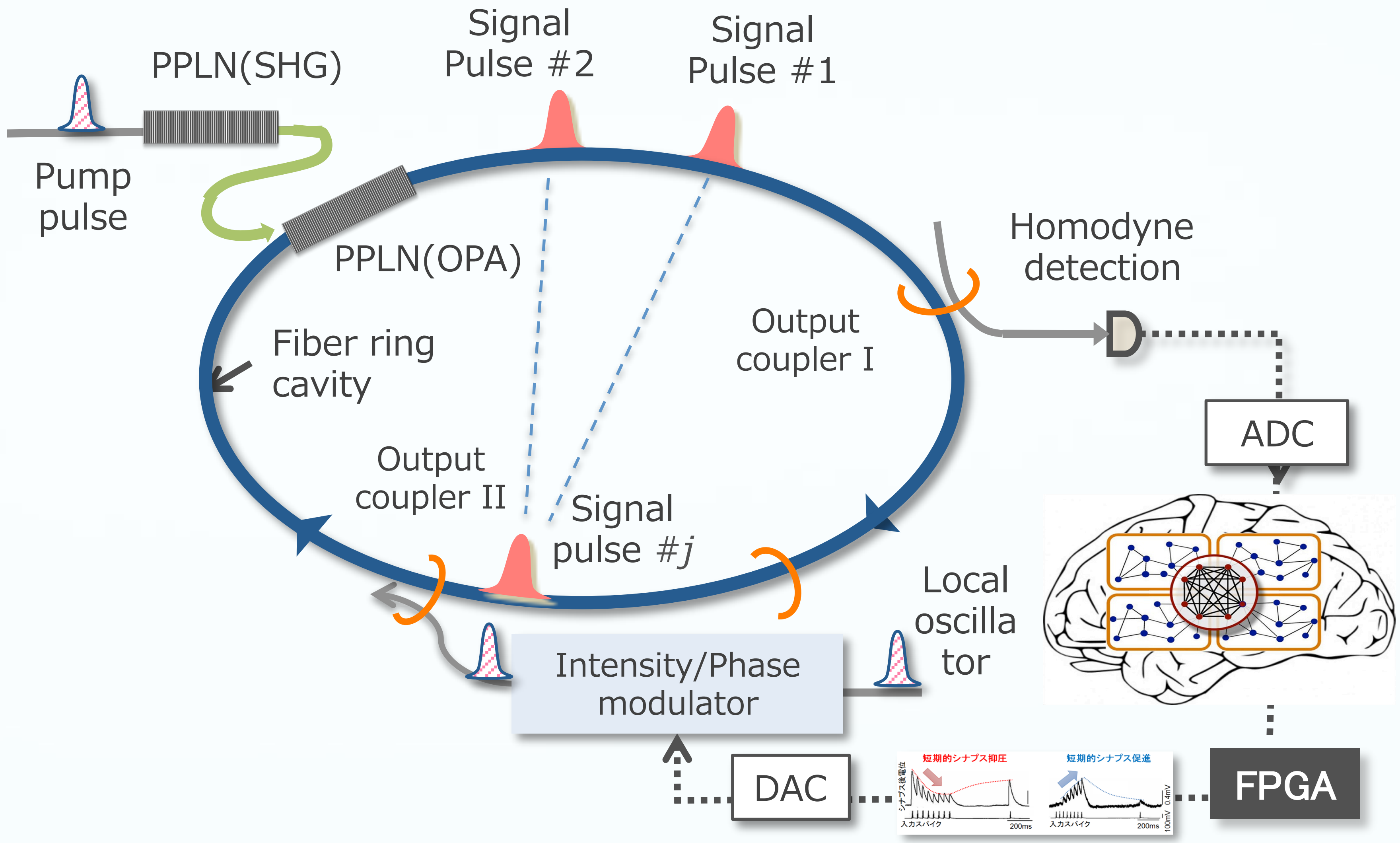


Coherent Ising machine using laser/DOPO network for solving combinatorial optimization problems

Shoko Utsunomiya

National Institute of Informatics

Outline

- Alternative computing architecture
  - Combinatorial optimization problems
  - non Von Neumann's computation architecture
  - Ising machine (Quantum annealing, Ising machine)
- Coherent Ising machine based on DOPO
  - Operational principles
  - Experimental progress (free space and fiber system)
  - Numerical simulation
    - computational scaling
- Coherent XY model based on laser network
  - 1D ring experiment based on fiber mode-locked laser
  - application to community detection
- Summary

Combinatorial optimization problems

✓ Combinatorial optimization problems

- Drug discovery, frequency allocation, routing optimization, etc.
- Computational hardest NP-hard problems (exponential scaling of computation time)

✓ Temporal networks (dynamic network structure) ※N>100 exact solution can't be obtained

- SNS analysis, risk prediction, brain network analysis, etc.
- Real time analysis of temporal network is necessary
- Huge amount of time series data set must be processed momentarily

Drug discovery Frequency allocation Routing (Distribution) SNS analysis risk prediction fMRI network analysis

Temporal network

Combinatorial optimization problems

- Heuristic algorithms (Simulated annealing, greedy algorithm for approximate solution)
- Neural network (Deep learning, machine learning, neural information processing)
- Quantum computing : D-wave based on QA (low tempe operation, scalability problem)

non Von Neumann computing architecture

✤ **<u>Synapse</u>** (Systems of Neuromorphic Adaptive Plastic Scalable Electronics)

- Neurochip project by IBM (DARPA program), inspired by human brain

✓ **TrueNorth** IBM, Cornel Univ, SUMSUNG (2014)

- with 5.4 billion transistors
- 1 million Neurons and 256 million synapses
- core also works as internal memory

-> low energy consumption : 70mW

extend Moore's law in Post-silicon-era

- Complicated software system

Deep learning (cognitive computing) …Siri (Apple)

- based on layered structure in brain
- outperforms in machine learning

- Specialized processor

FPGA (Field Programmable Gate Array) ,**GPU**(Graphics Processing Unit), **ASIC**..

Gate based quantum computing vs Quantum simulator

Gate based QC	Quantum simulator
Solves	Solves
Complicated mathematical problems	Quantum many body problems

• Grover's data search, Shor's factoring algorithm···

✦<u>Gate based quantum computing</u>

• Error correction correction

• Individual gate operation and error correction for many qubits are experimentally hard

• Hubbard model, Ising models, Heisenberg models···

✦<u>Quantum annealing</u>

• Error correction code

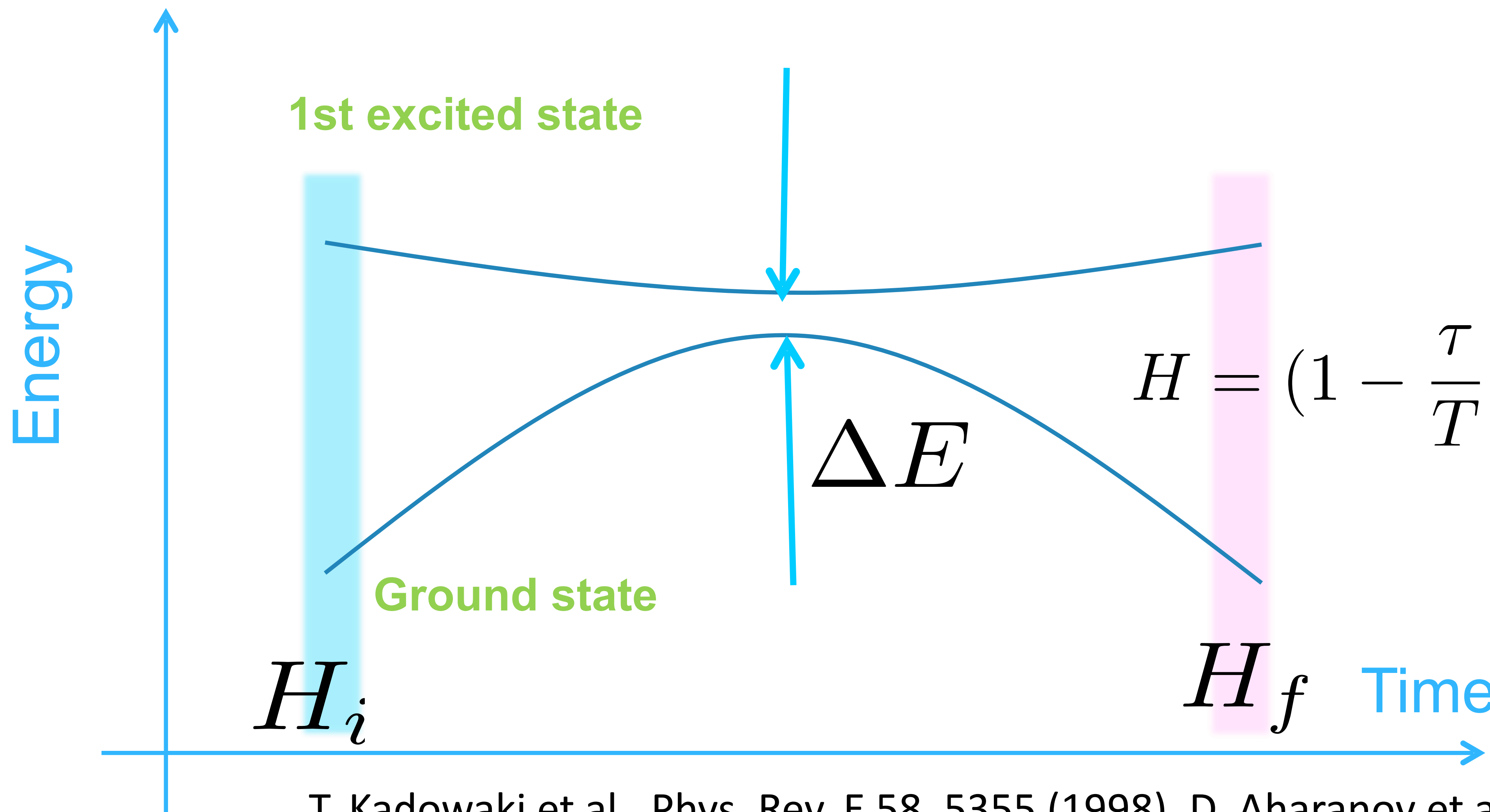
• Same computational complexity class as universal quantum computation

• Macroscopic operation for whole system is relatively easy to implement

Quantum annealing

- ✓ Ground state of H_i : determine as an easily accessible state
- ✓ Sweep Hamiltonian slowly enough (adiabatic) : system remains in ground state
- ✓ Ground state of H_f encodes the solution to a hard computational problem
- ✓ Energy gap decreases exponentially
- ✓ The number of excited states increases exponentially
- ✓ Equivalent computational power to quantum search ~O(N^(1/2))

- Macroscopic operation of Hamiltonian change : easier to implement



$$H = (1 - \frac{\tau}{T})H_i + \frac{\tau}{T}H_f$$

T. Kadowaki et al., Phys. Rev. E 58, 5355 (1998), D. Aharanov et al., SIAM Rev. (2008)

Ising model

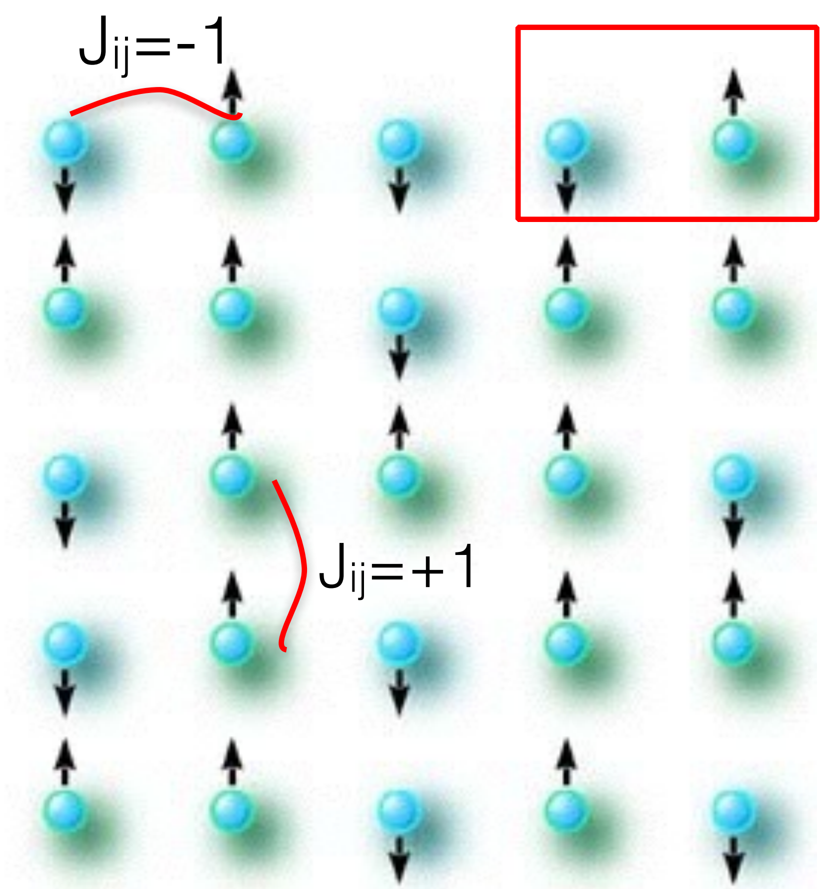
Mathematical abstraction of ferromagnetism in statistical mechanics

Total energy is determined by combination of Ising spins which takes discrete↑up or↓down

$$\mathcal{H} = -\sum_{i,j} J_{i<j}\sigma_{iz}\sigma_{jz}$$

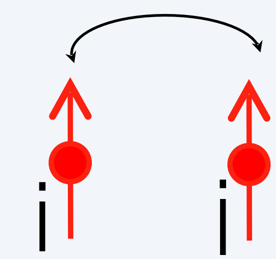
Ising spin σ_i: -1 or +1

Ising coupling J_{ij} : coupling coefficient



mutual interaction（J_{ij}）

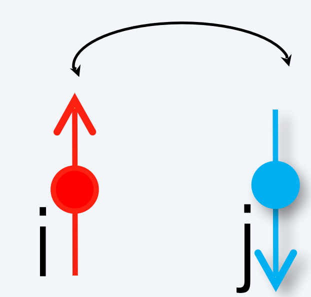
J_{ij} = +1



takes the same Ising spins when J_{ij} >0

$J_{ij}\sigma_{iz}\sigma_{jz} = +1$

J_{ij} = -1



takes opposite directions when J_{ij} <0

Ising problem

Barahona 1982

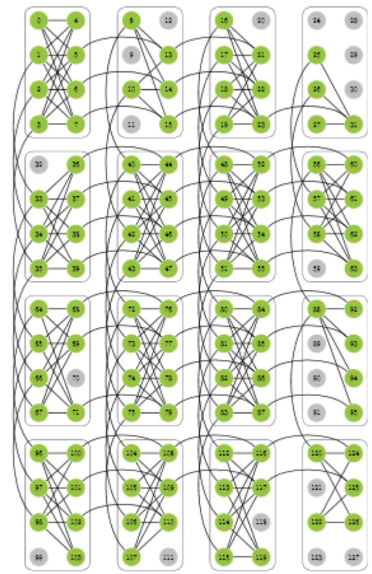
to find the ground state spin configuration of Ising model

1,2 dimensional Ising problem → class P

2 dimensional + Zeeman term
3 dimensional Ising problem → class NP-hard

Ising machine ~ non Von Neumann type computer architecture

D-wave II using quantum annealing (2013)



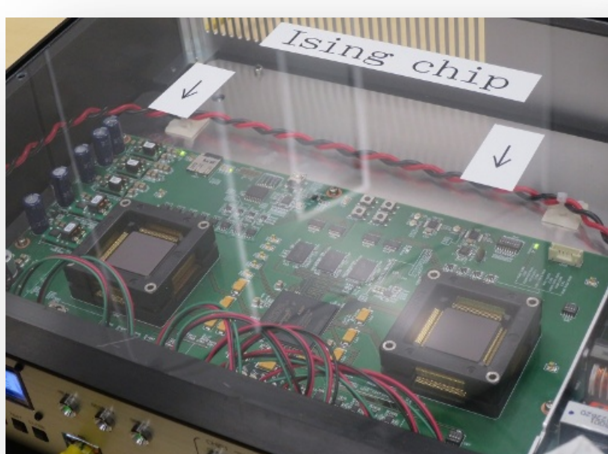
Chimera graph
(D-wave one N=128)

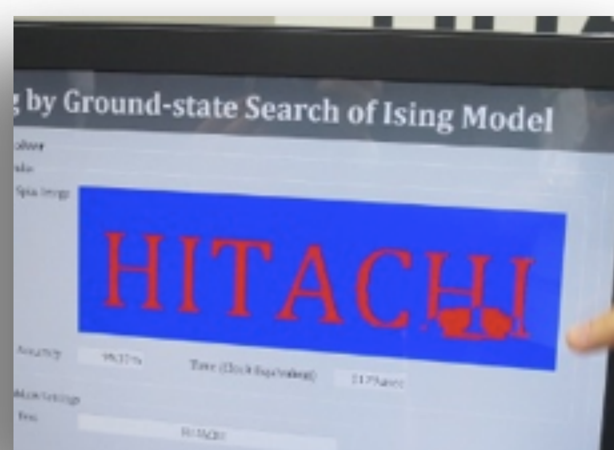
D-WAVE II Ising machine with QA installed at Google / NASA AMES quantum computing AI lab (2013)

- Ising machine with 512 flux qubit
- Cryogenic operation (~20mK)
- Sparse connectivity limitation
- N~20 complete graph implemented
- defeated SA for some instances

T. Kadowaki et al., Phys. Rev. E 58, 5355 (1998)
D. Aharanov et al., SIAM Rev. (2008)
S. Boixo et al, Nat. Phys. 10, 218–224 (2014)

Hitachi CMOS Ising chip (2015)





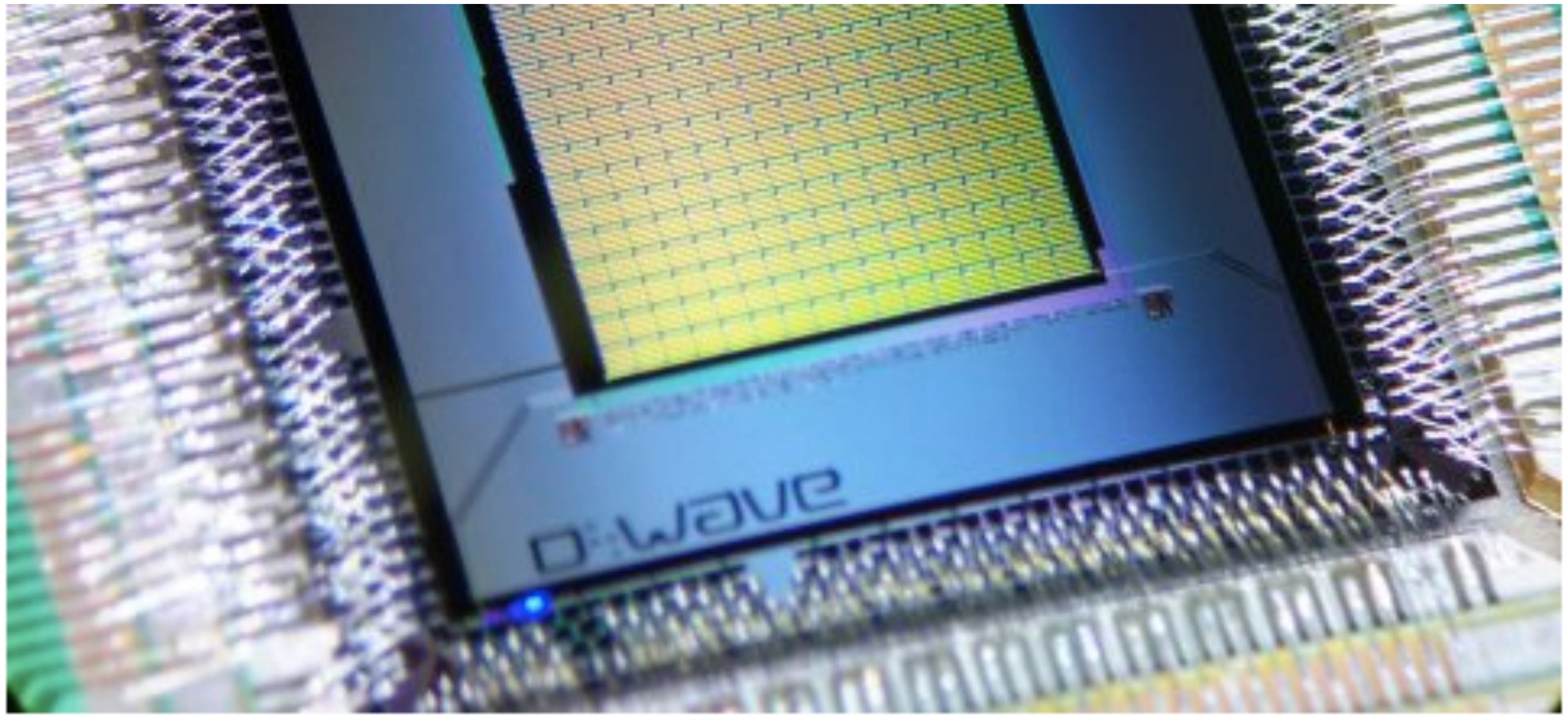
Graph cut problem

- N=20,480 Ising chip
- Computation time (approximate solution) : 1ms
- 4x3=12mm^2
- 65nm fabrication process
- low consumption power

http://itpro.nikkeibp.co.jp/atcl/column/14/346926/022000173/

C. Yoshimura et al., DOI: 10.1109/ECCTD.2013.6662276 (2013)

New generation D-wave 2X^{TM} : 1152 qubits



JUN 22, 2015

D–Wave Systems Breaks the 1000 Qubit Quantum Computing Barrier

New Milestone Will Enable System to Address Larger and More Complex Problems

Palo Alto, CA – June 22, 2015 – D–Wave Systems Inc., the world's first quantum computing company, today announced that it has broken the 1000 qubit barrier, developing a processor about double the size of D–Wave's previous generation and far exceeding the number of qubits ever developed by D–Wave or any other quantum effort. This is a major technological and scientific achievement that will allow significantly more complex computational problems to be solved than was possible on any previous quantum computer.

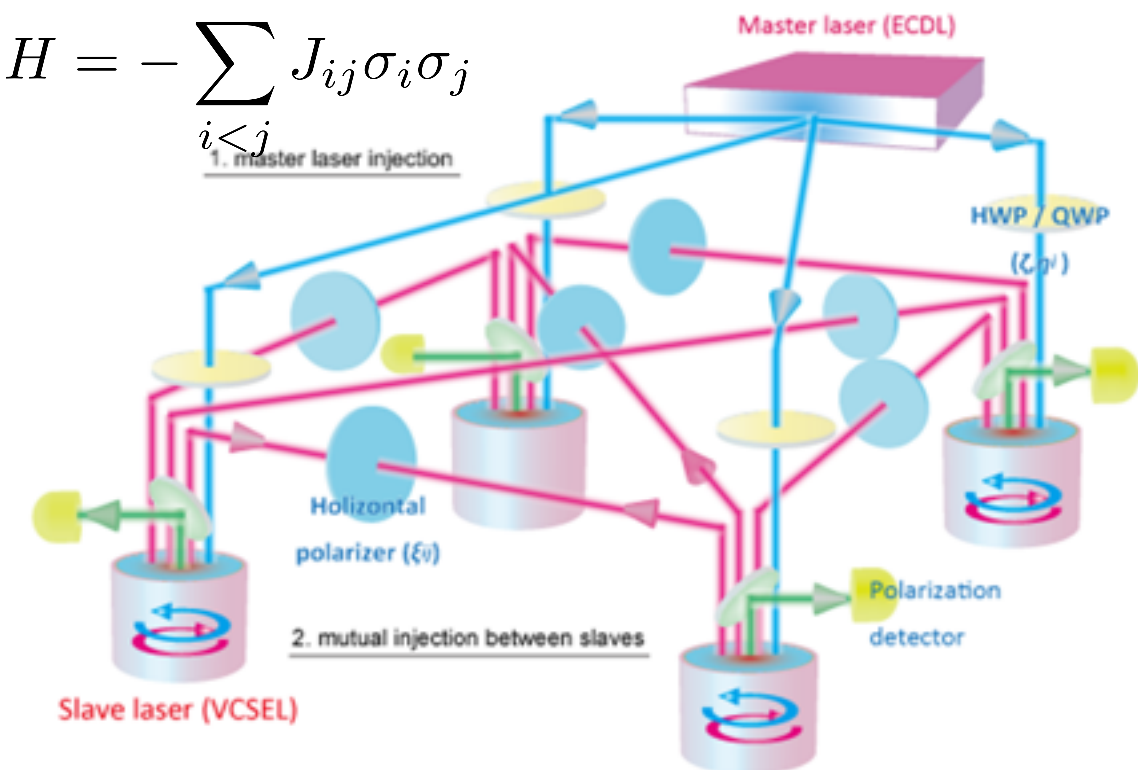
* from D-wave official website

Coherent Ising machine using Laser/DOPO network

Laser network

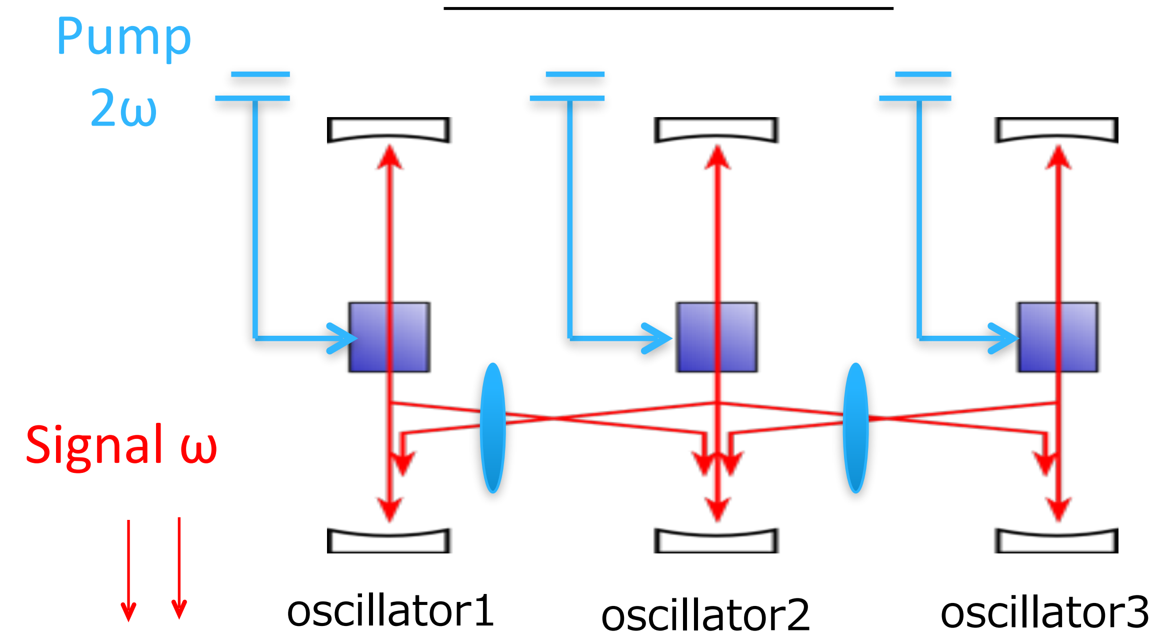
Mapping Ising Hamiltonian onto an injection-locked laser network

$$H = -\sum_{i<j} J_{ij}\sigma_i\sigma_j$$



S. Utsunomiya, et al. Optics Express 19(19) 18091-18108(2011)

DOPO network



Laser network	Ising model
Minimum loss	Ground state

Mutually coupled laser network modulates their effective total loss. Spontaneous oscillation modes tend to minimize the total loss.

Ising spin $\sigma_i^z = +1 \; or \; -1$: Oscillation modes of oscillators

Ising coupling J_{ij}: coupling coefficient between oscillators (Intensity and phase)

- Each oscillator spontaneously select their oscillation modes
- Oscillation modes among the 2^N possible combination corresponds to Ising ground state

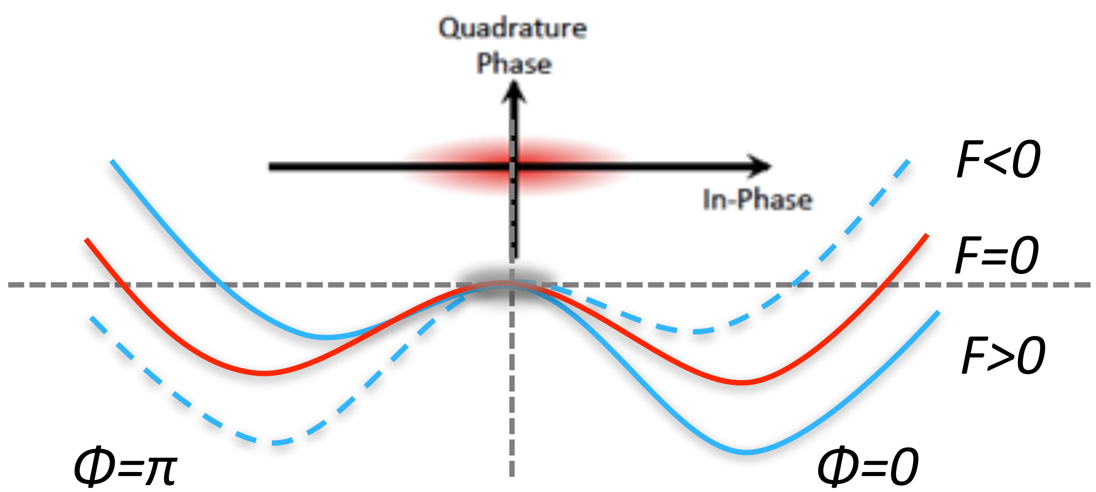
C-LGE for numerical simulation of DOPO network

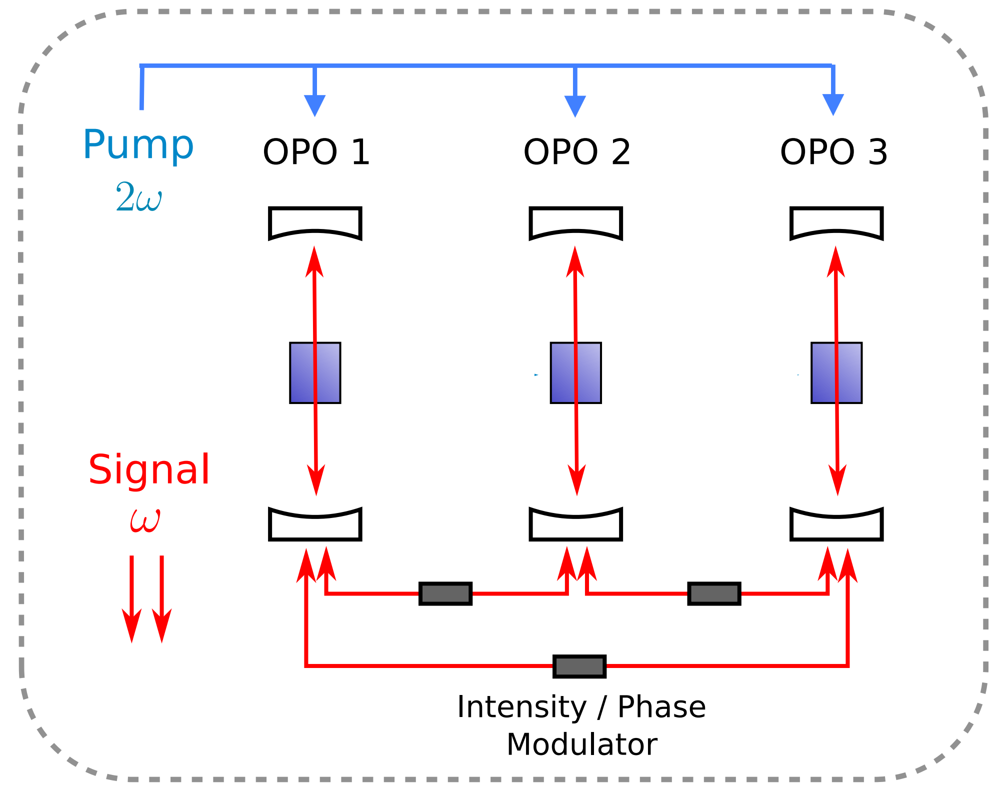
C-number Langevin equation of in-phase signal field of a laser network

$$dc_i = [\underbrace{(-1+p-c_i^2-s_i^2)c_i}_{\text{Single DOPO}} + \underbrace{\sum_j \xi_{ij}\tilde{c}_j}_{\text{Ising interaction}}]\, dt + \underbrace{\frac{1}{A_s}\sqrt{c_i^2+s_i^2+\frac{1}{2}}\,dW_i}_{\text{quantum noise}}.$$

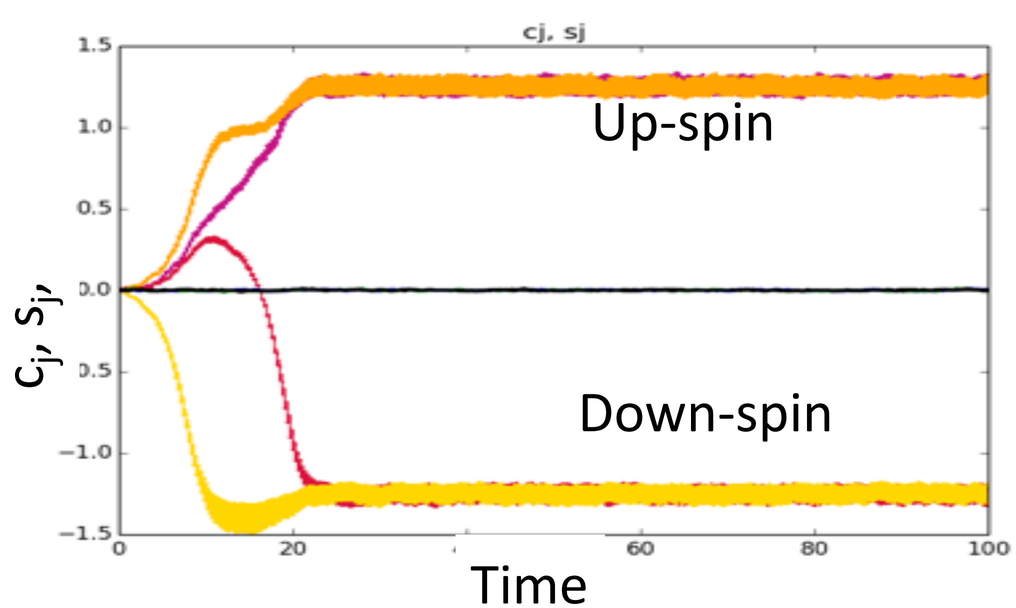
derived from Q-FPE by Ito stochastic integral with adiabatic elimination of pump

Injection signal breaks the balance





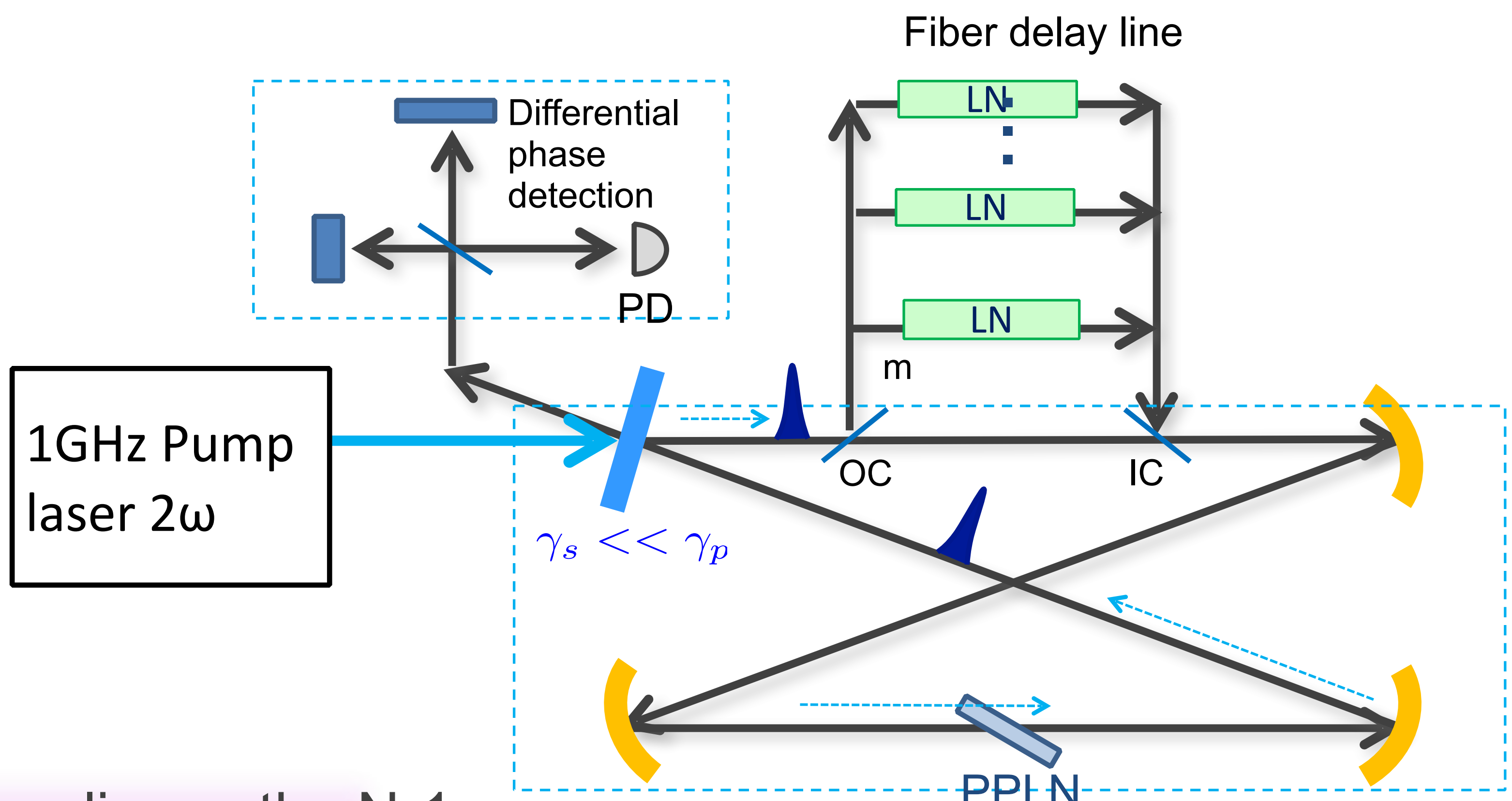
Mutual injection between lasers modulates total loss



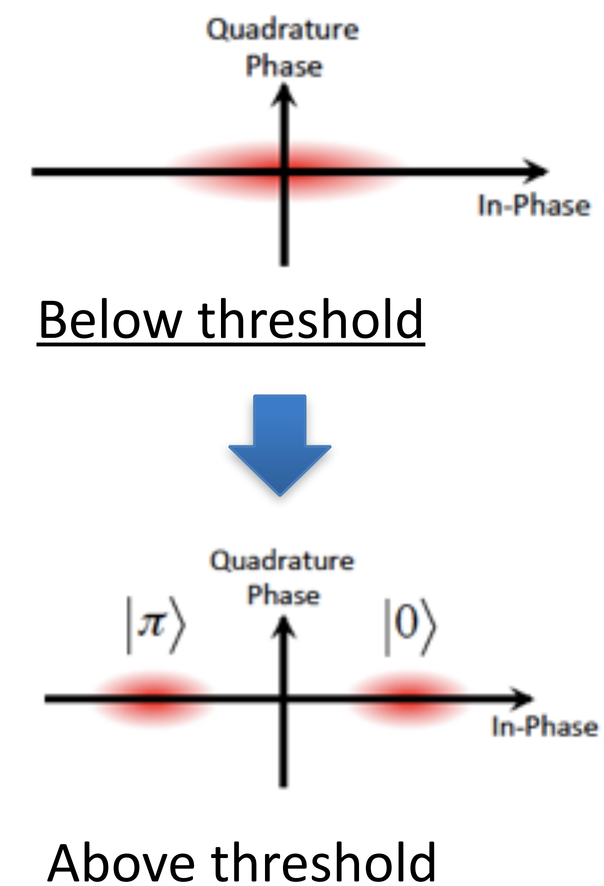
- ✓ Mutual injection modulates total loss of network
- ✓ Quantum noise is the driving force of phase amplification
- ✓ Mutual coupling fixes the wrong phase configuration
- ✓ Computation time is determined by the turn-on delay

Coherent Ising machine using DOPO network

Fiber delay lines to implement Ising interaction



Signal: ω: 1550nm



# of mutual coupling path : N-1

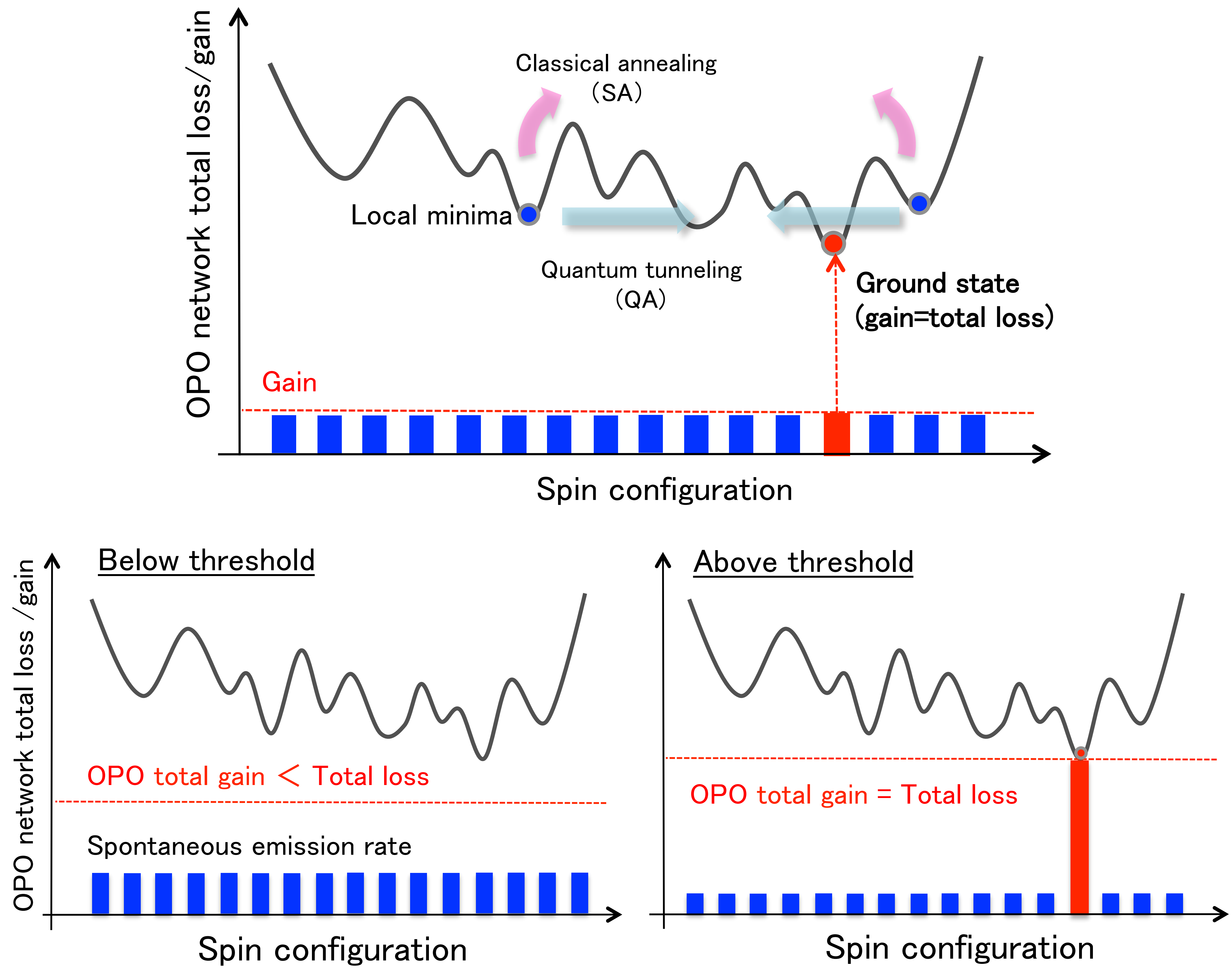
Ising Hamiltonian : $\hat{H} = \sum_j J_{ij} \sigma_i^z \sigma_j^z$

Ising spin $\sigma_i^z = +1 \ or \ -1$: oscillation mode（phase)

Ising interaction J_{ij} : Mutual coupling strength (phase, intensity)

Alireza et al., *Nat. Phot.* (2014) doi:10.1038/nphoton.2014.249

Operational principle of SA, QA, CIM



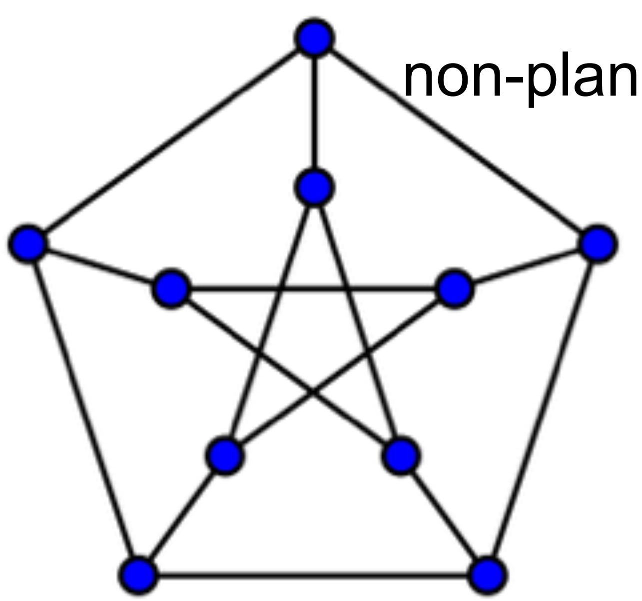
Success probability to find ground state of MAX-CUT-3 graphs

The ground state configuration of Ising Hamiltonian gives a MAX-CUT(maximum cut) between two vertex groups (S, V\S).

$$\min H_{\text{Ising}} = \sum_{i<j} J_{ij} - 2\text{maxcut(G)}$$

(if $J_{ij} = -w_{ij}$)

non-planar



Petersen Graph (N=10)

max cut =10

C-SDE for j-th signal pulse

$$\frac{d}{dt} c_j = \left(-1 + p - (c_j^2 + s_j^2)\right) c_j + \sum_{l=1, l\neq j}^{N} \xi_{jl} c_l$$

Number of possible cubic graphs

Order	4	6	8	10	12	14	16	18	20
Cubic graphs	1	2	5	19	85	509	4060	41301	510489

Worst Case Success Probability

	Order							
	4	6	8	10	12	14	16	20
q_{min}	0.93	1.00	0.41	0.54	0.52	0.37	0.33	0.11
(at well above oscillation threshold: p=1.1, ξ=-0.1)								
q_{opt}	1.00	1.00	0.70	0.74	1.00	1.00	1.00	0.74
(at optimum pump rates)								

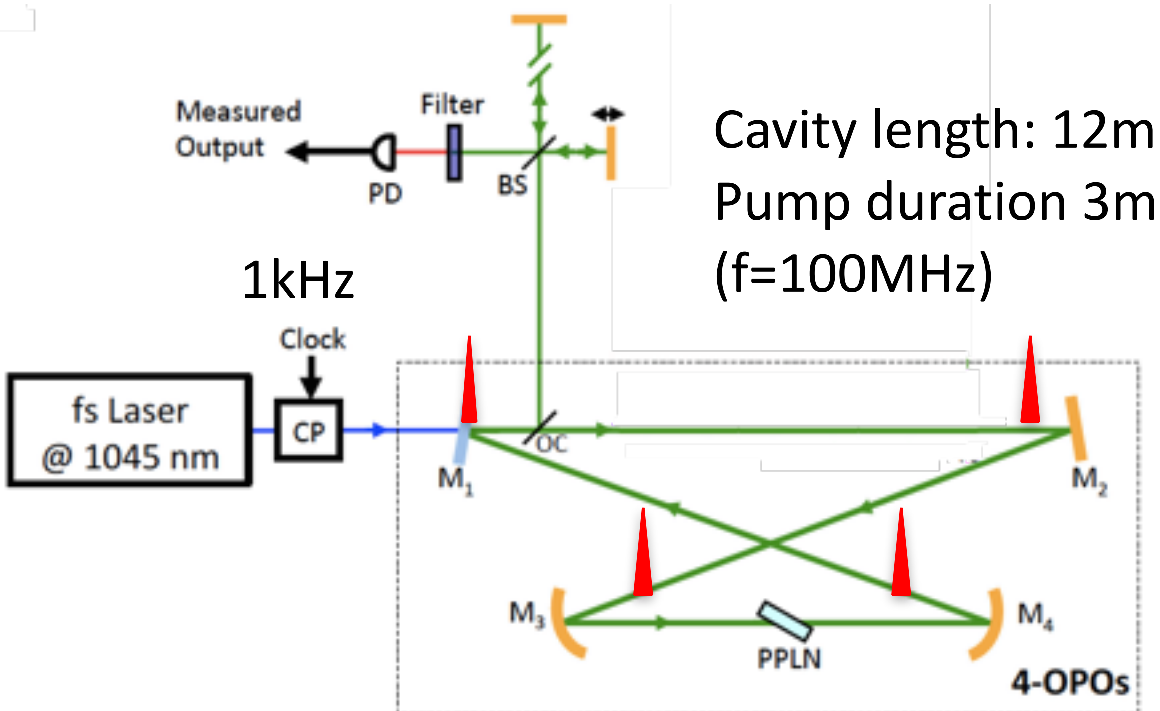
Brute Force Search 3×10^{-1} → 2×10^{-6}

(exponential scaling)

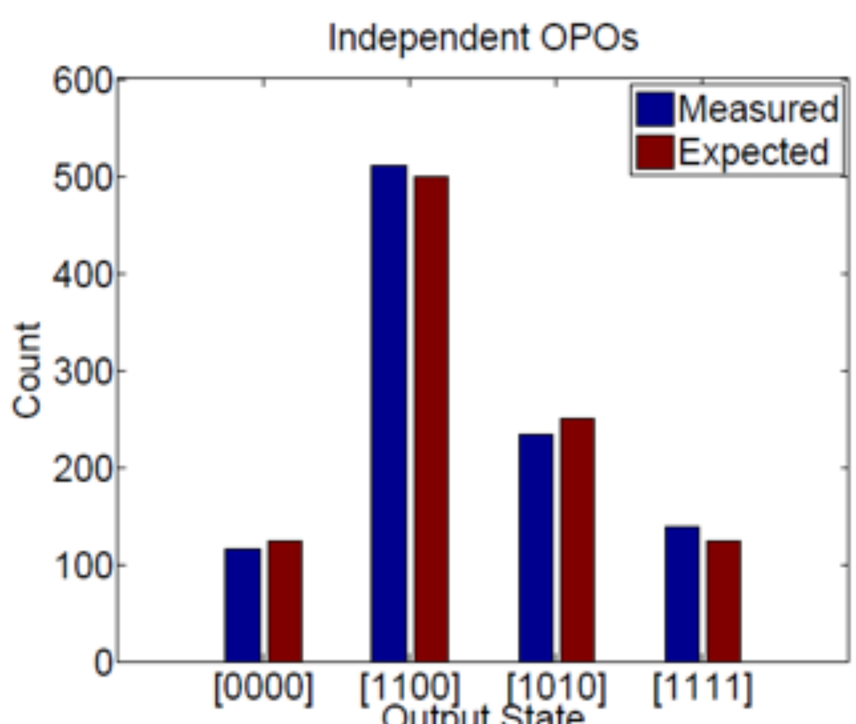
Z. Wang et al., Phys. Rev. A 88, 063853(2013)

M = 4 DOPO Ising machine experimental result

A. Marandi et al., Nature Photonics (2014)
doi:10.1038/nphoton.2014.249



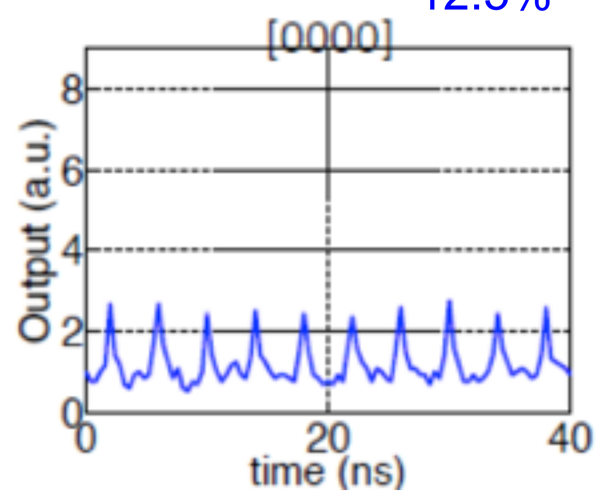
Cavity length: 12m
Pump duration 3m
(f=100MHz)



16 patterns of phase amplification in 4 optical pulses
|pulse1, pulse2, pulse3, pulse4>

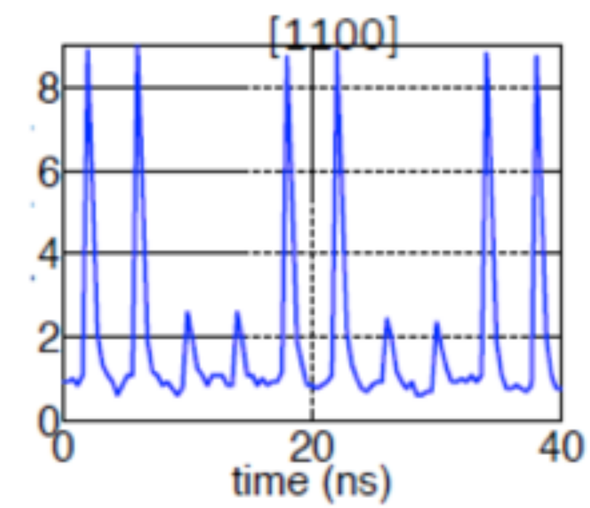
$|0\pi0\pi\rangle, |\pi0\pi0\rangle$

12.5%



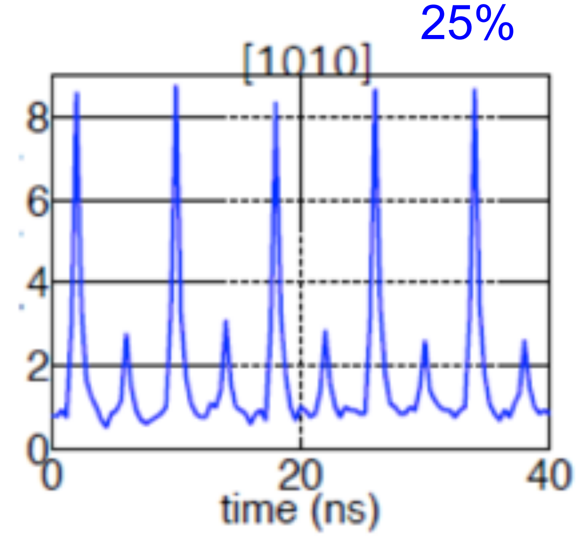
50%

$|\pi000\rangle, |0\pi00\rangle, |00\pi0\rangle, |000\pi\rangle,$
$|\pi\pi\pi0\rangle, |\pi0\pi\pi\rangle, |\pi\pi0\pi\rangle, |\pi\pi\pi0\rangle$



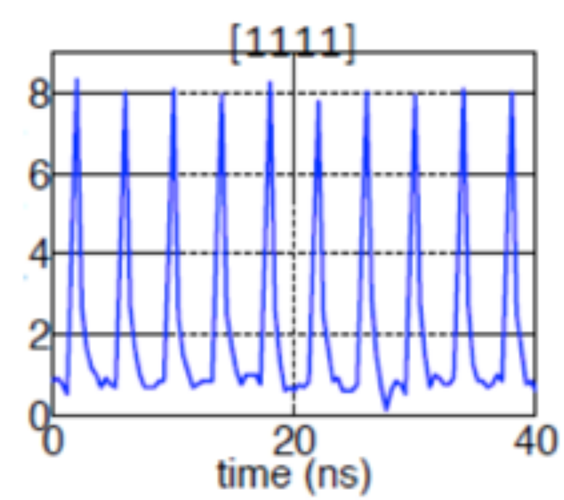
$|0\pi\pi0\rangle, |\pi\pi00\rangle,$
$|\pi00\pi\rangle, |00\pi\pi\rangle$

25%



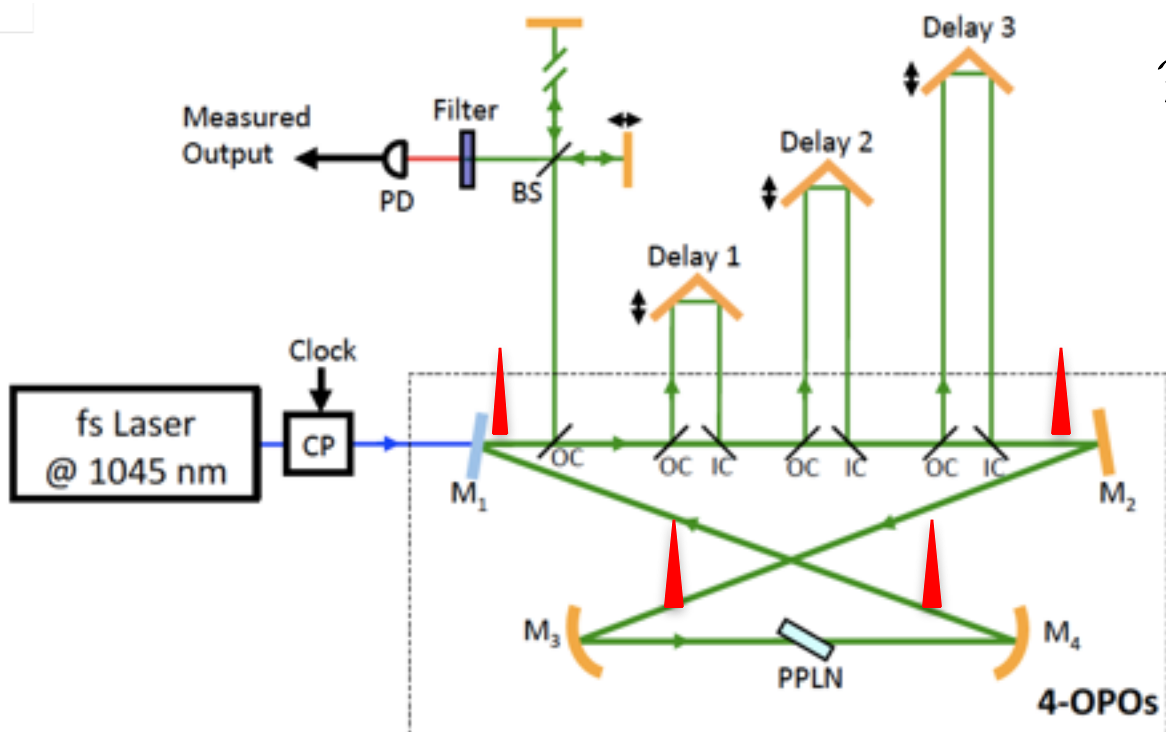
$|0000\rangle, |\pi\pi\pi\pi\rangle$

12.5%



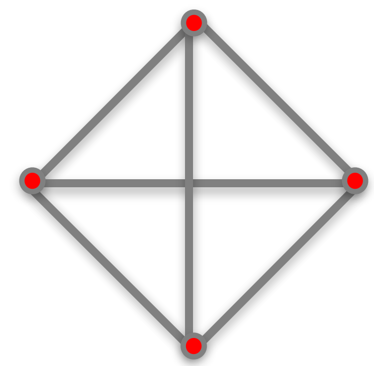
M = 4 DOPO Ising machine experimental result

A. Marandi et al., Nature Photonics (2014)
doi:10.1038/nphoton.2014.249



M=4 Ising model

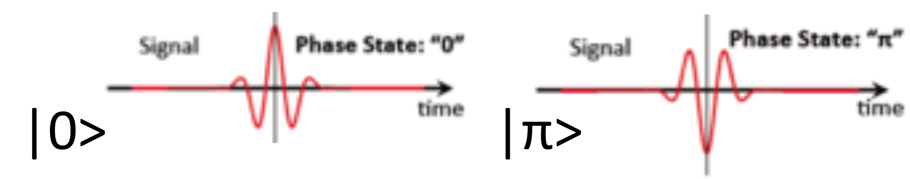
$$\mathcal{H} = -\sum_{i,j} J_{i<j}\sigma_{iz}\sigma_{jz}$$



Jij=-1 for all edges

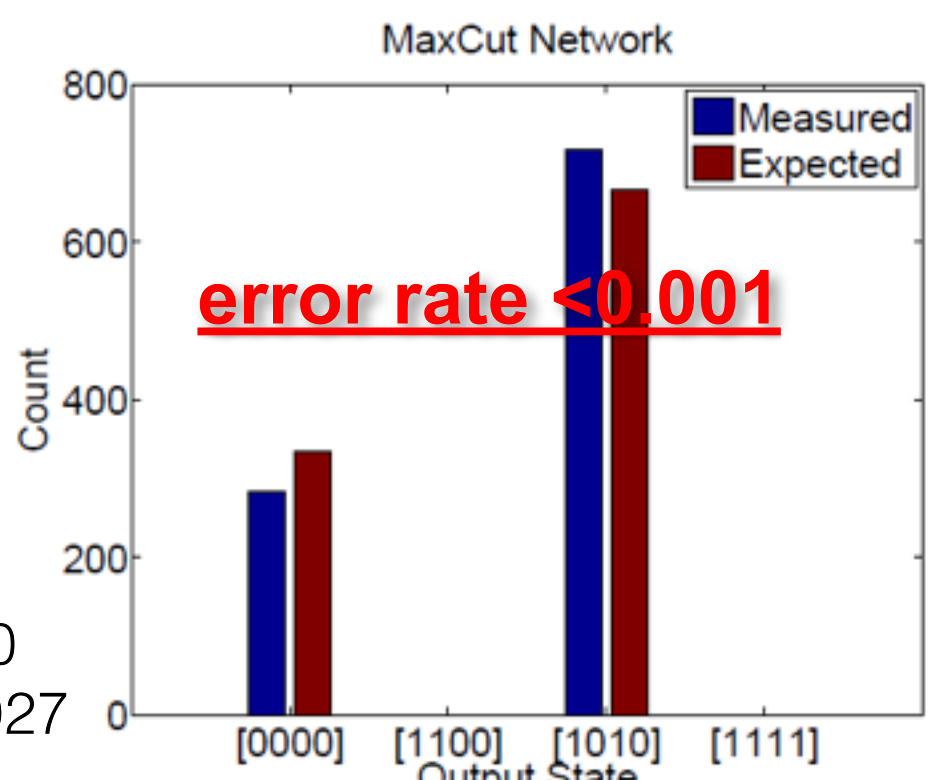
Out of phase coupling (J_{12} <0)

Anti-ferromagnetism :
(0,π) or (π, 0)



6 degenerate ground states

↑↑↓↓, ↑↓↑↓, ↑↓↓↑
↓↓↑↑, ↓↑↓↑, ↓↑↑↓

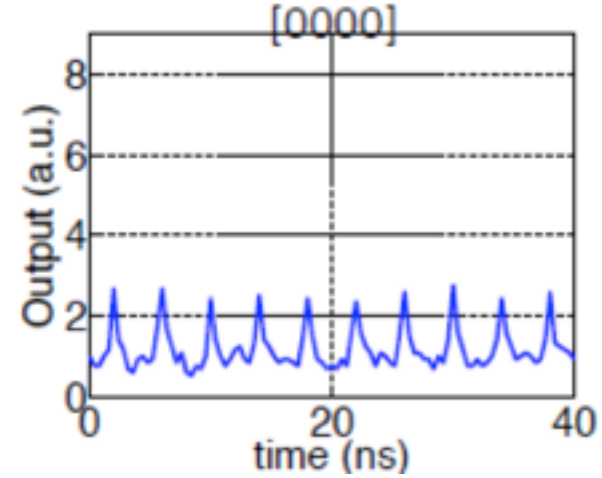


error rate <0.001

MM 1.0
SM 0.927

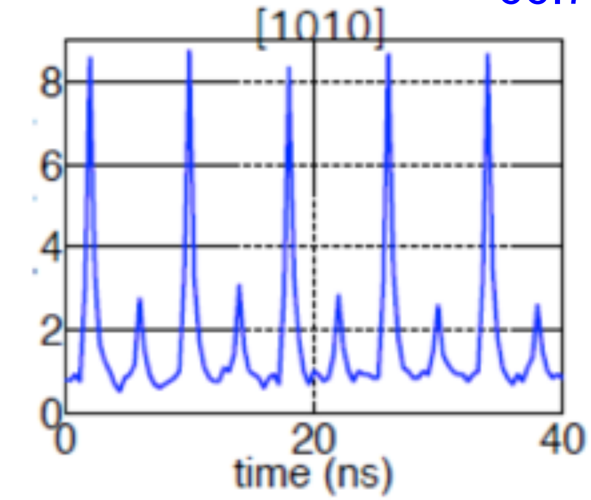
$|0\pi0\pi\rangle, |\pi0\pi0\rangle$

33.3%

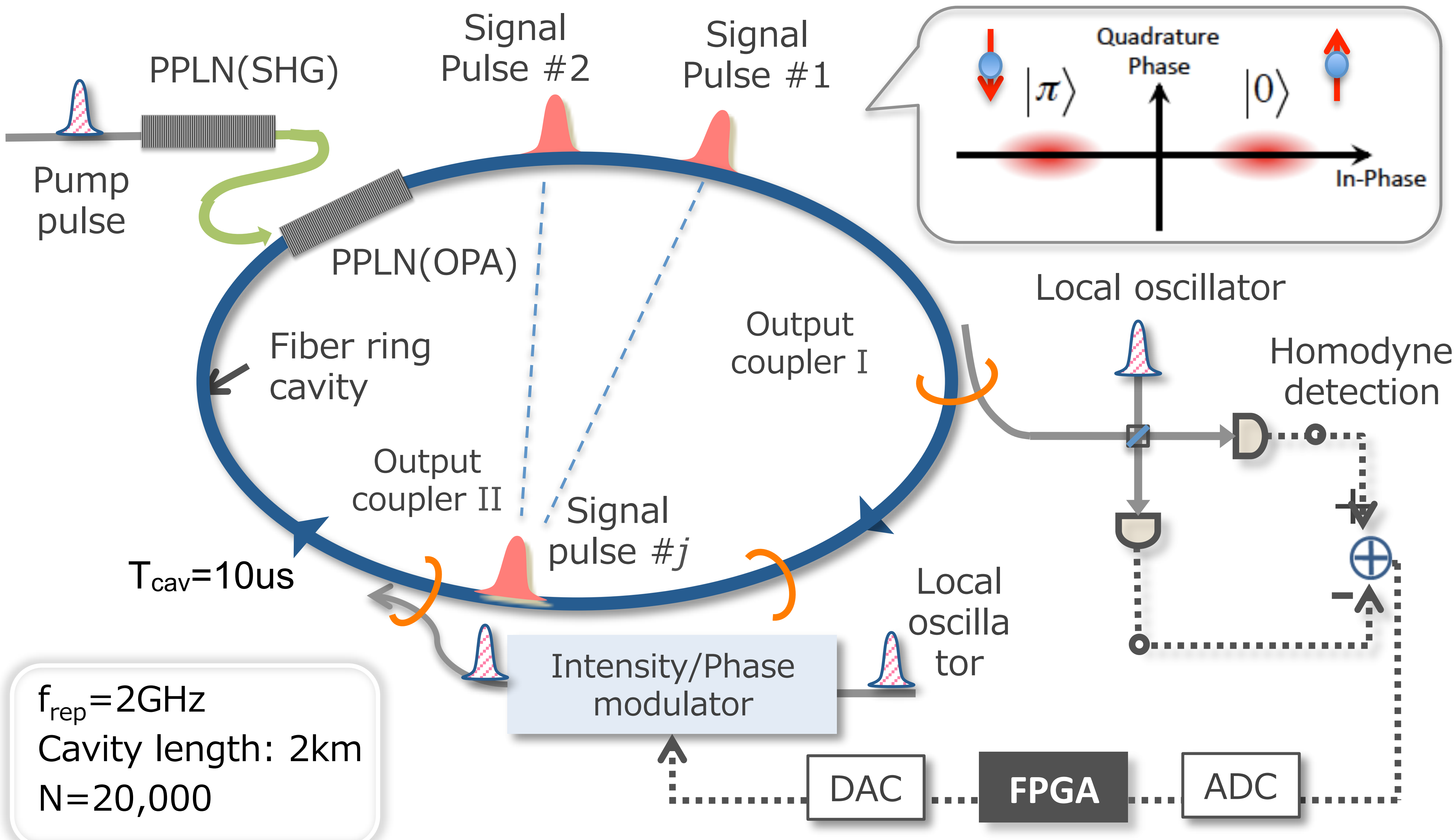


$|0\pi\pi0\rangle, |\pi\pi00\rangle,$
$|\pi00\pi\rangle, |00\pi\pi\rangle$

66.7%



Measurement based DOPO network towards large scale of CIM

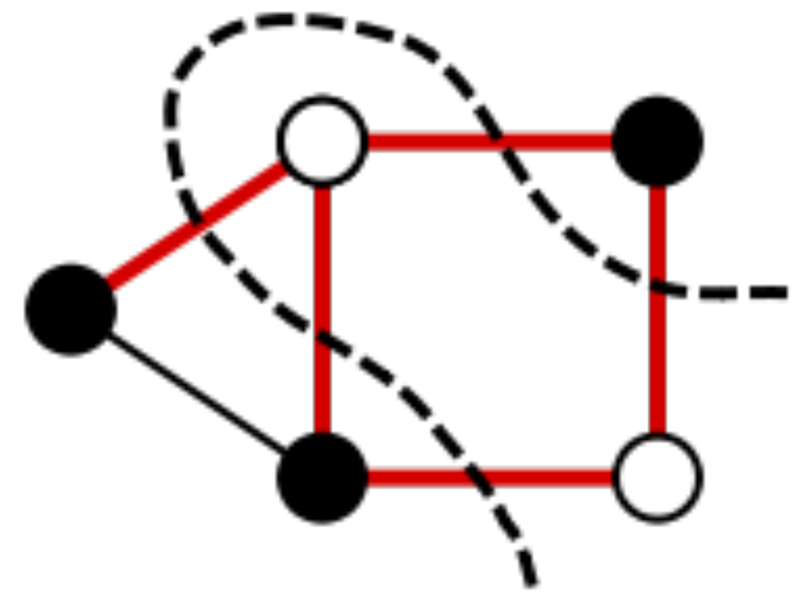


- ✓ Pulses are measured by Homodyne detection at every round trip
- ✓ based on measured σ_{iz},, feedback terms are calculated on FPGA and apply modulation to feedback pulse

MAX-CUT problem for benchmark study

$$\min H_{\text{Ising}} = \sum_{i<j} J_{ij} - 2\text{maxcut(G)}$$

- MAX-CUT problem : find the size of the largest cut in a given graph.
  - a cut is a partition of the vertices V into two sets $\{\sigma = +1\}$, $\{\sigma = -1\}$
  - the size of a cut is the number of edges crossing the cut
    $\underline{(1 - \sigma_i\sigma_j)/2 = 1}$



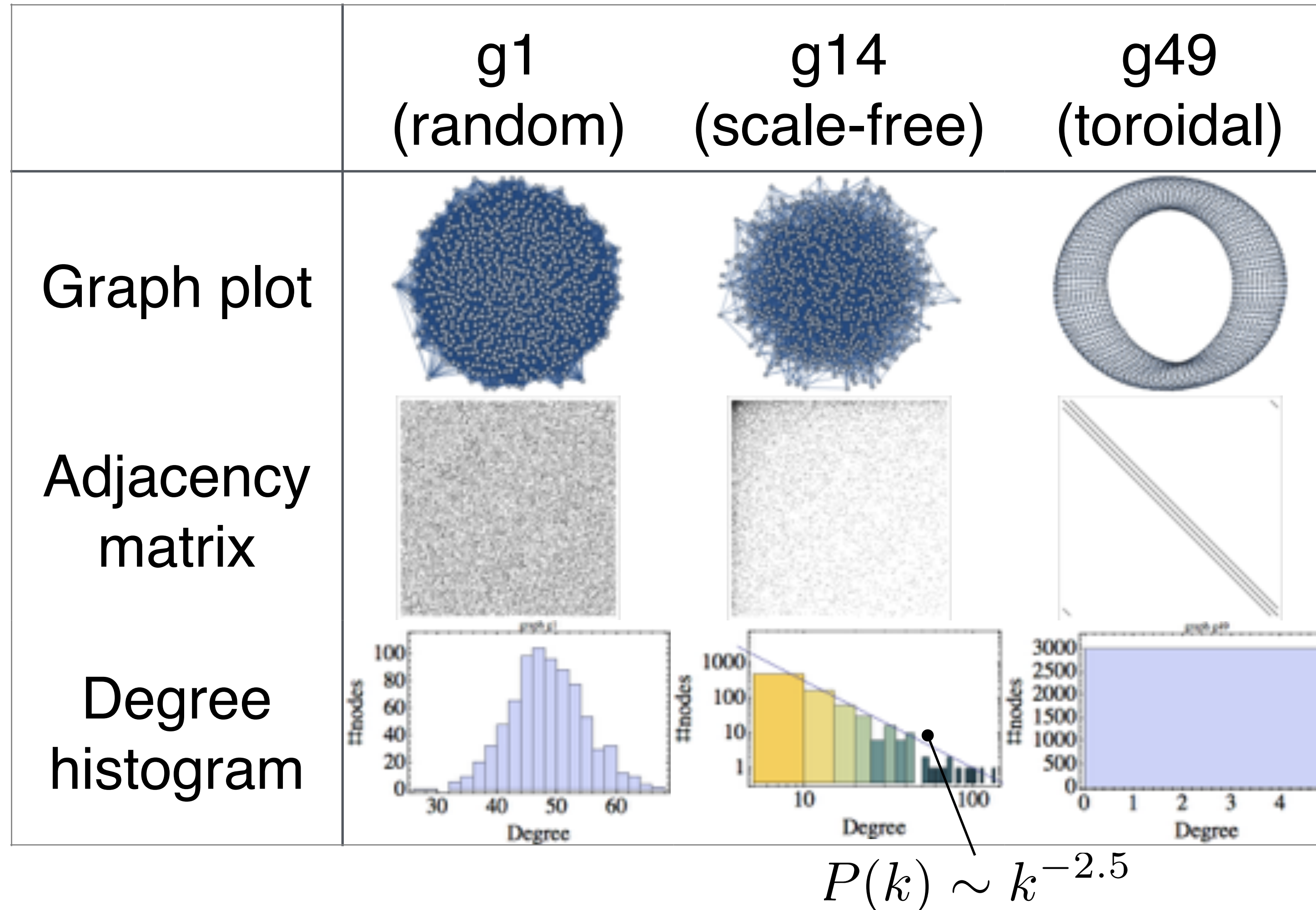
$$5 = \sum_{i<} w_{ij}(1 - \sigma_i\sigma_j)/2$$

- <u>NP-hard unless the graph is planar.</u>

	approx. ratio	complexity class
		NP-hard
P≠NP, PCP theorem	0.94117	
GW SDP, Unique Games Conjecture	0.87856	
		P

MAX-CUT Benchmark by G-set

- Problem set : G-set
  - 71 graph instances
  - 800 ≦ #V ≦ 20000
  - 1600 ≦ #E ≦ 42000
  - 2 ≦ avg deg ≦ 48
  - weight : {1} or {+1, -1}
  - 3 types
    - (Random, combination of planar, Toroidal)



- on Intel Xeon (2.67 GHz).

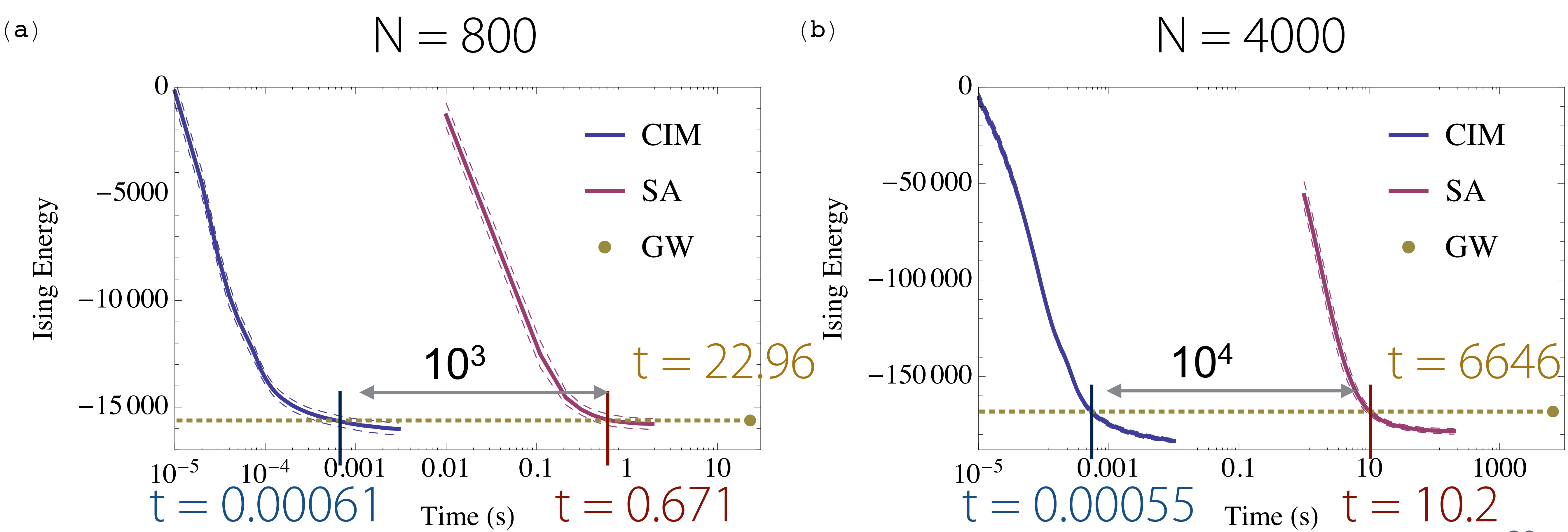
Coupling constant

$$\xi_{ij} = \xi w_{ij} = -\frac{0.4}{d_{\text{avg}}} w_{ij}$$

How fast Ising energy decreases?

- Computational time is defined as the time to **GW SDP** solution.
- Pure computational time (without I/O time).
- 10^3 faster than SA for the graph of order N = 800, 10^4 faster for N = 4000.

Y. Haribara et al.,arXiv:1501.07030v3 [quant-ph]

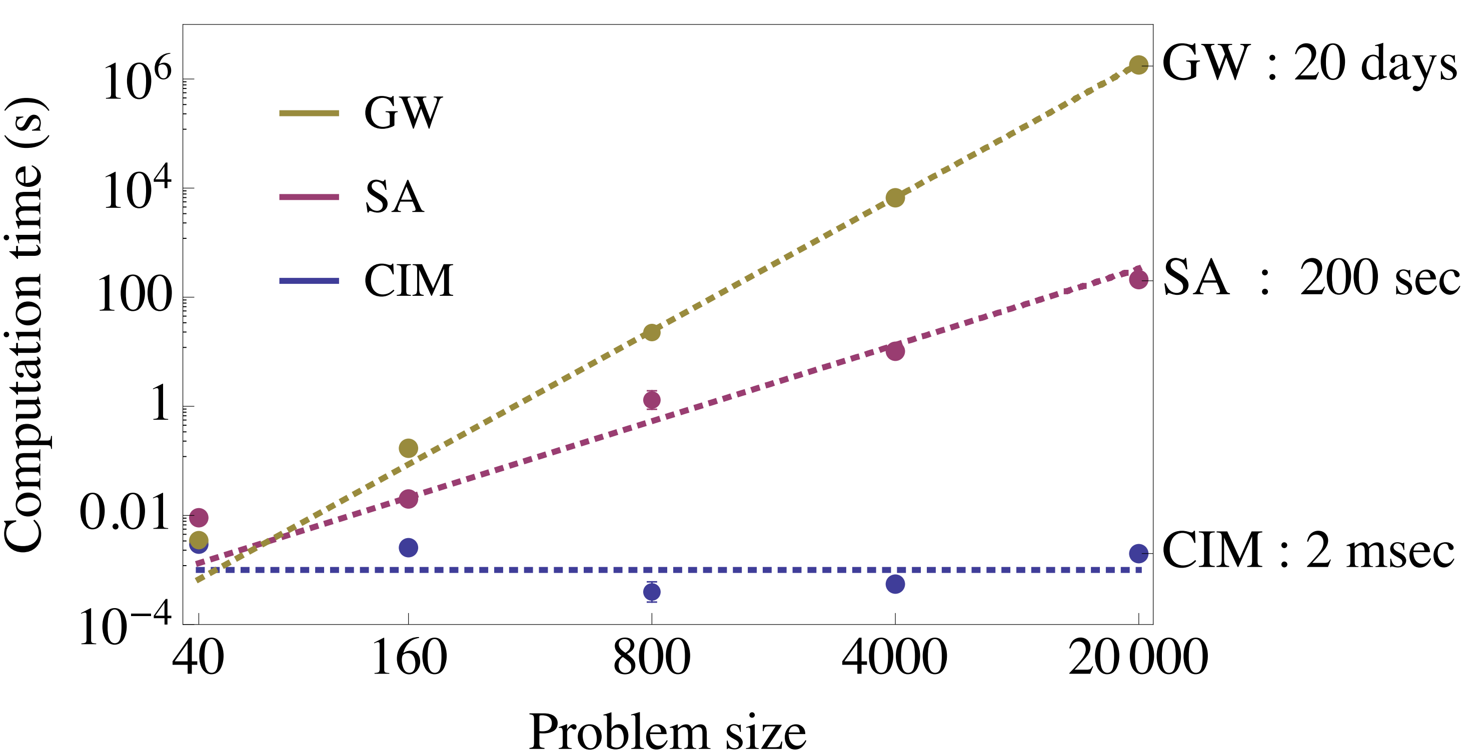


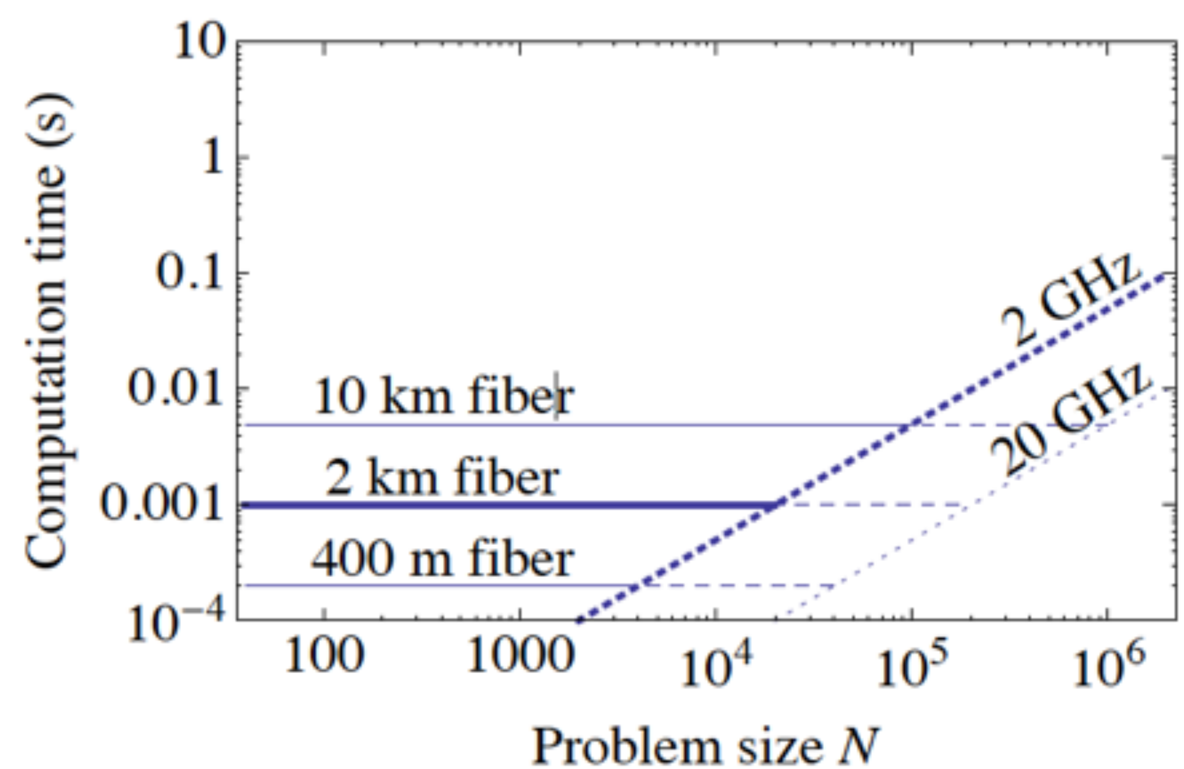
Computation Time Scaling

GW: Goemans-Williamson SDP algorithm
- 87.8% Performance guarantee,
- Interior point method for solving SDP $O(N^{3.5})$

SA : Simulated Annealing (Heuristic algorithm)
- N per Monte Carlo step × vertex degree (N). $O(N^2)$

CIM: Coherent Ising machine
- FPGA calculation is done less than 1um
- Turn on delay of DOPO . O(1)





Y. Haribara et al.,