Title and abstract for Mini workshop ``Various aspects of singularities"

Christopher Chiu

Title: On arc fibers of morphisms of schemes

Abstract: In this talk we aim to study the fibers of the map on arc spaces induced by a morphism of algebraic varieties. For finite morphisms, we prove that the arc fibers are topologically finite in general; and that they are scheme-theoretically finite away from the ramification locus. As a byproduct, we obtain a criterion for maps on arc spaces to be locally of finite type. We will discuss various consequences of these results, including that the local ring at stable points is topologically Noetherian and semicontinuity properties for embedding (co)dimension in the arc space. This is joint work with Tommaso de Fernex and Roi Docampo.

Makoto Enokizono

Title: Slope inequality of fibered surfaces and moduli of curves

Abstract: The slope inequality of fibered surfaces plays an important role in the classification problem of algebraic surfaces of general type.

It is known that some positive properties of a linear combination of tautological divisors on the moduli spaces of stable curves produce many slope inequalities of semi-stable fibered surfaces via the moduli maps.

In this talk, I will explain that many slope inequalities of not necessarily semi-stable fibered surfaces can be obtained by using the moduli space of curves which admit worse singularities than nodes.

If time permits, I will also explain related topics and problems.

Herwig Hauser(University of Vienna)

Title : "Algebraicity in Geometry (and Transcendence)"

Abstract

The singular locus of an algebraic variety is a Zariski-closed subvariety. This is not completely obvious, as regularity is defined pointwise via the local rings, or, alternatively, by requiring that their completions are formal power series rings.

More generally, one can then ask for the subsets where the completed local rings are isomorphic to an arbitrarily given complete local ring. Using Artin approximation, one can

show, as one would expect, that these sets are indeed Zariski-locally closed. We further show that the variety has a local product structure along them (joint with C. Chiu). The proof of both statements is, however, a bit involved.

(In a second part, and if time permits, we will address algebraicity criteria for a priori transcendent formal power series. They are given as solutions of linear differential equations, and yield to very deep and difficult conjectures. Some examples will illustrate the intricacy of the problem.)

Shinobu Hikami (OIST)

Title: Amida-kuji (阿弥陀籤) and singularities

Abstract: The title looks like for high-school students. It is, however related to the knots and quantum gravity.

We start from random matrix formulation in the replica limit N=0, where N is a size of matrix. This is equivalent to Seifert surface for 1-knot. The 2-knot is related to time-dependent matrix model, which is expressed by 2-matrix model. This knot is embedded into four dimensional sphere S^4 and is related to four dimensional gauge theory. This talk is aimed for the understanding of the singularity from matrix models.

Geometric aspects of ideals in a normal surface singularity

by Tomohiro Okuma (Yamagata University)

This is a joint work with Ken-ichi Yoshida and Kei-ichi Watanabe (M. E. Rossi for some part). Throughout this talk, let $(A, \mathfrak{m})$ be an excellent 2-dimensional normal local domain containing an algebraically closed field k, unless otherwise specified. Let $I \subset A$ be an $\mathfrak{m}$-primary integrally closed ideal. Then there exist a resolution of singularities $X \to \operatorname{Spec} A$ and an anti-nef cycle Z on X such that $I\mathcal{O}_X = \mathcal{O}_X(-Z)$, $I = H^0(X, \mathcal{O}_X(-Z))$; we say that I is *represented* by Z on X, and write $I = I_Z$. If we have a representation of an ideal $I = I_Z$, then the ring-theoretic properties of I are determined by the invertible sheaf $\mathcal{O}_X(-Z)$ on the resolution space X. In this talk, we introduce some results obtained from this viewpoint. For example, the normal reduction number of $I = I_Z$ is given in terms of cohomology of $\mathcal{O}_X(-nZ)$, and the characterization and the existence of the p_g-ideals are described by using the cohomological cycle (similar results hold for elliptic ideals). We also have a representation of the core of p_g-ideals and certain special elliptic ideals, and the finiteness of the ideals I with Gorenstein normal tangent cone $\overline{G}(I)$ for a Gorenstein elliptic singularity $(A, \mathfrak{m})$.

References

[OWY1] T. Okuma, K.-i. Watanabe, and K. Yoshida, *Good ideals and p_g-ideals in two-dimensional normal singularities*, Manuscripta Math. **150** (2016), no. 3-4, 499–520.

[OWY2] ——— *Rees algebras and p_g-ideals in a two-dimensional normal local domain*, Proc. Amer. Math. Soc. **145** (2017), no. 1, 39–47.

[OWY3] ——— *A characterization of 2-dimensional rational singularities via core of ideals*, J. of Algebra **499** (2018), 450–468.

[OWY4] ——— *Normal reduction numbers for normal surface singularities with application to elliptic singularities of Brieskorn type*, Acta Mathematica Vietnamica **44** (2019), no. 1, 87–100.

[OWY5] ——— *The normal reduction number of two-dimensional cone-like singularities*, Proc. Amer. Math. Soc. **149** (2021), no. 11, 4569–4581.

[OWY7] ——— *Gorensteinness for normal tangent cones of elliptic ideals*, submitted (available from arXiv: 2302.07991v1 [math.AC] 15 Feb 2023).

[ORWY] T. Okuma, M. E. Rossi, K.-i. Watanabe, and K. Yoshida, *Normal Hilbert coefficients and elliptic ideals in normal 2-dimensional singularities*, Nagoya Math. J. **248** (2022), 779–800.

Kenta Sato

Title: Geometrically log canonicity of log canonical surfaces in positive characteristic

Abstract:

In this talk, we give a sufficient condition for log canonical surface singularities over a field to be geometrically log canonical.

As an application, we prove that if a 3-dimensional quasi-projective variety X over an algebraically closed field of characteristic p>3 has only log canonical singularities, then so does a general hyperplane section H of X.

Kohsuke Shibata

Title: Shokurov's index conjecture for quotient singularities

Abstract

The minimal log discrepancy is an invariant of singularities defined in birational geometry. Shokurov conjectured that the Gorenstein index of a Q-Gorenstein germ can be bounded in terms of its dimension and minimal log discrepancy. This conjecture is useful to study minimal log discrepancies. In this talk I will explain basic properties for quotient singularities and show Shokurov's index conjecture for quotient singularities. This is joint work with Yosuke Nakamura.

Masataka Tomari and Tadashi Tomaru

Title : Comparison of fundamental cycles and maximal ideal cycles for normal surface double points. - due to Laufer decomposition

Abstract:

For a resolution space of a normal complex surface singularity (X,o), Artin's fundamental cycle Z (the minimal anti-nef non-zero cycle) and the maximal ideal cycle M (defined as the pull back of the maximal ideal) are defined and important geometric objects to study various properties of singularity. M. Artin proved that M = Z for all resolutions of all rational singularities. However, for non-rational singularities, it is a delicate problem whether M =Z or not. In this lecture, we will give the criterion for M = Z, under the assumption that the multiplicity is two. Then the singularity defined as z^2=f(x,y), and we prove that M >Z holds on the minimal resolution if and only if f has the Laufer decomposition ;a canonical decomposition f=f_{[L]}f_{[c]}f_{[o]}. We characterize these three factors f_{[L]}(Laufer type) , f_{[c]} (contact type)and f_{[o]} (odd type) in terms of Puiseux pair. Our results explain the Laufer's famous example z^2=y (x^4 + y^6) naturally. Here we would like to present several

related topics and basic techniques for interested beginners.

Shoji Yokura

Title: Ranks of homotopy and homology groups of rationally elliptic spaces and algebraic varieties

Abstract:

A rationally elliptic space is a simply connected space such that the ranks of its total homotopy group and (co)homology group are both finite. A conjecture by M.R.Hilali claims that the homotopy rank does not exceed the homology rank for a rationally elliptic space. In this talk I will explain about some fundamental properties of a rationally elliptic space, some examples for which the Hilali conjecture holds, and some comparisons of Poincare polynomials and mixed Hodge polynomials of homotopy and (co)homology groups of rationally elliptic complex algebraic varieties. This talk is partly based on joint works with Anatoly Libgober.

Introduction to ring-theoretic properties of geometric ideals

by Ken-ichi Yoshida (Nihon University)

This is a joint work with Tomohiro Okuma and Kei-ichi Watanabe (M. E. Rossi for some part). Throughout this talk, let $(A, \mathfrak{m})$ be an excellent 2-dimensional normal local domain containing an algebrically closed field k, unless otherwise specified. In this case, for any $\mathfrak{m}$-primary integrally closed ideal $I \subset A$, there exist a resolution of singularities $X \to \operatorname{Spec} A$ and an antinef cycle Z on X such that I can be represented as follows: $I\mathcal{O}_X = \mathcal{O}_X(-Z)$, $I = H^0(X, \mathcal{O}_X(-Z))$. Put $q(nI) = \dim_K H^1(X, \mathcal{O}_X(-nZ))$ for every integer $n \geq 1$. Then the *normal reduction number* $\bar{\mathrm{r}}(I)$ can be desribed in terms of $q(nI)$:

$$\begin{aligned}\bar{\mathrm{r}}(I) &= \min\{r \in \mathbb{Z}_{\geq 0} \mid \overline{I^{n+1}} = Q\overline{I^n}\ (n \geq r)\}\\ &= \min\{n \in \mathbb{Z}_{\geq 0} \mid q((n-1)I) = q(nI)\}.\end{aligned}$$

An $\mathfrak{m}$-primary integrally closed ideal I is called an *elliptic ideal* (resp. a *p_g-ideal*) if $\bar{\mathrm{r}}(I) = 2$ (resp. $\bar{\mathrm{r}}(I) = 1$). These are typical examples of **geometric ideals**. For any integrally lclosed ideal $I \subset A$, we consider the following geometric blow-up algebra:

$$\overline{G}(I) := \bigoplus_{n \geq 0} \overline{I^n}/\overline{I^{n+1}}.$$

We call this algebra the *normal tangent cone* of I.

In this talk, we introduce **characterizations**, some **ring-theoretic properties (e.g. Cohen-Macaulayness, Gorensteinness and so on)** of $\overline{G}(I)$ of such an ideal I.

A geometric aspect of this talk will be discussed in Okuma-san's talk.

References

[OWY1] T. Okuma, K.-i. Watanabe, and K. Yoshida, *Good ideals and p_g-ideals in two-dimensional normal singularities*, Manuscripta Math. **150** (2016), no. 3-4, 499–520.

[OWY2] ______ *Rees algebras and p_g-ideals in a two-dimensional normal local domain*, Proc. Amer. Math. Soc. **145** (2017), no. 1, 39–47.

[OWY3] ______ *A characterization of 2-dimensional rational singularities via core of ideals*, J. of Algebra **499** (2018), 450–468.

[OWY4] ______ *Normal reduction numbers for normal surface singularities with application to elliptic singularities of Brieskorn type*, Acta Mathematica Vietnamica **44** (2019), no. 1, 87–100.

[OWY5] ______ *The normal reduction number of two-dimensional cone-like singularities*, Proc. Amer. Math. Soc. **149** (2021), no. 11, 4569–4581.

[OWY7] ______ *Gorensteinness for normal tangent cones of elliptic ideals*, submitted (available from arXiv: 2302.07991v1 [math.AC] 15 Feb 2023).

[ORWY] T. Okuma, M. E. Rossi, K.-i. Watanabe, and K. Yoshida, *Normal Hilbert coefficients and elliptic ideals in normal 2-dimensional singularities*, Nagoya Math. J. **248** (2022), 779–800.