

# Workshop

## Knot theory, LMO invariants and related topics

at Okinawa Institute of Science and Technology (OIST)  
1919-1 Tancha, Onna-son, Kunigami-gun, Okinawa, Japan 904-0495

March 9 - 11, 2024

### 1 Schedule

	March 9	March 10	March 11
10:00-10:30 Tea time	Suzuki	10:00 - 10:45 Nozaki	Kitayama
10:45-11:15 Break	Bao	11:00 - 11:45 Cha	Sheng
11:30-12:00 Lunch	Ishikawa		Petrou
13:30-14:15 Tea time	Nozaki	Excursion	Cha
14:30-15:00 Break	Kitano		Sakasai
15:15-15:45 Tea time	Ozawa		Morifuji
16:00-17:00	discussion		discussion

### 2 Title and Abstract

1. Yuanyuan Bao (Tohoku University)  
 Title:  $gl(1|1)$  Alexander polynomial, Reidemeister torsion and lens space  
 Abstract: Based on nilpotent representations of quantum  $sl(2)$  at a root of unity of order  $2r$  ( $r \geq 2$ , not divisible by 4), Blanchet, Costantino, Geer and Patureau-Mirand constructed a series of TQFTs. They showed that the 3-manifold invariant of the  $r = 2$  case is a canonical normalization of Reidemeister torsion. In this talk, we show that the 3-manifold invariant constructed from quantum  $gl(1|1)$  is also equivalent to Reidemeister torsion, and we discuss its value for lens spaces.
2. Jae Choon Cha (POSTECH)  
 Minicourse: “Topological = smooth” in dimension 4

Abstract: In dimension 4, the works of Freedman and Donaldson led us to the striking discovery that the smooth (or piecewise linear) category behaves very differently from the topological category, compared to other dimensions. Since then, it has been a very successful direction to investigate the difference between the two categories in various contexts in dimension 4, using modern smooth invariants. On the other hand, little was understood, if any, about when smooth and topological are similar in dimension 4. In this talk, we will discuss some recent progress on new “topological = smooth” results in dimension 4. In particular, we show that topological and smooth isotopy classes of properly embedded disks admitting algebraic duals in the boundary are bijective.

3. Katsumi Ishikawa (Kyoto University)

Title: The trapezoidal conjecture for knots and links of braid index 3

Abstract: The trapezoidal conjecture is a classical famous conjecture posed by Fox, which states that the coefficient sequence of the Alexander polynomial of any alternating knot is trapezoidal. In this talk, we show this conjecture for any alternating links obtained by closing 3-braids.

4. Teruaki Kitano (Soka University)

Title: Some applications of a twisted Alexander polynomial to a symmetric union presentation of a knot

Abstract: This is based on joint works with Michel Boileau (Marseille) and Yuta Nozaki (Yokohama). A symmetric union presentation of a knot was originally studied by Kinoshita-Terasaka as a union of knots in 1958. In 2000 Lamm defined a symmetric union of even type or a skew-symmetric union. It is well known that the Alexander polynomial of a knot  $K$  has a special form if  $K$  has a symmetric union of even type. In this talk, we discuss that it can be generalized to a twisted Alexander polynomial. We show some applications to the knot genus, the existence of an epimorphism between knot groups, or relation with a ribbon knot as long as one has time.

5. Takahiro Kitayama (University of Tokyo)

Title: Blanchfield pairings and Gordian distance

Abstract: We present a lower bound of the Gordian distance of knots in terms of their Blanchfield pairings, extending a theorem of Borodzik and Friedl on the unknotting number of a knot. We also describe computational results, in particular, giving examples of pairs of knots with the Gordian distance equal to 3 which are difficult to treat otherwise. This talk is based on joint work with Stefan Friedl and Masaaki Suzuki.

6. Takayuki Morifuji (Keio University)

Title: A volume presentation of a hyperbolic fibered knot

Abstract: In this talk, we present a volume presentation of a hyperbolic fibered knot using the Bell polynomials. More precisely, we show that the hyperbolic

volume of a fibered knot can be expressed in terms of the traces of powers of a monodromy matrix.

7. Yuta Nozaki (Yokohama National University)

Mini course: On the LMO invariant of 3-manifolds

Abstract: In the realm of quantum topology, various invariants are studied in the framework of finite-type invariants. Le, Murakami and Ohtsuki introduced the LMO invariant of closed 3-manifolds, which is universal among certain finite-type invariants. Subsequently, this invariant has been extended to 3-manifolds with boundary and applied to the study of mapping class groups of surfaces. In these talks, I will provide background on the LMO invariant and discuss recent developments in related fields. These talks are partially based on joint work with Masatoshi Sato and Masaaki Suzuki.

8. Yuko Ozawa (Meiji University)

Title: Epimorphisms between genus two handlebody-knot groups

Abstract: For two prime knots, a relation that there exists an epimorphism between these knot groups is defined. This relation has been determined up to eleven crossings. In this talk, we define a relation for handlebody-knots in the same way and consider the relation for genus two handlebody-knots up to six crossings.

9. Andreani Petrou (OIST)

Title: Factorisation of the Harer-Zagier transform of the HOMFLY-PT polynomial

Abstract: I will talk about the Laplace transform of the HOMFLY-PT polynomial for knots, called the Harer-Zagier (HZ) transform, which is a function of two variables  $\lambda$  and  $q$ . For some special families of knots it admits a fully factorised form. This is not true, however, for the majority of knots, for which it can only be decomposed as a sum of factorised terms. An interesting relation between this decomposition and Khovanov homology will be discussed. Notwithstanding, we suggest that by fixing the variable  $\lambda = q^n$ , for some "magical" exponent  $n$ , the HZ transform of any knot can obtain a factorised form in terms of cyclotomic polynomials. Moreover, I will talk about the zeros of HZ transform which show an interesting behavior.

10. Takuya Sakasai (University of Tokyo)

Title: Invariants of homologically fibered knots

Abstract: A homologically fibered knot is an enlargement of a fibered knot. We discuss invariants of homologically fibered knots using Johnson-Morita theory. In particular, we see some distinction between the genus 1 case and higher.

11. Xiaobing Sheng (OIST)

Title: On sequences of alternating links constructed from elements of Thompson's group  $F$

Abstract: Thompson's groups  $F, T$  and  $V$  were first constructed by Richard Thompson from a logical context where these groups were later found to have connections

with many other branches of not only mathematics but also theoretical physics. Jones, in his late years, motivated by an attempt to develop new conformal field theory, obtained unitary representations of Thompson's groups  $F$  and  $T$ . as one of the results he discovered a rather concrete way of constructing knots and links from Thompson's group  $F$  as an analogue of results on braid groups.

In this talk, we focus on a particular sequence of elements of the group  $F$  that produces a sequence of alternating knots and we provide several different ways to look at the sequence through a selective of computational results. This is an ongoing joint work with Bao Yuanyuan.

12. Masaaki Suzuki (Meiji University)

Title: Twisted Alexander polynomials of knots associated to the regular representations of finite groups

Abstract: The twisted Alexander polynomial of a knot is defined associated to a linear representation of the knot group. If there exists a surjective homomorphism of a knot group onto a finite group, then we obtain a representation of the knot group by the composition the surjective homomorphism and the regular representation of the finite group. In this talk, we provide several formulas of the twisted Alexander polynomial of a knot associated to such representations.

### 3 Organizers

Shinobu Hikami (OIST), Masaaki Suzuki (Meiji University), Xiaobing Sheng (OIST)