Workshop: Singularities and related topics

Date: July 30th- Aug 2nd, 2019
Venue: The University of Tokyo (Komaba Campus), Graduate School of Mathematical Science Bldg. Room 002
https://www.ms.u-tokyo.ac.jp/access_e/index_e.html

Program:

July 30 (Tue) 10:00 -11:00 Shihoko Ishii
(Tsinghua University/University of Tokyo)
Birational invariants and arc spaces I.
11:00- 11:30 break
11:30-12:30 Shihoko Ishii
Birational invariants and arc spaces II.
12:30- 14:30 lunch
14:30-15:30 Masataka Tomari (Nihon Univ.) I
On the finiteness of Goto-Watanabe's a-invariant and weight types of complex weighted homogeneous singularities of dimension $\geqq 2$ for a fixed positive geometric genus"
15:30-16:00 Coffee break
16:00-17:00 Kazuhiko Kurano (Meiji Univ.)
Rationality of negative curves and finite generation of symbolic Rees rings

July 31 (Wed) 10:00 -11:00 Masataka Tomari II
On weighted homogeneous hypersurface singularities with Saito's regular system of weights.
11:00- 11:30 break
11:30-12:30 Shinobu Hikami (OIST) I
Arc space, jet scheme and super conformal field theory

12:30- 14:30 lunch
14:30-15:30 Kei-Ichi Watanabe (Nihon Univ.)
Ideal theory of 2 dimensional normal singularities via resolution of singularities I
15:30-16:00 Coffee break
16:00-17:00 Kei-Ichi Watanabe
Ideal theory of 2 dimensional normal singularities via resolution of singularities II
18:00- 20:00 Dinner

Aug 1 (Thu) 10:00 -11:00 Anne Moreau (Lille Univ.)
Arc spaces on spherical varieties I: stringy invariants
11:00- 11:30 break
11:30-12:30 Anne Moreau (Lille Univ.)
Arc spaces on spherical varieties II: on the satellites subgroups.
12:30- 14:30 lunch
14:30-15:30 Claus Hertling (Mannheim Univ.)
Marked singularities, their moduli spaces, distinguished bases and Stokes regions I.
15:30-16:00 Coffee break
16:00-17:00 Claus Hertling
Marked singularities, their moduli spaces, distinguished bases and Stokes regions II.

Aug 2 (Fri) 10:00 -11:00 Makiko Mase (Mannheim Univ.)
On dualities among families of K3 surfaces associated to strange duality of invertible polynomials.
11:00- 11:30 break
11:30-12:30 Makiko Mase II
On dualities among families of K3 surfaces associated to strange duality of invertible polynomials.
12:30- 14:30 lunch
14:30-15:30 Shinobu Hikami II
Singularities and moduli spaces of Riemann and Klein surfaces.

15:30-16:00 Coffee break
16:00-17:00 "Problems, discussions and prospect" (tentative)
17:00 End

Abstract:

Shihoko Ishii: Birational invariants and arc spaces. I, II.

J. F. Nash posed ``Nash problem" in his famous preprint in 1969.
The problem was solved in 2012, but it is not the end.
The problem plays an important role in singularity theory,
because the problem is a bridge between birational geometry and arc spaces.
Inspired by this idea, some birational invariants were proved to be described in terms of arc spaces by M. Mustata and others for the base field of characteristic 0. Later, these descriptions became also known to work for positive characteristic case.
In this talk, I will show the descriptions of minimal log discrepancies and log canonical thresholds in terms of arc spaces and show some applications.

Masataka Tomari I:
On the finiteness of Goto-Watanabe's a-invariant and weight types of complex weighted homogeneous singularities of dimension≧2 for a fixed positive geometric genus

In my talk, in some higher dimensional general settings, I will explain how the geometric genus of singularities give the bounds for several invariants of singularities; multiplicity for the fixed embedding dimension, the Goto-Watanabe's a-invarinat for graded cases, and their weights systems.
I will also discuss a classification of hypersurface 2-dim case by which I respond to Prof. Hikami's several questions.

Anne Moreau: Arc spaces on spherical varieties I: stringy invariants

In my first talk, I will explain how to prove a formula for the stringy E-function of an arbitrary Q-Gorenstein spherical G-variety X, where G is any connected reductive group, using the motivic integral over the arc space of X. Our formula for the horospherical case is a natural generalization of the one obtained by Batyrev for the toric case, although the proof is very different. As an application, we formulate and prove a new smoothness criterion for locally factorial horospherical varieties. We expect that this smoothness criterion holds for arbitrary spherical varieties.
This is based on a joint work with Victor Batyrev.

Anne Moreau: Arc spaces on spherical varieties II: on the satellites subgroups.

In my second talk, I will explain how to define new invariants, called the satellites, associated with any spherical homogeneous space. The satellites were introduced by Batyrev and myself. They enjoy nice properties and can be used (among other things) to compute some stringy invariants for general Q-Gorenstein spherical varieties. If time allows, I will mention other applications.
This is also based on a joint work (partly in progress) with Victor Batyrev.

Claus Hertling: Marked singularities, their moduli spaces, distinguished bases and Stokes regions I, II.

One part of the talks is on a global study of μ-constant families of holomorphic function germs with isolated singularities. Some new data are defined and discussed, μ-constant monodromy groups, marked singularities, their global moduli spaces, and a global Torelli type conjecture for their Brieskorn lattices. Another part of the talks is on universal unfoldings, Brieskorn lattices at semisimple points, their Stokes data and distinguished bases, and a global Lyashko-Looijenga map. For the simple singularities (by work of Looijenga and Deligne 73/74) and the simple elliptic singularities (joint work with C. Roucairol 18) this leads to an understanding of a certain global base space as an atlas of Stokes data.

Makiko Mase: On dualities among families of K3 surfaces associated to strange duality of invertible polynomials I, II

As a generalisation of Arnold's strange duality for unimodal singularities, Ebeling and Takahashi introduced a notion of strange duality for invertible polynomials. We focus on invertible polynomials in three variables that define bimodal singularities and their strange dual partners. It is known that such polynomials can be compactified into weighted K3 hypersurfaces classified into 95 families of simple K3 hypersurface singularities by Yonemura. Thus we may consider families of K3 surfaces associated to them.

In the talk, I will discuss about relations between the strange duality of singularities and polytope- and lattice duality of families of K3 surfaces.

Shinobu Hikami I: Arc space, jet scheme and super conformal field theory

Nash's arc space and jet scheme are applied to the super conformal field theory. We discuss several examples in algebraic aspects. (1) Yang-Lee edge singularity: In two dimensions, it is related to C_2 algebra and jet scheme. This model in d-dimensions is known to be equivalent to d+2 dimensional branched polymer critical phenomena. This equivalence is due to the dimensional reduction of the supersymmetry. (2) The random field Ising model(RFIM): This model in d dimension has been discussed whether dimensional reduction to pure Ising model is valid or not. The fermionic degree of freedom in super conformal theory makes the dimensional reduction from d to d-2 dimensions. Both cases will be discussed in this talk by the conformal bootstrap method using determinantal rings. (3) chiral rings of N=2 super conformal theory : This example has $C[x]/\partial W(x)$ with $W(x)= x^p + t_1x^{p-2}+ O(x^{p-3})$, which is Landau-Ginzburg potential. This model is related to the talk in II.

Shinobu Hikami II: Singularities and moduli spaces of Riemann and Klein surfaces

Topological field theory, starting from Kontsevich and Witten' works in 1992, exhibits the close relation to the moduli space of curves. Dolgachev gave a bridge between the singularity theory of Brieskorn type and the moduli space of Riemann surfaces with m-spin bundle in 1983. Recently, Natanzon and Pratoussevitch investigated this correspondence through Arf invariants for higher m-spin curves in 2017. In a joint work with E. Brezin, we developed an intersection theory for the m-spin curves by a random supermatrix, and discussed topological invariants with marked points and boundaries. In this talk I will discuss the link between the singularity theory and a random supermatrix theory with an external source.

On weighted homogeneous hypersurface singularities with Saitos regular system of weights.

M. Tomari

For a weight system $(a_1, , a_{d+1}; h)$ on $d+1$ variables $z_1, , z_{d+1}$ with the degree h, $(a; h)$ is a regular system of weights if the characteristic function

$$\chi_{a,h}(T) = (T^{h-a_1} - 1) \cdots (T^{h-a_{d+1}} - 1)/(T^{a_1} - 1) \cdots (T^{a_{d+1}} - 1)$$

is a polynomial of T. It is well-known that a generic weighted homogeneous polynomial f of type $(a; h)$ in $C[z]$ has an isolated singularity when $d = 2$ by V.I. Arnolds remark and K. Saitos theory of regular system of weights. However, for the cases $d \geq 3$, it seems that there are few informations on this class.

In my talk, I will show that Theorem. For a generic weighted homogeneous polynomial f of type $(a; h)$ with regular system of weights,

1. $\{f = 0\}$ is a normal singularity at o for $d \geq 2$.
2. $\{f = 0\} - \{o\}$ has only rational singularities for $d = 3$.

Further several finiteness results similar to my previous talk will shown in this case. I also present several new examples the regular system of weights $(a; h)$ which is not an isolated singularity other than Ivlevs classical one.

Kazuhiko Kurano: Rationality of the negative curve and finite generation of symbolic Rees rings

Let K be a field. Let a, b, c be pairwise coprime positive integers such that $\sqrt{abc} \notin \mathbb{N}$. Let X be the weighted projective space $\mathrm{Proj}(K[x,y,z])$ with $\deg(x) = a$, $\deg(y) = b$, $\deg(z) = c$, respectively. Let $f : Y \to X$ be the blow-up at the smooth point defined by the kernel P of the K-algebra map $K[x,y,z] \to K[T]$ defined by $x \mapsto T^a$, $y \mapsto T^b$, $z \mapsto T^c$. Let E be the exceptional divisor. If the symbolic Rees ring $R_s(P)$ (equivalently, the Cox ring of Y) is Noetherian, there exists a curve C ($\neq E$) such that $C^2 < 0$.

In this talk, we give some sufficient condition for the negative curve to be rational. All examples (that I know) of negative curves satisfy this condition. Therefore, I do not know any examples of non-rational negative curves.

Assume that there exists a negative curve C. Then $R_s(P)$ is Noetherian if and only if there exists a curve D on Y such that $C \cap D = \emptyset$. (The defining equations of C and D satisfy the Huneke's criterion for finite generation.) In the case where C is rational, it is possible to estimate the degree of $f(D)$. Using computers, it is possible to determine whether $R_s(P)$ is Noetherian in some cases.

IDEAL THEORY OF 2-DIMENSIONAL NORMAL SINGULARITIES VIA RESOLUTION OF SINGULARITIES

KEI-ICHI WATANABE (NIHON UNIVERSITY, TOKYO)

I will explain that ideal theory of integrally closed ideals can be explained by cycles on a resolution of singularities and explain some new concepts introduced in a joint work in progress with Tomohiro Okuma and Ken-ichi Yoshida.

Let $(A, \mathfrak{m}, k)$ be an excellent normal 2-dimensional local ring and I be an integrally closed $\mathfrak{m}$-primary ideal. We assume k is algebraically closed and $k \subset A$.

For any resolution of singularity $f : X \to \mathrm{Spec}(A)$, we define

$$p_g(A) = h^1(\mathcal{O}_X) := \dim_k H^1(X, \mathcal{O}_X),$$

which does not depend on the resolution. We put

$$f^{-1}(\mathfrak{m}) = E = \cup_{i=1} E_i,$$

where each E_i a irreducible projective curve and we call $Z \in \sum_{i=1}^{r} \mathbb{Z}E_i$ a cycle on X.

The intersection theory of cycles on X plays very important role. It is fundamental that the intersection matrix $(E_iE_j)_{i,j=1}^{r}$ is negative definite. A cycle Z is called *anti-nef* if $ZE_i \le 0$ for every E_i. There is unique minimal anti-nef cycle $Z_0 > 0$ on X called the *fundamental cycle*. If $p_g(A) > 0$, there exists unique minimal cycle Z such that $h^1(\mathcal{O}_Z) = p_g(A)$. Such cycle is called the *cohomology cycle* after M. Reid.

For a positive cycle Z we define

$$p_a(Z) = \frac{Z^2 + K_X}{2} + 1 \quad \text{and} \quad p_a(A) = \max\{p_a(Z) \mid Z > 0\}$$

Definition 0.1. (1) A is a rational singularity if $p_g(A) = 0$, or, equivalently, $p_a(Z_0) = 0$.

(2) A is an elliptic singularity if $p_a(A) = 1$, or equivalently, $p_a(Z_0) = 1$. (In this case $p_g(A)$ can be any positive integer.)

Now, take an integrally closed $\mathfrak{m}$-primary ideal I of A. Then we can take X so that $I\mathcal{O}_X = \mathcal{O}_X(-Z)$ is invertible. In this case we will write $I = I_Z$. We say $Q \subset I$ is a *minimal reduction* of I if Q is a parameter ideal and I is integral over Q. If $I = I_Z$, then we have $Q\mathcal{O}_X = \mathcal{O}_X(-Z)$.

Definition 0.2. If $I = I_Z$, then we define

(1) $q(I) = h^1(\mathcal{O}_X(-Z))$. We can show that $0 \le q(I) \le p_g(A)$.
(2) $q(nI) = h^1(\mathcal{O}_X(-nZ))$. $\{q(nI)\}_{n\ge 0}$ is a non-increasing sequence.
(3) I_Z is called a p_g-ideal if $q(I) = p_g(A)$.

We will show that p_g-ideals have very nice properties. This topic is one of the main themes of my lecture. Other important topic is the concept of normal reduction nimbers.

For a minimal reduction Q of I, we calculate the numbers;

$$\mathrm{nr}(I) = \min\{n \mid \overline{I^{n+1}} = Q\overline{I^n}\}, \quad \bar{r}(I) = \min\{n \mid \overline{I^{N+1}} = Q\overline{I^N}, \forall N \ge n\}$$

and

$$\mathrm{nr}(A) = \max\{\mathrm{nr}(I)\}, \bar{r}(A) = \max\{\bar{r}(I)\} \quad (I \subset A, \mathfrak{m} - \text{primary integrally closed ideals})$$

Example. $\bar{r}(A) = 1$ if and only A is a rational singularity and if A is an elliptic singularity, then $\bar{r}(A) = 2$. In general, we can show $\bar{r}(A) \leq p_g(A) + 1$.

We discuss $\bar{r}(A)$ in case of cone singularities (we say A is a cone singularity if the exceptional set of the minimal resolution of A is a single smooth irreducible curve) using a vanishing theorem of Röhr. In particular, we will show;

Theorem. If A is a hypersurface singularity of dimension 2 defined by a homogeneous polynomial of degree $d \geq 4$, then $\bar{r}(A) = \mathrm{nr}(\mathfrak{m}) = d - 1$.

Also, we will show an example of A and I such that $\mathrm{nr}(I) = 1$ and $\bar{r}(I) = p_g(A) + 1$ for every $p_g(A) \geq 2$.