Workshop: Singularities and related topics

Date: July 30 th- Aug 2nd, 2019

Venue: The University of Tokyo (Komaba Campus), Graduate School of Mathematical Science Bldg. Room 002

https://www.ms.u-tokyo.ac.jp/access_e/index_e.html

Tentative Schedule:

July 30 (Tue) 10:00 -11:00 Shihoko Ishii
(Tsinghua University/University of Tokyo)
Birational invariants and arc spaces I.
11:00- 11:30 break
11:30-12:30 Shihoko Ishii
Birational invariants and arc spaces II.
12:30- 14:30 lunch
14:30-15:30 Masataka Tomari (Nihon Univ.)
On the finiteness of Goto-Watanabe's a-invariant and weight types of complex weighted homogeneous singularities of dimension≧2 for a fixed positive geometric genus"
15:30-16:00 Coffee break
16:00-17:00 Kazuhiko Kurano (Meiji Univ.)
Rationality of negative curves and finite generation of symbolic Rees rings

July 31 (Wed) 10:00 -11:00 Anna Pratoussevitch (Liverpool Univ.)
Quasi-Homogeneous Gorenstein Surface Singularities, Higher Spin Bundles and Their Real Forms I.
11:00- 11:30 break
11:30-12:30 Anna Pratoussevitch
Quasi-Homogeneous Gorenstein Surface Singularities, Higher Spin Bundles and Their Real Forms II.

12:30- 14:30 lunch
14:30-15:30 Kei-Ichi Watanabe (Nihon Univ.)
Ideal theory of 2 dimensional normal singularities via resolution of singularities I
15:30-16:00 Coffee break
16:00-17:00 Kei-Ichi Watanabe
Ideal theory of 2 dimensional normal singularities via resolution of singularities II
18:00- 20:00 Dinner

Aug 1 (Thu) 10:00 -11:00 Anne Moreau (Lille Univ.)
Arc spaces on spherical varieties I: stringy invariants
11:00- 11:30 break
11:30-12:30 Anne Moreau (Lille Univ.)
Arc spaces on spherical varieties II: on the satellites subgroups.
12:30- 14:30 lunch
14:30-15:30 Claus Hertling (Mannheim Univ.)
Marked singularities, their moduli spaces, distinguished bases and Stokes regions I.
15:30-16:00 Coffee break
16:00-17:00 Claus Hertling
Marked singularities, their moduli spaces, distinguished bases and Stokes regions II.

Aug 2 (Fri) 10:00 -11:00 Makiko Mase (Mannheim Univ.)
On dualities among families of K3 surfaces associated to strange duality of invertible polynomials I.
11:00- 11:30 break
11:30-12:30 Makiko Mase
On dualities among families of K3 surfaces associated to strange duality of invertible polynomials II.
12:30- 14:30 lunch

14:30-15:30 Shinobu Hikami (OIST)
Singularities and moduli spaces of Riemann and Klein surfaces I.
15:30-16:00 Coffee break
16:00-17:00 Shinobu Hikami
Singularities and moduli spaces of Riemann and Klein surfaces II.

Abstract:

Shihoko Ishii: Birational invariants and arc spaces. I, II.

J. F. Nash posed ``Nash problem" in his famous preprint in 1969.
The problem was solved in 2012, but it is not the end.
The problem plays an important role in singularity theory,
because the problem is a bridge between birational geometry and arc spaces.
Inspired by this idea, some birational invariants were proved to be described in terms of arc spaces by M. Mustata and others for the base field of characteristic 0. Later, these descriptions became also known to work for positive characteristic case.
In this talk, I will show the descriptions of minimal log discrepancies and log canonical thresholds in terms of arc spaces and show some applications.

Masataka Tomari: On the finiteness of Goto-Watanabe's a-invariant and weight types of complex weighted homogeneous singularities of dimension $\geqq 2$ for a fixed positive geometric genus

In my talk, in some higher dimensional general settings, I will explain how the geometric genus of singularities give the bounds for several invariants of singularities; multiplicity for the fixed embedding dimension, the Goto-Watanabe's a-invarinat for graded cases, and their weights systems.
I will also discuss a classification of hypersurface 2-dim case by which I respond to Prof. Hikami's several questions.

Anna Pratoussevitch: Quasi-Homogeneous Gorenstein Surface Singularities, Higher Spin Bundles and Their Real Forms I, II.

We study spaces of hyperbolic Gorenstein quasi-homogeneous surface singularities (GQHSS). We describe a correspondence between GQHSS and m-spin bundles, i.e. m-th roots of the cotangent bundle, of a hyperbolic Riemann surface. Using this correspondence and a generalization of the Arf invariants of quadratic forms, we will describe the topology of the space of GQHSS, enumerating the connected components and showing that any connected component is homeomorphic to a quotient of R^d by a discrete group. We then consider real forms of GQHSS, i.e. singularities equipped with an anti-holomorphic involution, and their correspondence with higher spin bundles on Klein surfaces. For certain types of anti-holomorphic involutions, we will describe the spaces of real forms of GQHSS as a branched covering spaces and determine the number of connected components and the types of ramification points. This is joint work with S. Natanzon and H. Riley.

Anne Moreau: Arc spaces on spherical varieties I: stringy invariants

In my first talk, I will explain how to prove a formula for the stringy E-function of an arbitrary Q-Gorenstein spherical G-variety X, where G is any connected reductive group, using the motivic integral over the arc space of X. Our formula for the horospherical case is a natural generalization of the one obtained by Batyrev for the toric case, although the proof is very different. As an application, we formulate and prove a new smoothness criterion for locally factorial horospherical varieties. We expect that this smoothness criterion holds for arbitrary spherical varieties.
This is based on a joint work with Victor Batyrev.

Arc spaces on spherical varieties II: on the satellites subgroups.

In my second talk, I will explain how to define new invariants, called the satellites, associated with any spherical homogeneous space. The satellites were introduced by Batyrev and myself. They enjoy nice properties and can be used (among other things) to compute some stringy invariants for general Q-Gorenstein spherical varieties. If time allows, I will mention other applications. This is also based on a joint work (partly in progress) with Victor Batyrev.

Claus Hertling: Marked singularities, their moduli spaces, distinguished bases and Stokes regions I, II.

One part of the talks is on a global study of μ-constant families of holomorphic function germs with isolated singularities. Some new data are defined and discussed, μ-constant monodromy groups, marked singularities, their global moduli spaces, and a global Torelli type conjecture for their Brieskorn lattices. Another part of the talks is on universal unfoldings, Brieskorn lattices at semisimple points, their Stokes data and distinguished bases, and a global Lyashko-Looijenga map. For the simple singularities (by work of Looijenga and Deligne 73/74) and the simple elliptic singularities (joint work with C. Roucairol 18) this leads to an understanding of a certain global base space as an atlas of Stokes data.

Makiko Mase: On dualities among families of K3 surfaces associated to strange duality of invertible polynomials I, II

As a generalisation of Arnold's strange duality for unimodal singularities, Ebeling and Takahashi introduced a notion of strange duality for invertible polynomials. We focus on invertible polynomials in three variables that define bimodal singularities and their strange dual partners. It is known that such polynomials can be compactified into weighted K3 hypersurfaces classified into 95 families of simple K3 hypersurface singularities by Yonemura. Thus we may consider families of K3 surfaces associated to them.
In the talk, I will discuss about relations between the strange duality of singularities and polytope- and lattice duality of families of K3 surfaces.

Shinobu Hikami: Singularities and moduli spaces of Riemann and Klein surfaces I, II

Topological field theory, starting from Kontsevich and Witten' works in 1992, exhibits the close relation to the moduli space of curves. Dolgachev gave a bridge between the singularity theory of Brieskorn type and the moduli space of Riemann surfaces with m-spin bundle in 1983. Recently, Natanzon and Pratoussevitch investigated this correspondence through Arf invariants for higher m-spin curves in 2017. In a joint work with E. Brezin, we developed an intersection theory for the m-spin curves by a random supermatrix, and discussed topological invariants with marked points and boundaries.

In this talk I will discuss the link between the topological field theory and the singularity theory by a random supermatrix theory.

Kazuhiko Kurano: Rationality of the negative curve and finite generation of symbolic Rees rings

Let K be a field. Let a, b, c be pairwise coprime positive integers such that $\sqrt{abc} \notin \mathbb{N}$. Let X be the weighted projective space $\mathrm{Proj}(K[x, y, z])$ with $\deg(x) = a$, $\deg(y) = b$, $\deg(z) = c$, respectively. Let $f : Y \to X$ be the blow-up at the smooth point defined by the kernel P of the K-algebra map $K[x, y, z] \to K[T]$ defined by $x \mapsto T^a$, $y \mapsto T^b$, $z \mapsto T^c$. Let E be the exceptional divisor. If the symbolic Rees ring $R_s(P)$ (equivalently, the Cox ring of Y) is Noetherian, there exists a curve C ($\neq E$) such that $C^2 < 0$.

In this talk, we give some sufficient condition for the negative curve to be rational. All examples (that I know) of negative curves satisfy this condition. Therefore, I do not know any examples of non-rational negative curves.

Assume that there exists a negative curve C. Then $R_s(P)$ is Noetherian if and only if there exists a curve D on Y such that $C \cap D = \emptyset$. (The defining equations of C and D satisfy the Huneke's criterion for finite generation.) In the case where C is rational, it is possible to estimate the degree of $f(D)$. Using computers, it is possible to determine whether $R_s(P)$ is Noetherian in some cases.