Silver workshop VI

Date: Aug. 7-9, 2023

Venue: OIST Lab.4, F01 and Zoom

Organized by Noriko Yui (Queen's Univ.), Kyoji Saito (RIMS), Shinobu Hikami (OIST)

Silver Workshop VI (2023, Aug.) (talk 50 min and discussion 10 min.)

		10:00-11:00	11:20-12:20	12:20-14:00	14:00-15:00	15:20-16:20
7th(Mon)		Kyoji Saito (zoom)	J. Sekiguchi (zoom)	(Lunch)	T. Suwa (zoom)	Yoshihisa Saito (zoom)
8th(Tue)		M. Mase (zoom)	K. Watanabe (zoom)	(Lunch)	Seoyoung Kim	Tea time (chat) and Strings*
9th(Wed)		N. Yui	Minkyu Kim	(Lunch)	A. Petrou	S. Hikami

* La Primavera (A. Vivaldi) by Hokulani orchestra (OIST member).

Abstract:

Seoyoung Kim

Title: From the Birch and Swinnerton-Dyer conjecture to Nagao's Conjecture

Abstract: In 1965, Birch and Swinnerton-Dyer formulated a conjecture on the Mordell-Weil rank r of elliptic curves which also implies the convergence of the Nagao-Mestre sum. We show that if the Nagao-Mestre sum converges, then the limit equals $-r+1/2$, and study the connections to the Riemann hypothesis for E. We also relate this to Nagao's conjecture for elliptic curves. Furthermore, we discuss a generalization of the above results for the Selberg classes and hence (conjecturally) for larger classes of L-functions.

Yui, Noriko (Queen's University)

Title: Calabi–Yau threefolds of Borcea–Voisin type and Arithmetic Mirror Symmetry

Abstract: We consider Calabi–Yau threefolds of Borcea–Voisin type defined over the field of rationals. Such a Calabi–Yau threefold is constructed as the quotient of the product of a K3 surface and an elliptic curve by a specific involution.

For appropriate choices of K3 surfaces, we can establish the modularity/automorphy of the L-series of the Calabi–Yau motive. We also consider mirror pairs of Calabi–Yau threefolds of Borcea–Voisin type, and discuss arithmetic mirror symmetry and its variants. Finite path integral model and toric code based on

homological algebra.

Minkyu KIM

Finite path integral is a mathematical methodology to construct TQFT’s (topological quantum field theories) from finite gauge theory. In three dimensions, it is generalized to state sum models closely related with finite Hopf algebra gauge theory. Toric code originated from topological quantum computation provides a local and combinatorial description of state sum models. This talk concerns a refinement of finite path integral models by replacing finite (Hopf algebra) gauge theory with homological algebra based on bicommutative Hopf algebras. We introduce and study a relative version of Haar integrals for Hopf algebras to formulate finite path integral in our framework. It leads to a construction of a homotopy-theoretic version of projective TQFT’s from Mayer-Vietoris functors valued in the category of bicommutative Hopf algebras. The projectiveness gives rise to an obstruction problem, which is solved for some Mayer-Vietoris functors such as the dimension reduction and bounded homology theories. Furthermore, we relate toric code to chain complexes of bicommutative Hopf algebras.

Jiro Sekiguchi

Simple singularity of type E_7 and the complex reflection group No.34

In this paper, we shall treat Arnold’s problem for the complex reflection group called Mitchell group G whose number is 34 in the list of Shephard and Todd [2]. First of all, we recall the formulation of the problem. In the book [1], p.20, it is written that

“1974-5 Find applications of the (Shephard-Todd) complex reflection groups to singularity theory.”

We introduce a family F of surfaces S_τ : f_τ (x, y, z) = 0 of C^3 depending on parameters $\tau = (t_1, t_2, t_3, t_4, t_5, t_7)$. where

33 234 22 $f_\tau(x,y,z)=y +x y+t_7x+t_5x +t_3x +t_1x +y(t_4x+t_2x)-z$. (1)

We point out some of basic properties of F:

(a) The surface (1) has a simple singularity of type E_7, if $t_1 = \cdots = t_5 = t_7 = 0$. (b) If the weights attached to x,y,z,t_j (j = 1,2,3,4,5,7) are 2,3,9/2,j

(j = 1, 2, 3, 4, 5, 7), respectively, then f_τ is weighted homogeneous.

(c) If $t_7 = 0$, the origin is an isolated singular point of the surface (1).

(d) There is an irreducible polynomial Δ of τ such that S_τ is smooth if and only if $t_7 \cdot \Delta \neq 0$.

Since it is known that the degrees of basic invariants of G are 6, 12, 18, 24, 30, 42, the set of weights of $t_1, \cdots, t_5, t_7$ and that of degrees of basic invariants are parallel. It can be shown that if $t_1, \cdots, t_5, t_7$ are regarded as basic invariant of G, then Δ is the discriminant of G.

This is regarded as an answer to Arnold’s Problem for G. If I have a time, I am going to explain the reason why I recognized the existence of the family above which is not the versal family of E_7 singularity and Arnold’s Problem for other complex reflection groups.

References
[1] V.I. Arnold: Arnold’s Problems, Edited by V.I. Arnold. Springer-Verlag. 2005.
[2] G. C. Shephard and A. J. Todd: Finite unitary reflection groups. Canadian J. Math. (1954), 6, 274-304.

Tatsuo Suwa (Hokkaido University)

Title: Relative Bott-Chern cohomology

Abstract: The Bott-Chern cohomology of a complex manifold refines both the de Rham and Dolbeault cohomologies. The relative versions of the latters have many applications, including a recent one to the Sato hyperfunction theory. In this talk, a relative Bott-Chern cohomology theory will be presented together with applications and examples (based on a joint work with M. Corrˆea).

Makiko Mase

Abstract: Introducing coupling duality between weight systems, Ebeling proved there is a "Saito's duality" of reduced zeta functions associated to simple K3 singularities.
Hinted by this, we discuss some dualities that may explain a relation between the Picard lattice of a family of K3 surfaces and the Milnor lattice of a simple K3 singularity.

Shinichi Tajima

Title: Holonomic D-modules and non-isolated singularities.

Abstract: First, I give a quick introduction s-parametric annihilators, Bernstein-Sato polynomials and local cohomology Second, I consider holonomic D-modules associated toa hypersurface with non-isolated singularities. I give a new method for computing holonomic D-modules and microlocal b-functions. The keys of the method are the use of a Poincare-Birkhoff-Witt algebra and local cohomology. As applications, I discuss relations between the structure of holonomic D-modules and the vertical monodromy introduced by Siersma and Le cycles introduced by Massey.

Kyoji Saito joint work with Benoit Guerville

Title: Elliptic Artin braid relations and Zariski-van Kampen relations

Abstract: The fundamental group of the regular orbit space of a finite Weyl group is calculated by E.Brieskorn by gallarly of chambers. It is called an Artin group and is determined by the Artin braid relations (E.Brieskorn-K.Saito). It is shown that the Artin braid relations are also given by Zariski-van Kampen method (K.Saito)
The fundamental group of the regular orbit space of an elliptic Weyl group is recently determined as the elliptic Artin group by a system of, what we call, elliptic Artin braid relations in a joint work with Yoshihisa Saito and Kyoji Saito.
However, it is not known whether the elliptic Artin braid relations are given by Zariski-van Kampen method (it is a conjecture). For some special case of type E_6^{(1,1)}, Zariski-van Kampen relations are calculated joint with Benoit Guerville. In the present talk, we show that some of the Zariski-van Kampen relations are

generated by elliptic Artin braid relations.

Shinobu Hikami

Title: Arithmeticity in knot polynomials

Abstract: We discuss the factorization in Harer-Zagier transform (HZ) of Homply polynomial of knot continuing to the previous talk by A. Petrou. By the multiplication of a factor a^m (a= q^N) to Homply polynomial or by setting a parameter lambda to be q^m (m is integer), HZ obeys a factorized form where the zeros are roots of unity. This is analogous to the dimensional formula in arithmetic theory such as Igusa cusp form. The link case is also studied.

(Joint work with Andreani Petrou, arXiv:2307.05919).

Yoshihisa Saito

Title: Marked elliptic root systems with non-reduced affine quotients.

Abstract: In the middle of 1980's, motivated by study of singularity theories, K. Saito introduced the notion of "elliptic root systems". Roughly speaking, they are root systems with two null directions. Furthermore, he had classified elliptic root systems R with one-dimensional subspace of G of two dimensional null directions, under the assumption that the quotient affine root system R/G is reduced. In our joint work with A. Fialowski and K. Iohara, we take off the assumption above, and give the classification of the pair (R, G) with no assumption. In addition, we give an overview of the theory of elliptic root systems in this talk, and certain applications of this theory are also discussed.

IDEAL THEORETIC AND GEOMETRIC PROPERTIES OF INTEGRALLY CLOSED IDEALS IN 2 DIMENSIONAL NORMAL SINGULARITIES

KEI-ICHI WATANABE

Introduction

This is a joiint work in progress with T. Okuma (Yamagata Univ.) and K. Yoshida (Nihon Univ.).

Let $(A, \mathfrak{m})$ be a Noetherian normal local ring and I be an integrally closed ideal of A. Take resolution of singularities $f : X \to \mathrm{Spec}(A)$ so that $I\mathcal{O}_X = \mathcal{O}_X(-Z)$. So the ideal I is represented by a positive cycle Z on X.

Let $Q \subset I$ be a minimal reduction of I, namely, $Q = (a, b)$ is a parameter ideal with $I\mathcal{O}_X = Q\mathcal{O}_X$. In this paper, we will study the following invariants;

Definition 0.1.

$$\begin{aligned} \mathrm{nr}(I) &= \min\{r \in \mathbb{Z}_{\geq 1} \mid \overline{I^{r+1}} = Q\overline{I^r}\}, \\ \bar{\mathrm{r}}(I) &= \min\{r \in \mathbb{Z}_{\geq 1} \mid \overline{I^{n+1}} = Q\overline{I^n} \text{ for all } n \geq r\}. \end{aligned}$$

Also we define $\mathrm{nr}(A)$ (resp. $\bar{\mathrm{r}}(A)$) to be the maximal of all $\mathrm{nr}(I)$ (resp. $\bar{\mathrm{r}}(I)$ for all $I \subset A$.

These invariants reflects the "goodness" of the singularity A. For example,

Example 0.2. (1) $\bar{\mathrm{r}}(A) = 1$ if and only if A is a rational singularity.
(2) If A is an elliptic singularity, then $\bar{\mathrm{r}}(A) = 2$.

There are several invariants for the singularity A. Let $f : X \to \mathrm{Spec}(A)$ is a resolution of singularities of A and we put $\mathbb{E} := \bigcup_{i=1}^r E_i$ be irreducible decomposition of $\mathbb{E}$. The invariants below are independent of the resolution X.

Definition 0.3. (1) $p_g(A) := h^1(X, \mathcal{O}_X)$.
(2) $p_a(A) = \sup\{p_a(Z) \mid Z$ is a positive cycle supported on $E\}$.

It is well known that $p_a(A) \leq p_g(A)$ and if $p_a(A) = 0$, then $p_g(A) = 0$ and in this case A is **rational singularity** by definition. Also, A is called an **elliptic singularity** if $p_a(A) = 1$. There are elliptic singularities with arbitrary high $p_g(A)$. Note that $p_a(A)$ is a **topological invariant** of A.

The following is the main result of this talk proved by improved Röhr's Vanishing Theorem.

Theorem 0.4. $\bar{\mathrm{r}}(A) \leq p_a(A) + 1$.

We will discuss by several examples the behavior of $\mathrm{nr}(I), \bar{\mathrm{r}}(I)$ and $\bar{\mathrm{r}}(A)$.

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