

# Workshop ‘Super-Riemann surfaces and related topics’

**Organizers:** Shinobu Hikami(OIST), Nariya Kawazumi (U. Tokyo) and Takuya Sakasai (U. Tokyo )

**Workshop title:** Super-Riemann surfaces and related topics

**Dates:** December 2 (Mon) - December 4 (Thu), 2019

**Venus:** Graduate School of Mathematical Sciences, University of Tokyo.  
(December 2,3: Room 123, December 4: Room 128).  
3-8-1 Komaba Meguro-ku Tokyo 153-8914, Japan.

**Workshop website:** <https://groups.oist.jp/mtpu>

**Support:** This workshop will be held as part of a joint project between the Okinawa Institute of Science and Technology and the University of Tokyo.

## Schedule

### December 2 (Mon)

**10:30 - 11:30 Yusuke Kuno (Tsuda University)**

On spin structures associated with a trivalent fatgraph spine

**13:00 - 15:00 Anton Zeitlin (Louisiana State University)**

Introduction to the theory of super Riemann surfaces.

### December 3 (Tue)

**10:30 - 11:30 Casey Blacker (East China Normal University)**

Polysymplectic Reduction and the Moduli Space of Flat Connections

**13:00 - 15:00 Anton Zeitlin (Louisiana State University)**

$N = 1$  and  $N = 2$  super-Teichmueller theory

**15:30 - 16:30 Shinobu Hikami (OIST)**

Ramond punctures and p-spin curves in a random matrix theory

**17:00 - 18:30 Tuesday Seminar on Topology**

**Anton Zeitlin (Louisiana State University)**

Homotopy Gerstenhaber algebras, Courant algebroids, and Field Equations

### December 4 (Wed)

**10:30 - 11:30 Motoko Kato (Ehime University)**

Thompson’s groups and their actions on non-positively curved spaces

**13:00 - 15:00 Anton Zeitlin (Louisiana State University)**

Super-Teichmueller theory: some recent results

## Abstracts

### **Casey Blacker (East China Normal University) Polysymplectic Reduction and the Moduli Space of Flat Connections**

In a landmark paper, Atiyah and Bott showed that the moduli space of flat connections on a principal bundle over an oriented closed surface is the symplectic reduction of the space of all connections by the action of the gauge group. By appealing to polysymplectic geometry, a variant of symplectic geometry in which the symplectic form takes values in a fixed vector space, we extend this result to the case of higher-dimensional base manifolds. In this setting, the space of connections exhibits a natural polysymplectic structure and the reduction by the action of the gauge group yields the moduli space of flat connections equipped with a 2-form taking values in the cohomology of the base manifold. In this talk, I will first review the polysymplectic formalism and then outline its role in obtaining the moduli space of flat connections.

### **Shinobu Hikami (OIST)**

#### **Ramond punctures and p-spin curves in a random matrix theory**

The intersection numbers of the moduli space of p-spin curves have been investigated by matrix models for positive integer values of p, with spin component L taking the values  $L = -1, 0, 1, 2, \dots, p-2$ . The punctures (marked points) on the Riemann surface are classified into two types: Ramond ( $L=-1$ ) and Neveu-Schwarz ( $L=0, 1, \dots, p-2$ ). Although the Ramond type does not contribute to the intersection numbers for positive integer p, it does appear in the cases of half-integer p (such as  $p=1/2$ ) and negative odd integer p, as well as in the case of logarithmic potentials with boundaries. The Ramond type intersection numbers are computed by residues from the integral representations of the n-point correlation functions. This is a joint work with E. Brezin.

### **Motoko Kato (Ehime University)**

#### **Thompson's groups and their actions on non-positively curved spaces**

Thompson's groups are infinite groups which are studied in various areas of mathematics, including geometric group theory, low-dimensional topology and mathematical physics. There are three types of Thompson's groups:  $F$ ,  $T$ , and  $V$ . These groups are defined as subgroups of the group of homeomorphisms of the unit interval, the circle and the Cantor space, respectively. Thompson's groups  $T$  and  $V$  are known as the first examples of finitely presented infinite simple groups.

In this talk, we consider group actions of Thompson's groups on non-positively curved spaces. Providing good actions on non-positively curved spaces, for example proper actions on proper CAT(0) spaces, can be useful in relation with a big open problem which asks whether Thompson's

group  $F$  is amenable. We show that such actions do not exist in the case where non-positively curved spaces are of finite dimension. This result is a generalization of Farley's result which shows that Thompson's groups  $T$  and  $V$  do not have Serre's property FA.

**Yusuke Kuno (Tsuda University)**

**On spin structures associated with a trivalent fatgraph spine**

For a given trivalent fatgraph spine on a once bordered surface, one can naturally define three spin structures on the surface. I will explain some background, how these spin structures arise, and the proof of the fact that they differ from each other. (Based on "A homology valued invariant for trivalent fatgraph spines", AGT 17, 1785–1811 (2017))

**Anton Zeitlin (Louisiana State University)**

**Lecture 1: Introduction to the theory of super Riemann surfaces**

This lecture serves as an introduction to the theory of super Riemann surfaces. No previous knowledge of supermanifold theory is required. The primary objects of discussion will be  $N = 1$  super-Riemann surfaces with punctures and their moduli. In addition, we will also discuss  $N = 2$  super-Riemann surfaces and their duality with  $(1|1)$  complex supermanifolds.

**Lecture 2:  $N = 1$  and  $N = 2$  super-Teichmüller theory**

This lecture is devoted to the recent results obtained in collaboration with R. Penner and I. Ip regarding a construction of Penner-like coordinates on  $N = 1$  and  $N = 2$  super-Teichmüller spaces. We will discuss all necessary parts leading to this construction including a combinatorial approach to spin structures and super-Ptolemy transformations.

**Lecture 3: Super-Teichmüller theory: some recent results**

This lecture will be split between two topics. The first one is devoted to the application of the material from lecture 2, namely to the construction of the super-analogue of McShane identity on a 1-punctured torus following the recent work in collaboration with Y. Huang and R. Penner. The second part is devoted to the Strebel-like approach approach to the study of  $N = 1$  super-Riemann surfaces (based on the unpublished work with A. S. Schwarz).

**Tuesday Seminar on Topology: Homotopy Gerstenhaber algebras, Courant algebroids, and Field Equations**

I will talk about the underlying homotopical structures within field equations, which emerge in string theory as conformal invariance conditions for sigma models. I will show how these, often hidden, structures emerge from the homotopy Gerstenhaber algebra associated to vertex and Courant algebroids, thus making all such equations the natural objects within vertex algebra theory.