OIST 2020 workshop "Quantum Math, Singularities and Applications" Program

Kyoji Saito

Title: Undifferentiated elliptic flat structure

Abstract: Associated with an elliptic root system, we have defined the elliptic flat structure (1990, nowadays also called, elliptic Frobenius manifold structure by Dubrovin and Manin). More precisely, according to a choice of rank 1 subspace G, called the marking, of the radical of the elliptic root system, we introduce elliptic Weyl group action on the elliptic period domain. Then the Weyl group quotient space, i.e. the orbit space, is isomorphic to a smooth family of affine space over the upper half plane. Then, the flat structure is a flat metric structure J_G on the quotient space.

In the present talk, we introduce the "undifferentiated flat structure $\tilde{J}$" in the following sense. It is defined on a principal $\mathbb{C}^\times$-bundle equipped with a degenerate metric $\tilde{J}$. Then, according to a choice of a marking G, we choose one direction in the bundle so that the quotient metric specializes to J_G. The modular group $SL(2, Z)$ acts on $\tilde{J}$ (and on the set of markings) so that different flat structures are connected to each other by its action.

Motoko Kato

Title: Acylindrical hyperbolicity of some Artin-Tits groups of infinite type

Abstract: In this talk, we treat Artin-Tits groups of infinite type and their actions on non-positively curved spaces. Artin-Tits groups are groups defined by finite presentations of specific form. Braid groups, free abelian groups and free groups are examples of Artin-Tits groups. It is conjectured that the central quotient of every irreducible Artin-Tits group is either virtually cyclic or acylindrically hyperbolic. We prove this conjecture under some combinatorial conditions on generators of groups. In the proof of the main result, we observe group actions on

CAT(0) spaces, and detect group elements acting as rank one isometries. This talk is based on a joint paper arXiv:2004.03914 with Shinichi Oguni.

Shunsuke Takagi

Title: Arithmetic deformations of F-pure singularities

Abstract: In this talk, I discuss a connection between singularities in complex binational geometry and singularities defined via the Frobenius map. Let $x \in X$ be a Q-Gorenstein normal singularity defined over the field of complex numbers. Hara and Watanabe proved that if its modulo p reduction is F-pure for infinitely many primes p, then it is a log canonical singularity. I will argue that the same conclusion can be drawn by looking at a single p. This is joint work with Kenta Sato.

Takehiko Yasuda

Title: Quotient singularities via stringy invariants

Abstract: Batyrev introduced stringy invariant for singular varieties, motivated by study of the mirror symmetry. It turned out that this invariant was useful for the study of singularities themselves, in particular, from the viewpoint of minimal model program. In this talk, we discuss recent results about application of stringy invariants to quotient singularities in positive characteristic. In those applications, there naturally appears the arithmetic problem of controlling extensions of a local field.

Kosuke Shibata

Title: Minimal log discrepancies in positive characteristic

Abstract: The minimal log discrepancy is an important invariant of singularities in birational geometry. In characteristic zero, the existence of a prime divisor computing the minimal log discrepancy is shown using resolution of singularities and many properties of minimal log discrepancies are proved using the existence of such divisors. In this talk, I will show the existence of prime divisors computing minimal log discrepancies in positive characteristic except for a special case. Moreover I will prove the lower semicontinuity of minimal log discrepancies for smooth varieties in positive characteristic if the exponent of an ideal is less than the log canonical threshold of the ideal.

Yasuyuki Kawahigashi

Title: Topological order, tensor networks and operator algebras

Abstract: We will explain studies of 2-dimensional topological order in terms of tensor networks and subfactors arising from commuting squares in the sense of Jones. Appearance of braiding structure from 3-dimensional topological quantum field theory is demonstrated from a viewpoint of tensor categories. We will give higher relative commutants of a subfactor as spaces on which gapped Hamiltonians act.

Makiko Mase

Title: On Orlik ' s conjecture for the Milnor lattice of isolated hypersurface singularities

Abstract: An isolated hypersurface singularity admits the Milnor lattice together with a classical monodromy action. It is conjectured by Orlik that the Milnor lattice of an isolated hypersurface singularity admits a " standard decomposition into Orlik blocks ", which is roughly a decomposition corresponding to elementary divisors. In this talk, I will report our results that we can obtain a sufficient condition for several types of isolated hypersurface singularities to admit such a nice decomposition. This is a joint work with Professor Claus Hertling.

Mayuko Yamashita

Title: The classification problem of non-topological invertible QFT ' s and a " differential " model for the Anderson duals

Abstract: Freed and Hopkins conjectured that the deformation classes of non-topological invertible quantum field theories are classified by a generalized cohomology theory called the Anderson dual of bordism theories. The main difficulty of this problem lies in the fact that we do not have the axioms for QFTs. In this talk, I will explain the ongoing work to give a new approach to this conjecture. We construct a new, " physicists-friendly " model for the Anderson duals. This model is constructed so that it abstracts a certain property of invertible QFT ' s which physicists believe to hold in general. I will start from basic motivations for the classification problem, report the progress of our

work and explain future directions. This is the joint work with Yosuke Morita (Kyoto, math) and Kazuya Yonekura (Kyushu, physics).

Benoir Collins

Title: Matrix integrals in a tensor setup

Abstract: Matrix integrals has been heavily developed in the context of theoretical physics and more recently, it was further studied from a mathematical point of view. In the meanwhile, random tensors have become the object of important activity in theoretical physics (part of the work presented here started from previous investigations by Rivasseau and Gurau). Viewing matrix integrals as an aspect of random matrix theory, we extend this point of view to the case of random tensors. In particular, we introduce matrix integrals that generalize the HCIZ integral, whose symmetry group has a tensor structure and obtain new expansion results. We discuss applications to quantum information theory and tensor valued free probability. This talk is based on a series of joint works with Luca Lionni and Razvan Gurau.

Shinobu Hikami

Title: Singularities and Conformal Bootstrap for Curves

Abstract: The singularities of A_{p-1} type are related to p-spin curves of the moduli space of Riemann surfaces. These curves are derived by the conformal field theory (CFT) of matrix model, where the case of $p = 4$ represents the critical phenomena of Ising (Z_2) model. A random magnetic field Ising model (RFIM) has a conjecture of the dimensional reduction $d \to d - 2$ by the supersymmetry, which may hold above $d = 4$ space dimensions. We examine the breakdown of this conjecture near $d = 4$ by the conformal bootstrap method. Extension of the integer p to half-integer value is formulated in a matrix model with Ramond punctures. The half-integer case $p = 3/2$ corresponds to β-γ system.
We discuss arc spaces for spin curves of half integer and negative p. (This talk is based on works with E. Brézin, arXiv: 2001.09267, 2010.01444).

Terry Gannon

Title: Modular invariant classifications: From ADE to ...?

Abstract: The classification of modular invariants, more precisely that of quantum subgroups, is essentially the classification of conformal field theories associated to quantum groups at roots of unity. This problem became famous in the 1980s thanks to the ADE classification of Cappelli-Itzykson-Zuber, but little hard progress has been achieved since then. The early 2000's were tantalised by announcements by Ocneanu of the existence of an upper bound for the orders of those roots of unity possessing exceptional quantum subgroups. Unfortunately, details were not forthcoming. But a breakthrough happened in 2017 when Schopieray made this explicit for the rank 2 cases: e.g. he found about 18 million potentially exceptional orders (levels) for G_2. In my talk I'll explain that combining Ocneanu-Schopieray's idea with Galois considerations yields much smaller bounds (e.g. for G_2 it gives only 2 potentially exceptional levels, and for e.g. $sl(7)$ there are only 36 such levels). I'll then use this bound to give the classification of quantum subgroups for all ranks up to and including 6.

Shihoko Ishii

Title: Uniform bound of the number of weighted blow up to compute the minimal log discrepancy for smooth 3-folds.

Abstract: We study a pair consisting of a smooth 3-fold defined over an algebraically closed field and a real ideal. We show that the minimal log discrepancy of every such a pair with a "general" real ideal is computed by at most two weighted blow ups. This uniform bound is regarded as a weighted blow up version of Mustata-Nakamura's conjecture.

Masahiko Yoshinaga

Title: The primitive derivation and discrete integrals

Abstract: The modules of logarithmic derivations for the (extended) Catalan and Shi arrangements associated with root systems are known to be free. However, except for a few cases, explicit bases for such modules are not known. In this paper, we construct explicit bases for type A

root systems.
Our construction is based on Bandlow-Musiker's integral formula for a basis of the space of quasiinvariants. The integral formula can be considered as an expression for the inverse of the primitive derivation introduced by K. Saito. We prove that the discrete analogues of the integral formulas provide bases for Catalan and Shi arrangements. This is a joint work with Daisuke Suyama.

Lawrence Ein

Title: Singularities and syzygies of secant varieties of curve

Abstract: Let X be a smooth curve of genus g and L be a a line bundle of degree greater or equal to $2g+2k+1+p$, where $p \geq 0$. Let $\Sigma_k(X,L)$ be the $k-$th secant variety of (X,L). Then $\Sigma_k(X,L)$ satisfies $N_{k+2,p}$. It means that the homogenous ideal is generated by forms of degree $k+2$ and the next p steps of the minimal resolutions are given by matrices of linear forms. Furthermore, $\Sigma_k(X,L)$ is a normal projectie Cohen Macaulay variety with Du Bois singularities.

Vladimir Dotsenko

Title: Brick manifolds and wonderful models

Abstract: I will talk about a remarkable series of toric varieties that resemble the Deligne-Mumford compactifications of moduli spaces of genus zero curves with marked points. Homology groups of these spaces assemble into an algebraic operad which hints at a yet undiscovered notion of a non-commutative cohomological field theory. This is joint work with Sergey Shadrin and Bruno Vallette.

Zhenghan Wang

Title: Quantum mathematics: counting, computing, and reasoning with quantum numbers

Abstract: We count things with numbers classically. But in the quantum world, such as electrons in a material, the distinction between one thousand electrons and two more could be blurred by quantum superposition. As a result, quantum counting is better done using wave

functions. It turns out that information can be processed much more efficiently using wave functions on quantum computers. I will explain the basics of quantum computing and speculate on implications for future mathematics.

NORMAL HILBERT COEFFICIENTS AND ELLIPTIC IDEALS IN NORMAL 2-DIMENSIONAL LOCAL DOMAINS

KEI-ICHI WATANABE (NIHON UNIVERSITY AND MEIJI UNIVERSITY)

This is a joiint work with T. Okuma (Yamagata Univ.), M.E. Rossi (Univ. Genova) and K. Yoshida (Nihon Univ.).

Let $(A, \mathfrak{m})$ be an excellent two-dimensional normal local domain and let I be an integrally closed $\mathfrak{m}$-primary ideal.

Then the normal Hilbert coefficients $\bar{e}_i(I)$ $(i = 0, 1, 2)$ are defined by

$$\ell_A(A/\overline{I^{n+1}}) = \bar{e}_0(I)\binom{n+2}{2} - \bar{e}_1(I)\binom{n+1}{1} + \bar{e}_2(I)$$

for $n \gg 0$.

Let I be an $\mathfrak{m}$ primary integrally closed ideal in A and $f : X \to \operatorname{Spec}(A)$ be a resolution of A such that $I\mathcal{O}_X = \mathcal{O}_X(-Z)$ is invertible. Let Q be a minimal reduction of I (= a parameter ideal with $Q\mathcal{O}_X = I\mathcal{O}_X$). Then the reduction numbers

$$\mathrm{nr}(I) = \min\{n \mid \overline{I^{n+1}} = Q\overline{I^n}\}, \quad \bar{r}(I) = \min\{n \mid \overline{I^{N+1}} = Q\overline{I^N}, \forall N \geq n\}$$

are important invariants of the ideal and the singularity.

We have studied p_g ideals which satisfy $\bar{(}I) = 1$ or, equivalently $\bar{e}_2(I) = 0$. To say that A is a rational singularity is equivalent to say that all integrallyv closed $\mathfrak{m}$ primary ideals of A are p_g ideals.

These concepts are closely related to $p_g(A) := h^1(X, \mathcal{O}_X)$ and if A is an elliptic singularity, then it is shown by Okuma that $\bar{r}(I) \leq 2$ for every I.

Inspired by these facts we define I to be an *elliptic ideal* if $\bar{r}(I) = 2$ and *strongly elliptic ideal* if $\bar{e}_2 = 1$.

In this talk I will explain about the properties of elliptic and strongly elliptic ideals.

This talk is based on our joint work appeared in arXiv 2012.05530.