Accelerator Laboratory
Advanced Research Center for Beam Science
Institute for Chemical Research
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Thin Foil on Bulk Conductor

Y. Iwashita, Kyoto U.
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Contents:

- Normal Conducting Case
- Super Conducting Case

A. M. Clogston, Reduction of Skin-Effect Losses by the Use of Laminated Conductors, Proc. of the IRE, 39-7, July 1951, pp.767-782

Reduction of Skin-Effect Losses by the Use of Laminated Conductors*

A. M. CLOGSTON†, SENIOR MEMBER, IRE

Summary—**It has recently been discovered that it is possible to reduce skin effect losses in transmission lines by properly laminating the conductors and adjusting the velocity of transmission of the waves. The theory for such laminated transmission lines is presented in the case of planar systems for both infinitesimally thin laminae and laminae of finite thickness. A transmission line completely filled with laminated material is discussed. An analysis is given of the modes of transmission in a laminated line, and of the problem of terminating such a line.**

I. Introduction

IT HAS LONG been recognized that an electromagnetic wave propagating in the vicinity of an electrical conductor can penetrate only a limited distance into the interior of the material. This phenomenon is known as "skin effect" and is usually measured by a so-called "skin depth" δ. If y is measured from the surface of a conductor into its depth, the amplitude of the electromagnetic wave and the accompanying current density decreases as $e^{-y/\delta}$, provided the conductor is several times δ in thickness, so that for $y=\delta$ the amplitude has fallen to $1/e=0.367$ times its value at the surface. The skin depth δ is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}, \tag{1}$$

* This is one of a class of papers published through arrangements with certain other journals. It is appearing also in the July, 1951, issue of the *Bell System Technical Journal*.

† Bell Telephone Laboratories, Inc., Murray Hill, N. J.

where σ is the conductivity of the material, μ is its permeability, and ω is 2π times the frequency f under consideration. Throughout this paper rationalized mks units are used.

From one point of view, skin effect serves a most useful purpose; for instance, in shielding electrical equipment or reducing cross talk between communication circuits. On the other hand, the effect severely limits the high-frequency performance of many types of electrical apparatus, including in particular the various kinds of transmission lines.

Surprisingly enough, it has been discovered that it is possible, within limits, to increase the distance to which an electromagnetic wave penetrates into a conducting material. This is done essentially by fabricating the conductor of many insulated laminae or filaments of conducting material arranged parallel to the direction of current flow. If the transverse dimensions of the laminae or filaments are small compared to the skin depth δ at the frequency under consideration, and if the velocity of the electromagnetic wave along the conductor is close to a certain critical value, the wave will penetrate into the composite conductor a distance great enough to include a thickness of conducting material many skin depths deep. Physically speaking, the lateral change of the wave through the conducting regions is very nearly cancelled by the change through the insulating regions.

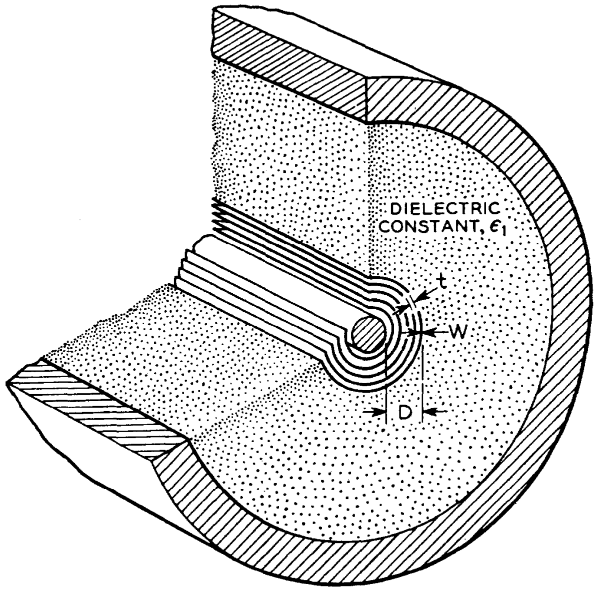

Fig. 1—Laminated transmission line.

But no product...

RF Power Loss

Current Distribution: $$j(x)=\frac{1+i}{\delta}H_z(0)e^{i\omega t}e^{-(1+i)x/\delta}$$

Total Current in conductor: $$J=\int_0^\infty j(x)\,dx=H_z(0)e^{i\omega t}$$

Power loss：

$$P_{bulk}=\int_0^\infty |j|^2/\sigma\,dx=\frac{H_z(0)^2}{\sigma\delta}=\sqrt{\frac{\omega\mu}{2\sigma}}H_z(0)^2$$

Higher conductance σ→ Thinner skin depth δ

→ Higher current density *j(x)* → Less loss reduction

Independent current distribution control possible?

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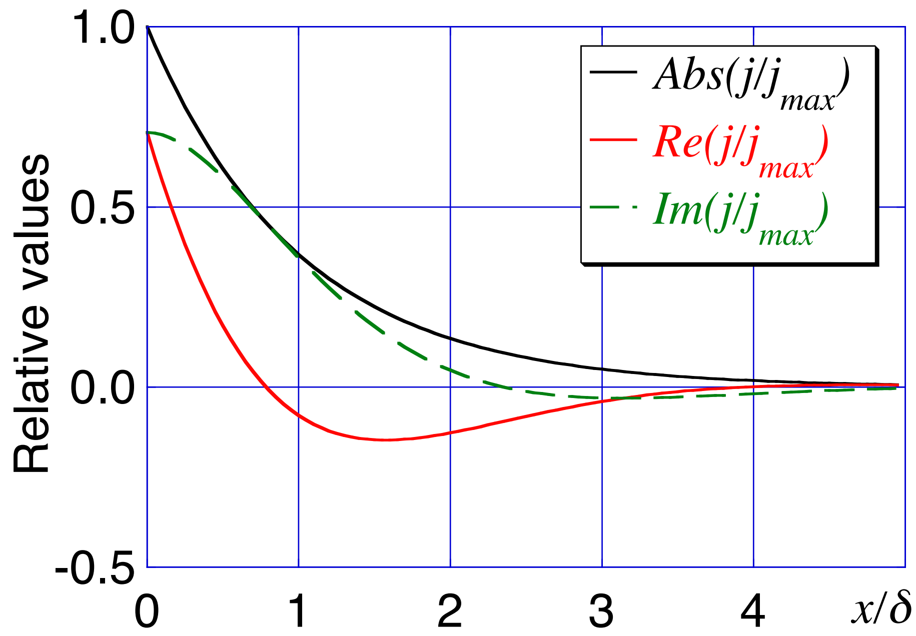
Current Distribution in Conductor

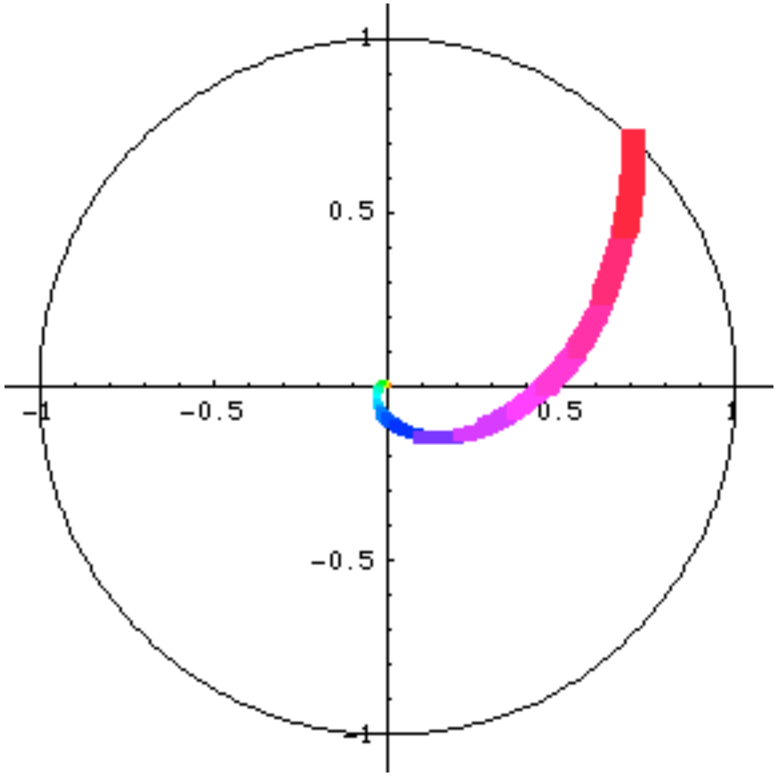
$$j(x) = \frac{1+i}{\delta} H_z(0) e^{i\omega t} e^{-(1+i)x/\delta}$$

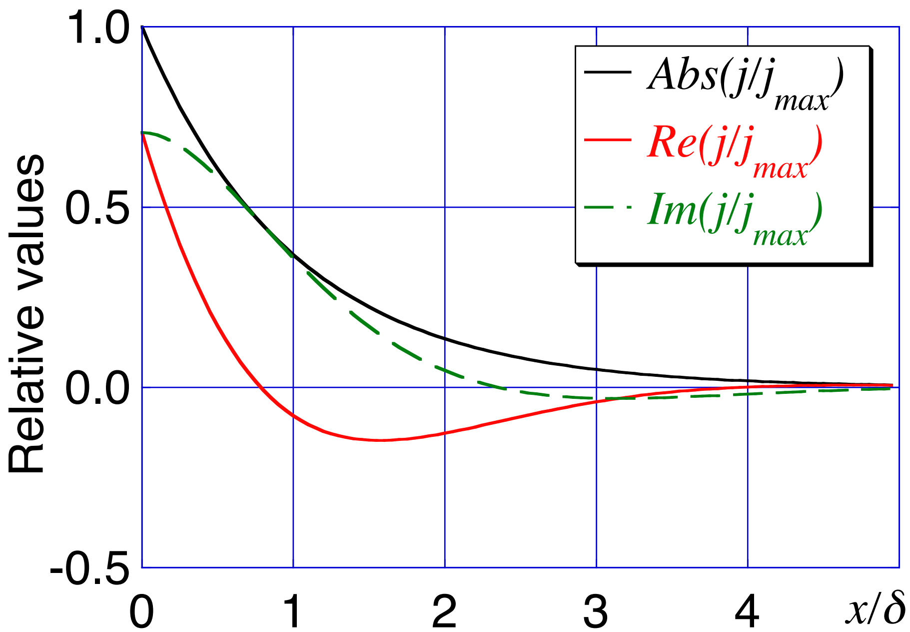


δ : Skin Depth

Current Distribution in Conductor

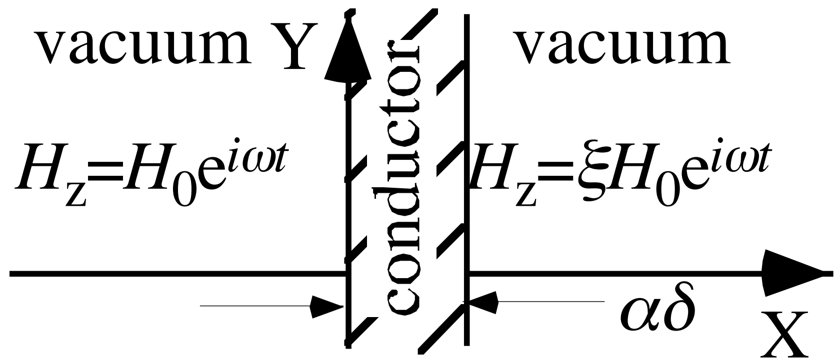
$$j(x) = \frac{1+i}{\delta} H_z(0) e^{i\omega t} e^{-(1+i)x/\delta}$$





δ : Skin Depth

EM Field in a Thin Foil Conductor

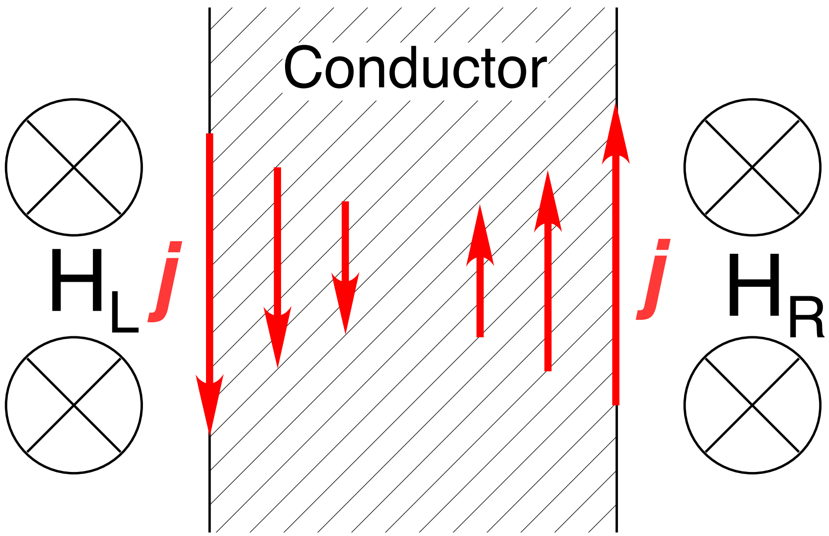


$$j(x) = H_z(0)\left(j_f e^{-(1+i)x/\delta} - j_b e^{-(1+i)(\alpha\delta - x)/\delta}\right)$$

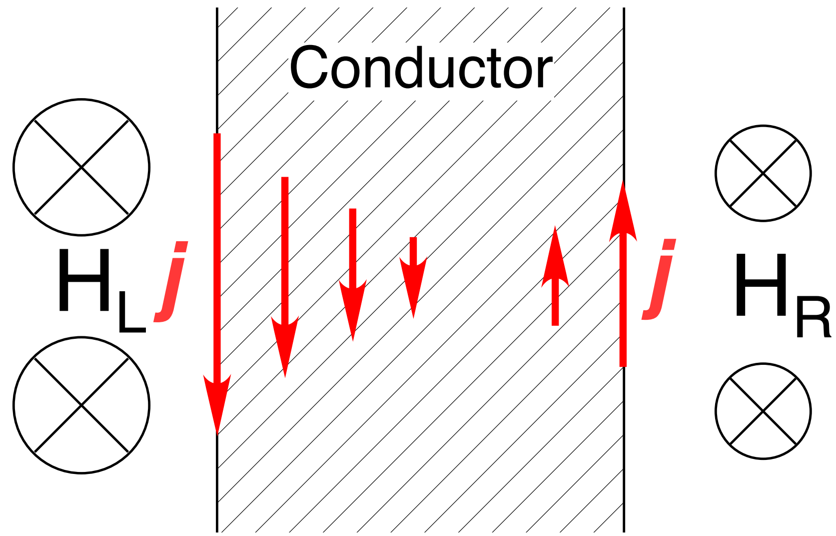
$$j_f = \frac{(1+i)e^{(1+i)\alpha}\left(e^{(1+i)\alpha} - \xi\right)}{\delta\left(e^{2(1+i)\alpha} - 1\right)}, j_b = \frac{(1+i)e^{(1+i)\alpha}\left(\xi e^{(1+i)\alpha} - 1\right)}{\delta\left(e^{2(1+i)\alpha} - 1\right)}.$$

Superposition of left and right traveling waves.

Current Distribution



$H_L = H_R$

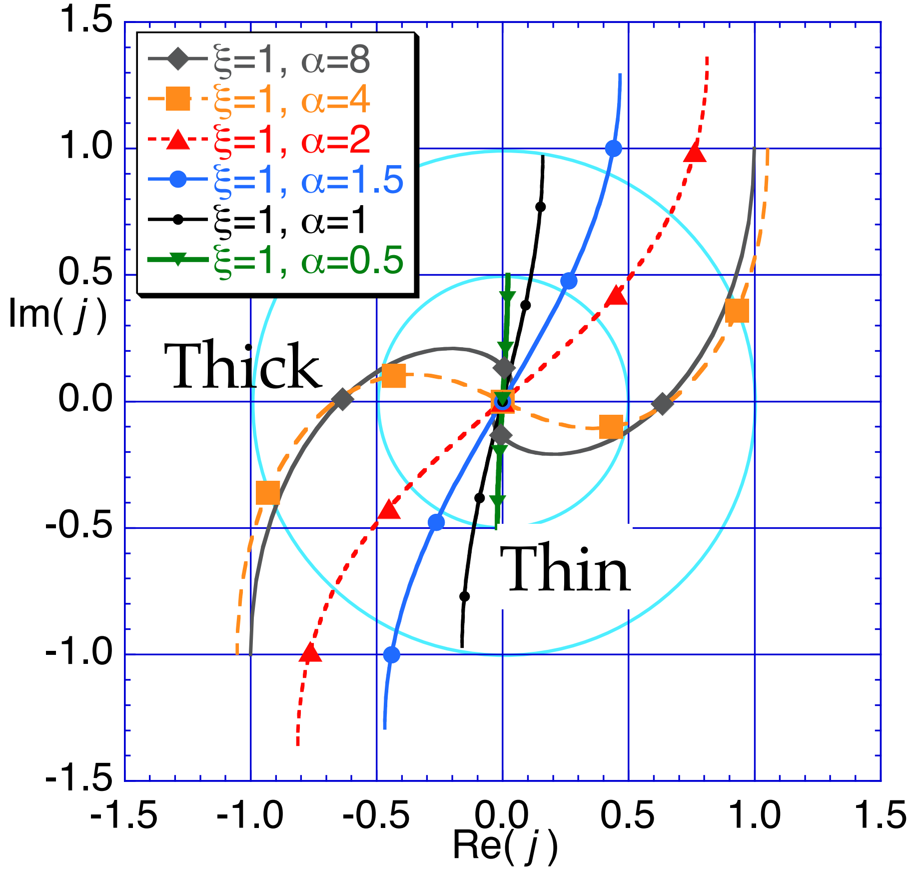


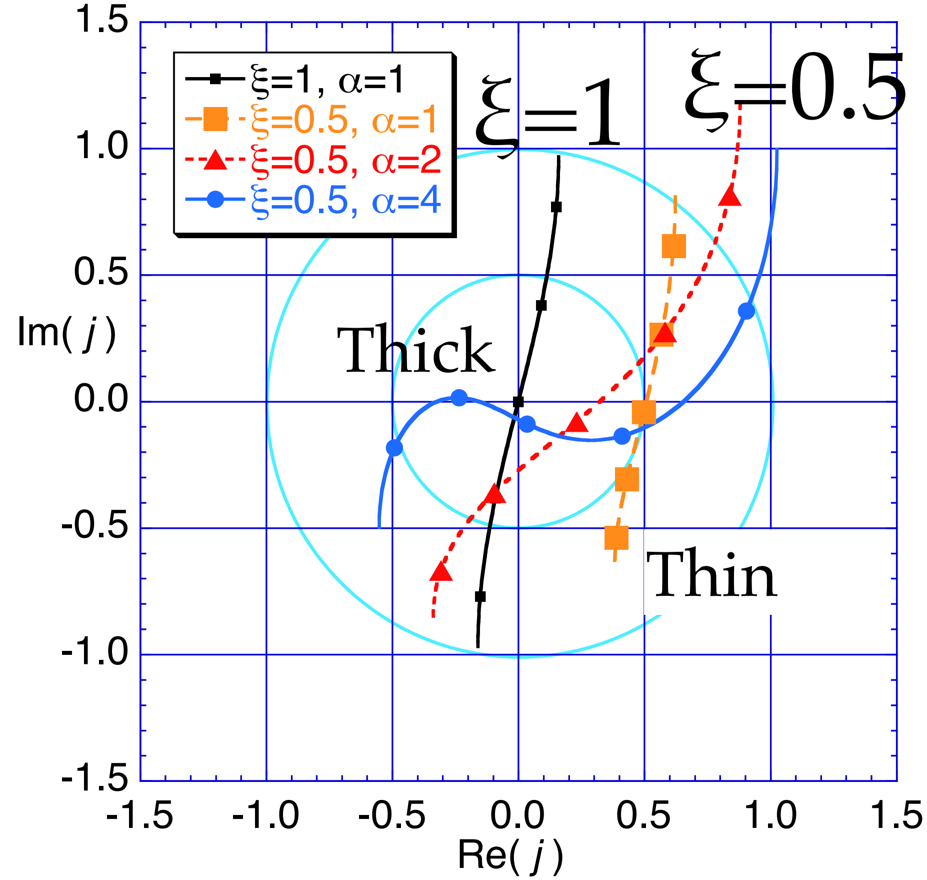
$H_L > H_R$

Currents cancel each other in-between

(simplified schematics
... have to consider phase)

Current Distribution in a Thin Foil





Same B: Zero net Current

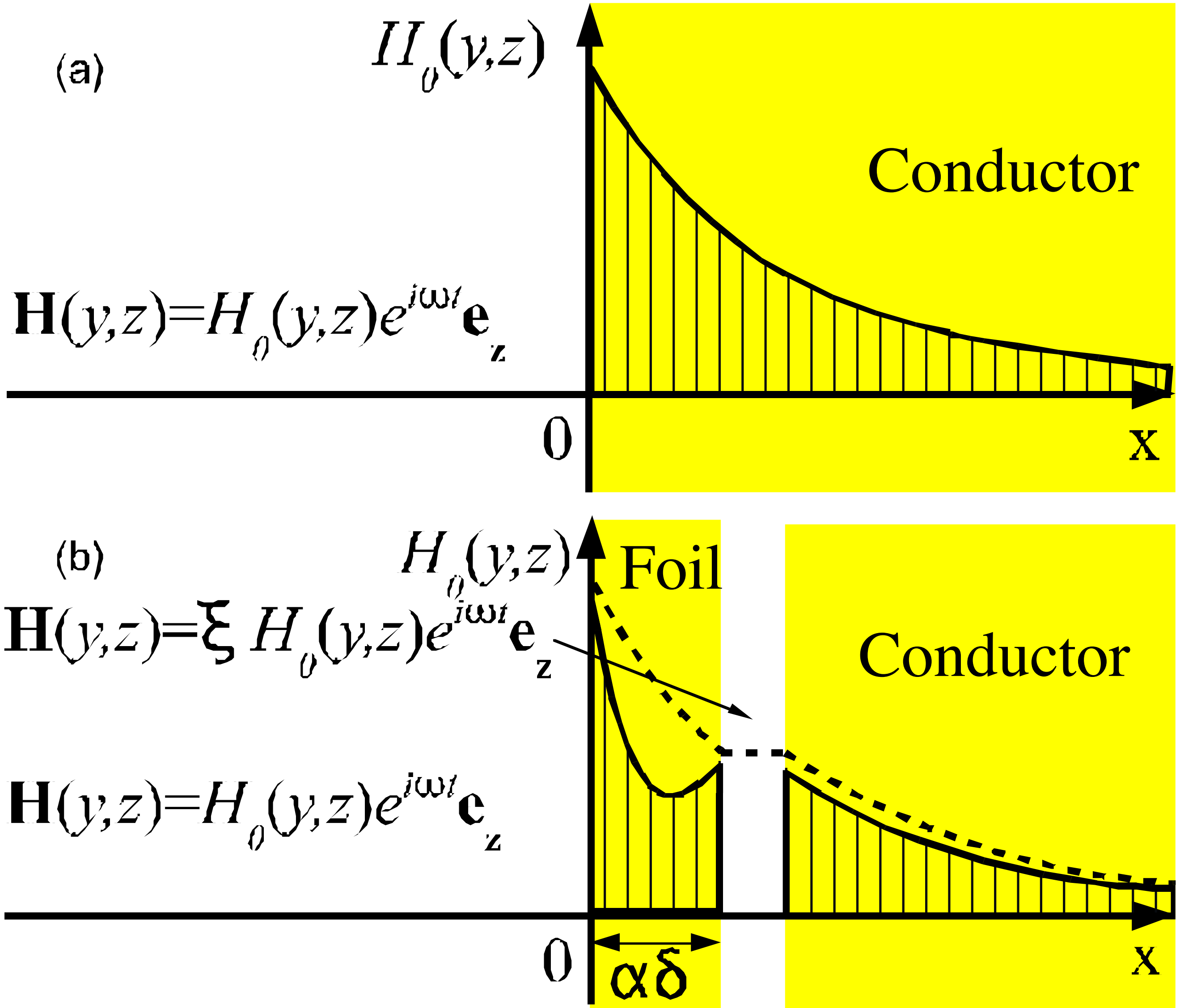
Thinner→Real part vanishes quickly while imaginary part remains

Half strength at right

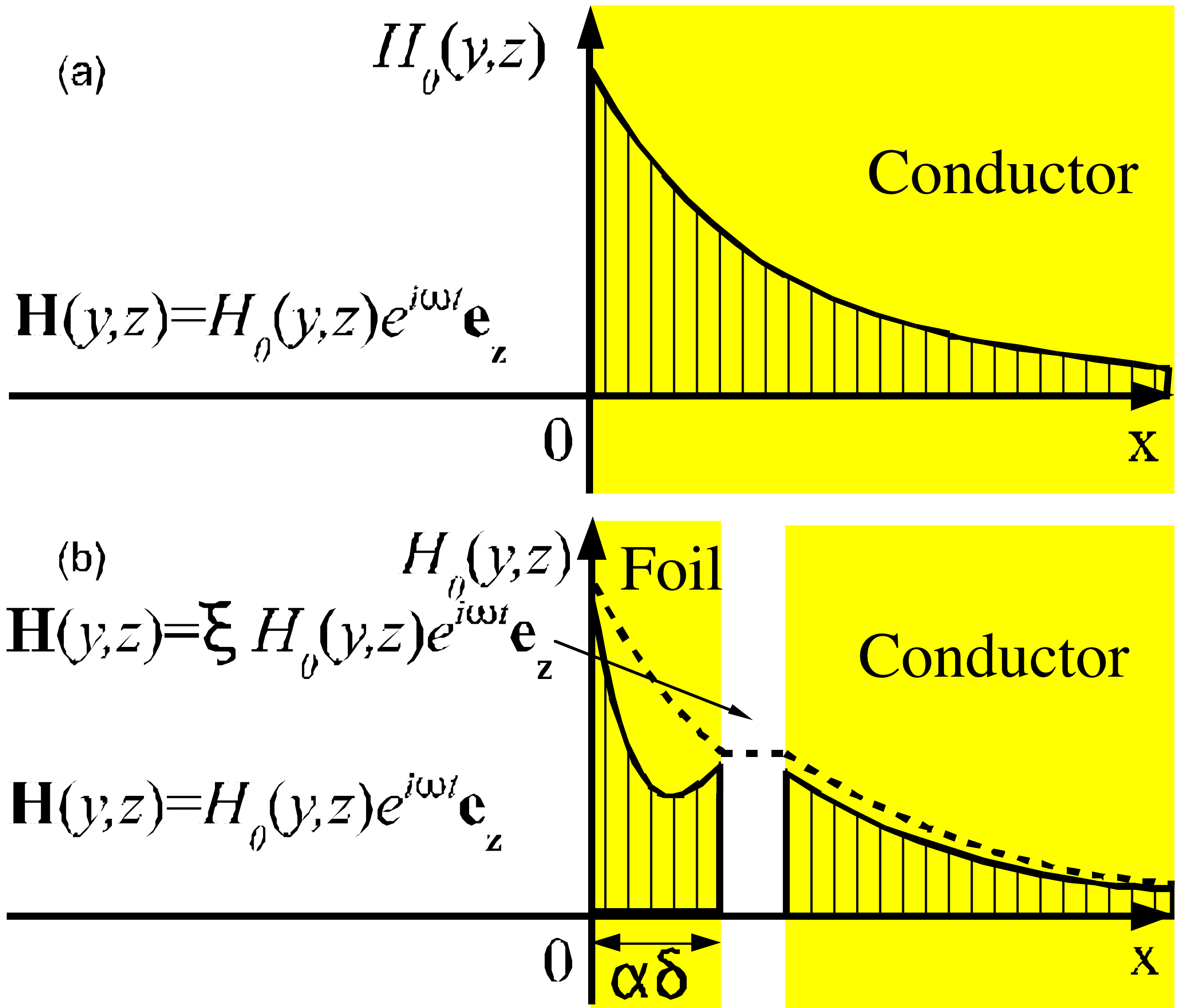
Thinner→Uniform real part Linear in imaginary part

Net current in imaginary part is Zero.

Magnetic Field(current)distribution



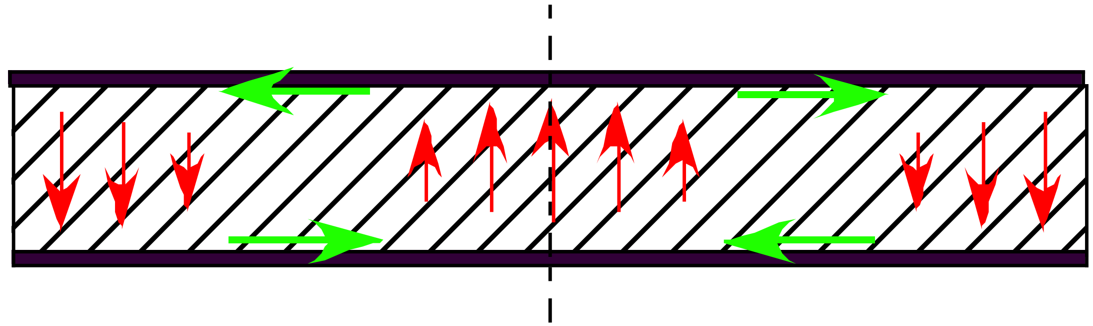
Magnetic Field(current)distribution



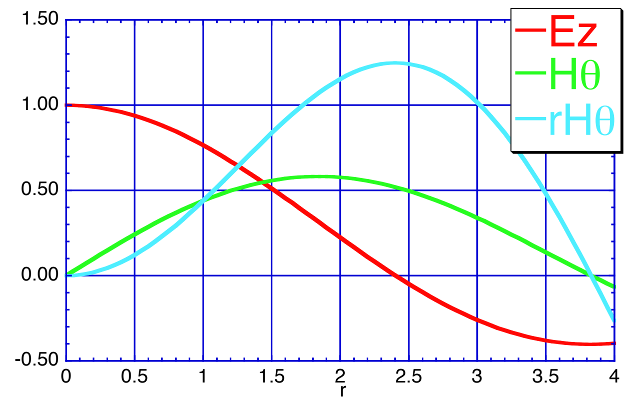
How to re-distribute the currents?

Example: Dielectric Resonator

$\varepsilon \gg 1$

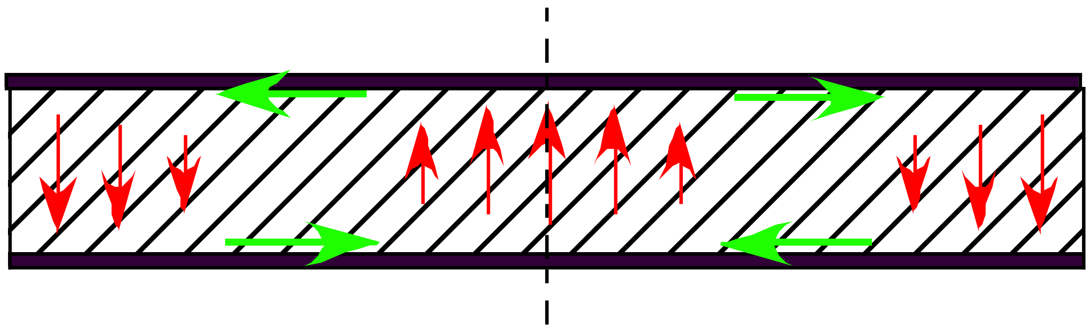


$$E_z(r) = E_z(0)\,\mathrm{J}_0(kr), \quad k = x_1'/R$$

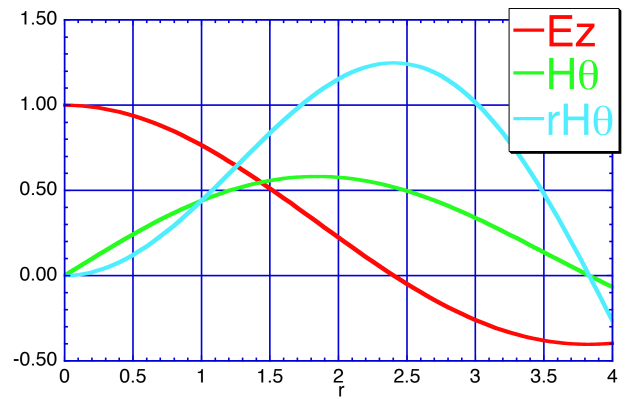


Example: Dielectric Resonator

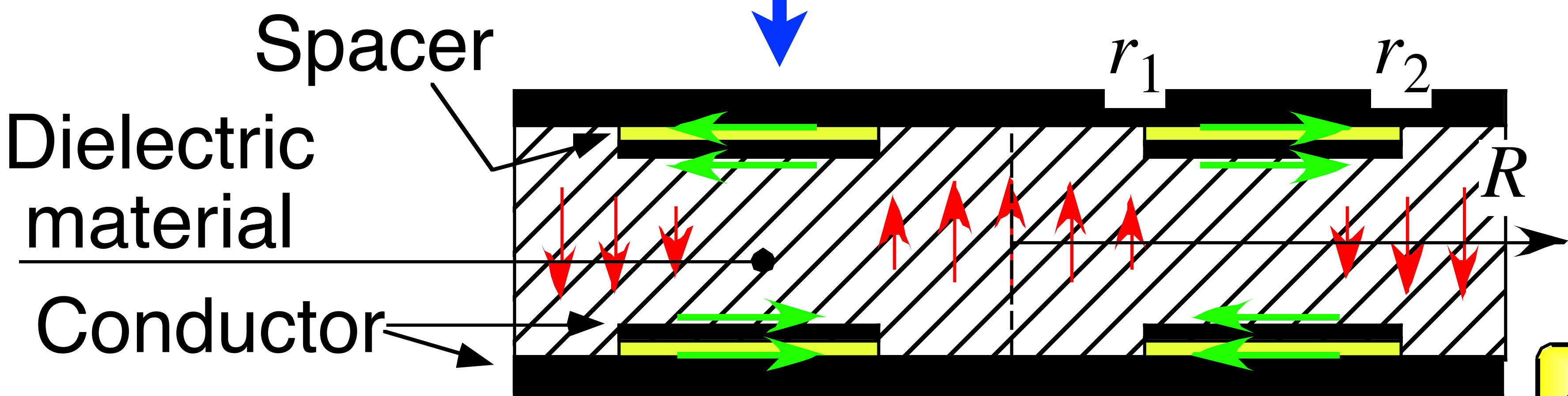
ε >> 1



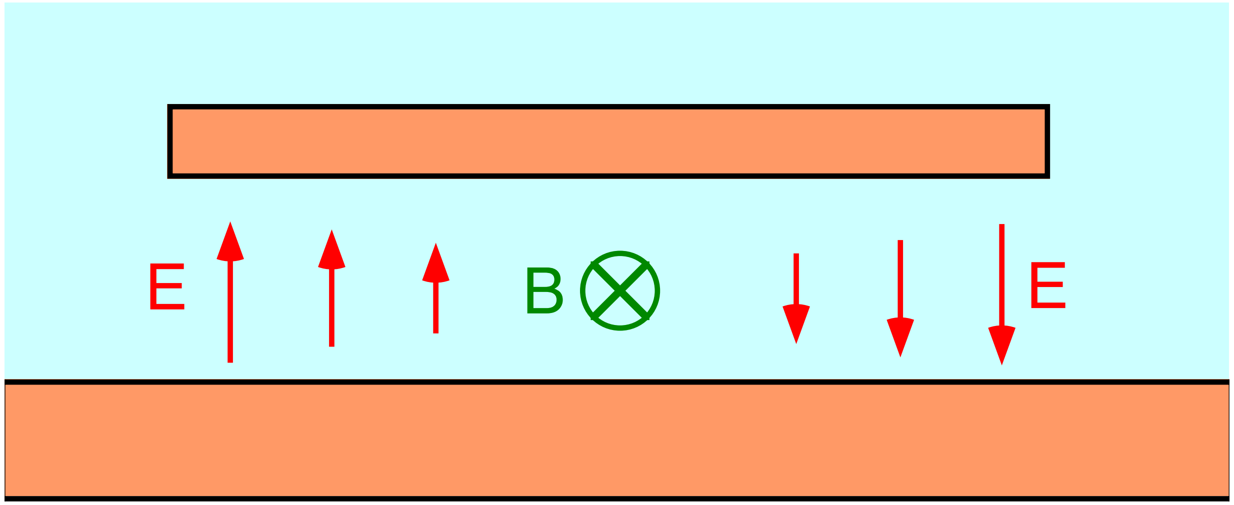
$$E_z(r) = E_z(0)\,\mathrm{J}_0(kr), \quad k = x_1'/R$$

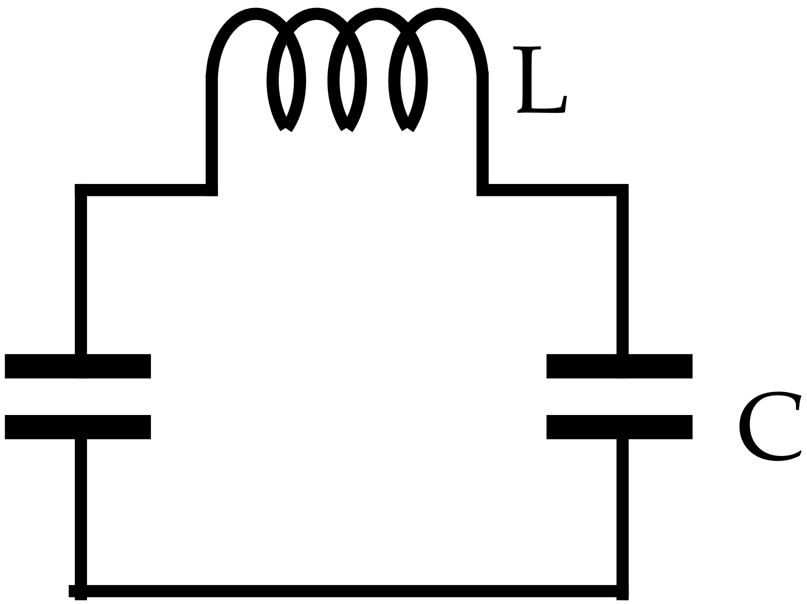


$$\int_{r_1}^{r_2} \dot{D}\, 2\pi r dr = 0$$



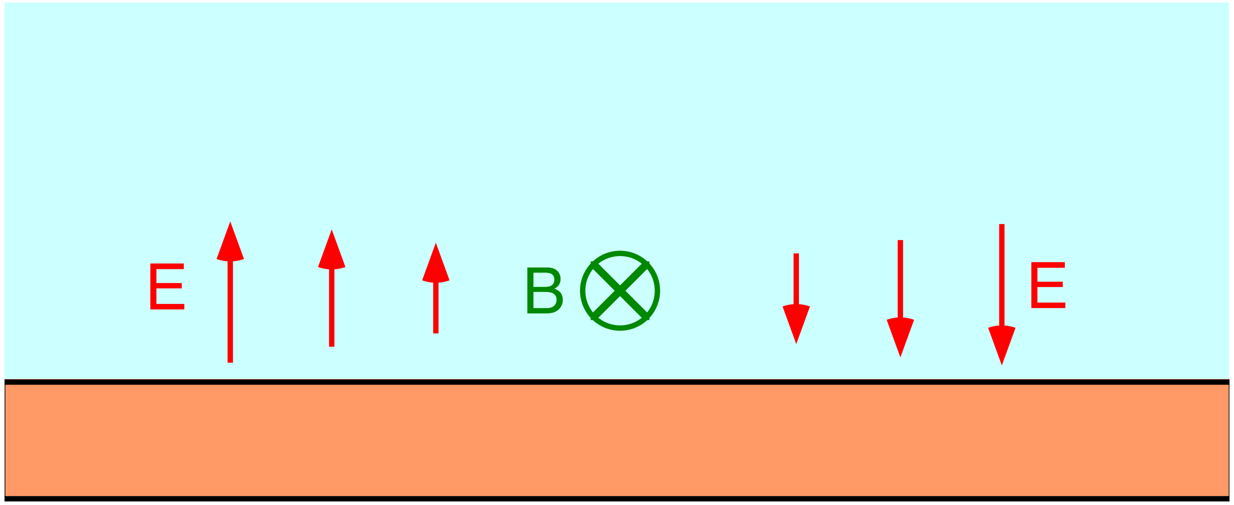
Equivalent Circuit

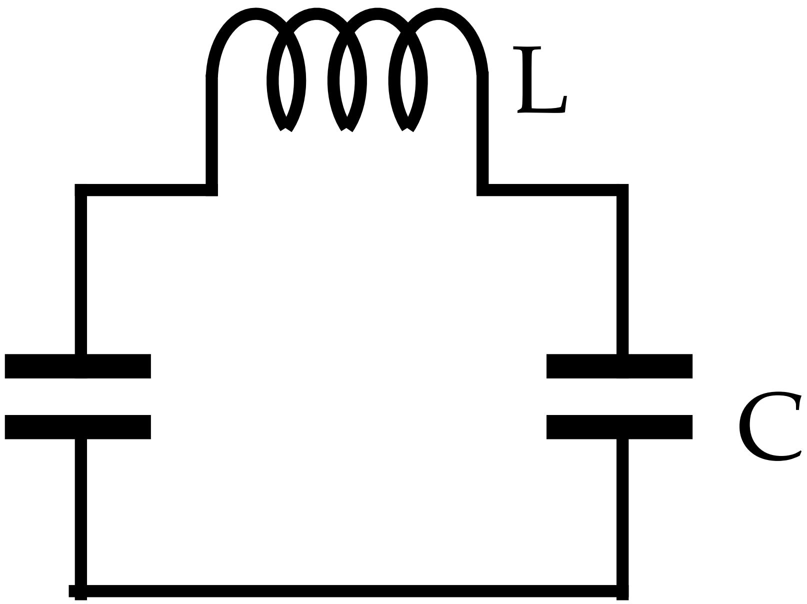




Raise freq. by reducing L and C

Equivalent Circuit





Raise freq. by reducing L and C

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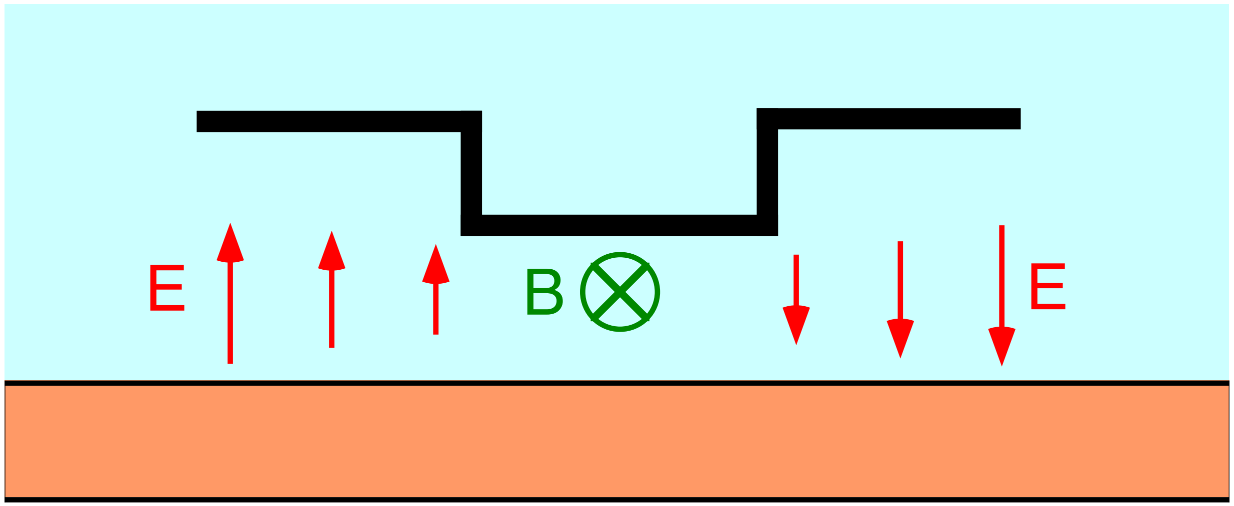
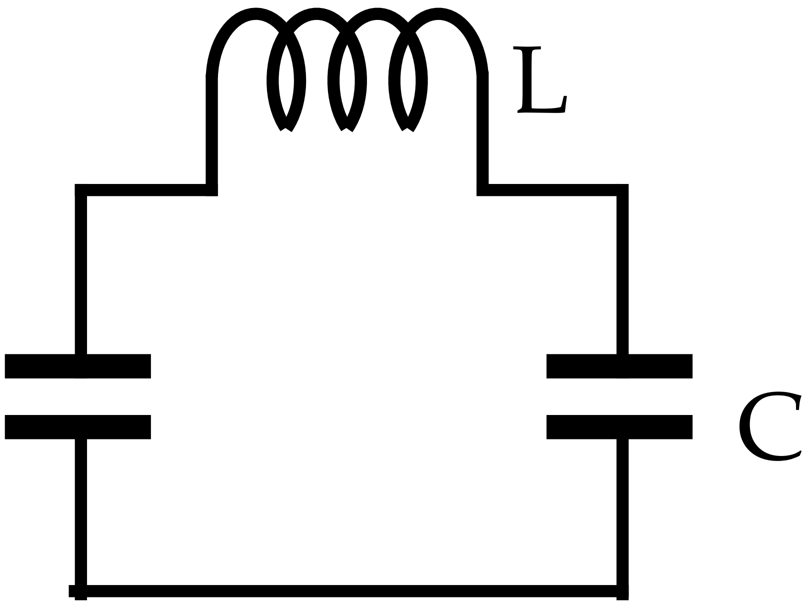
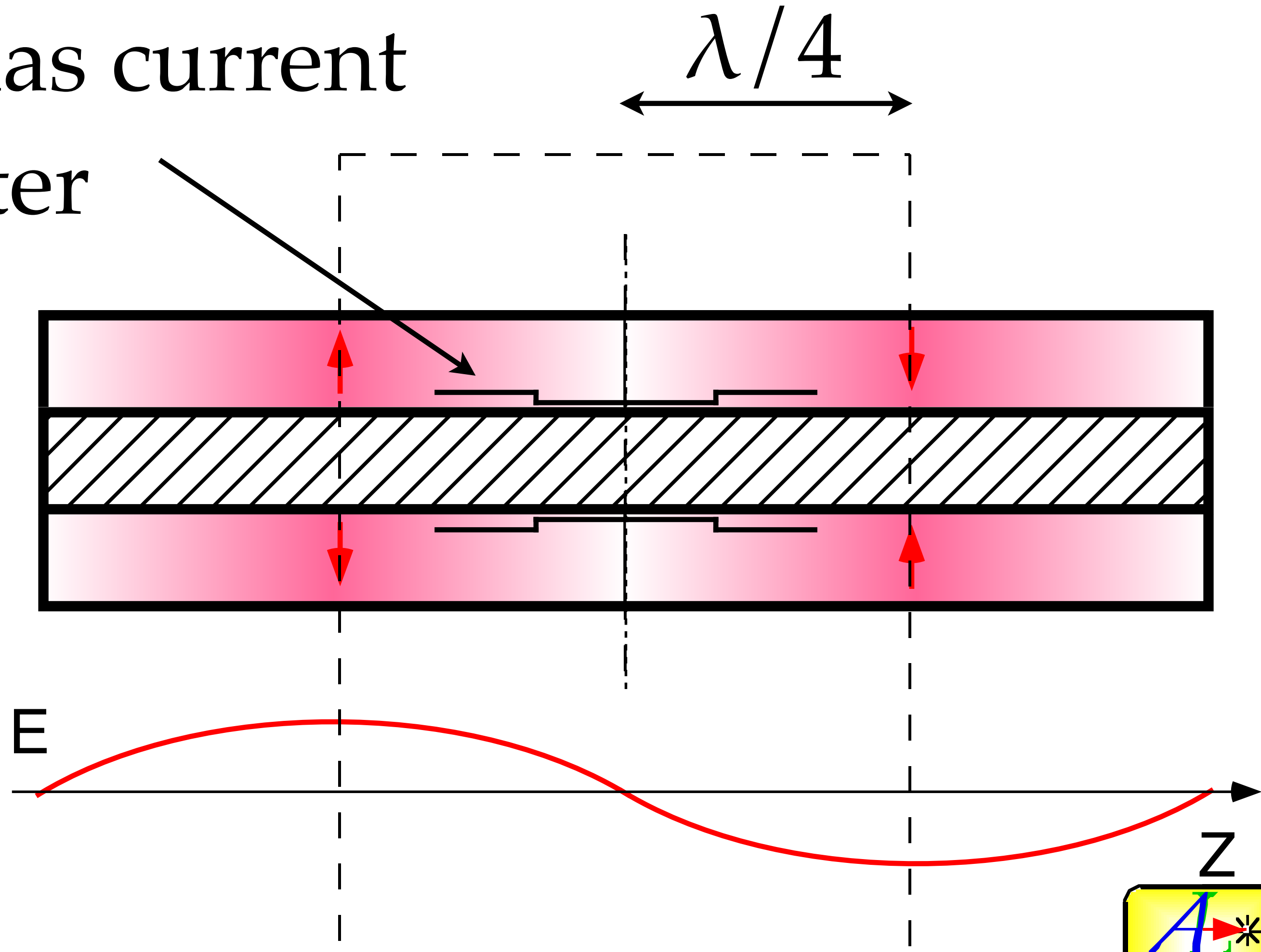




Raise freq. by reducing L and C

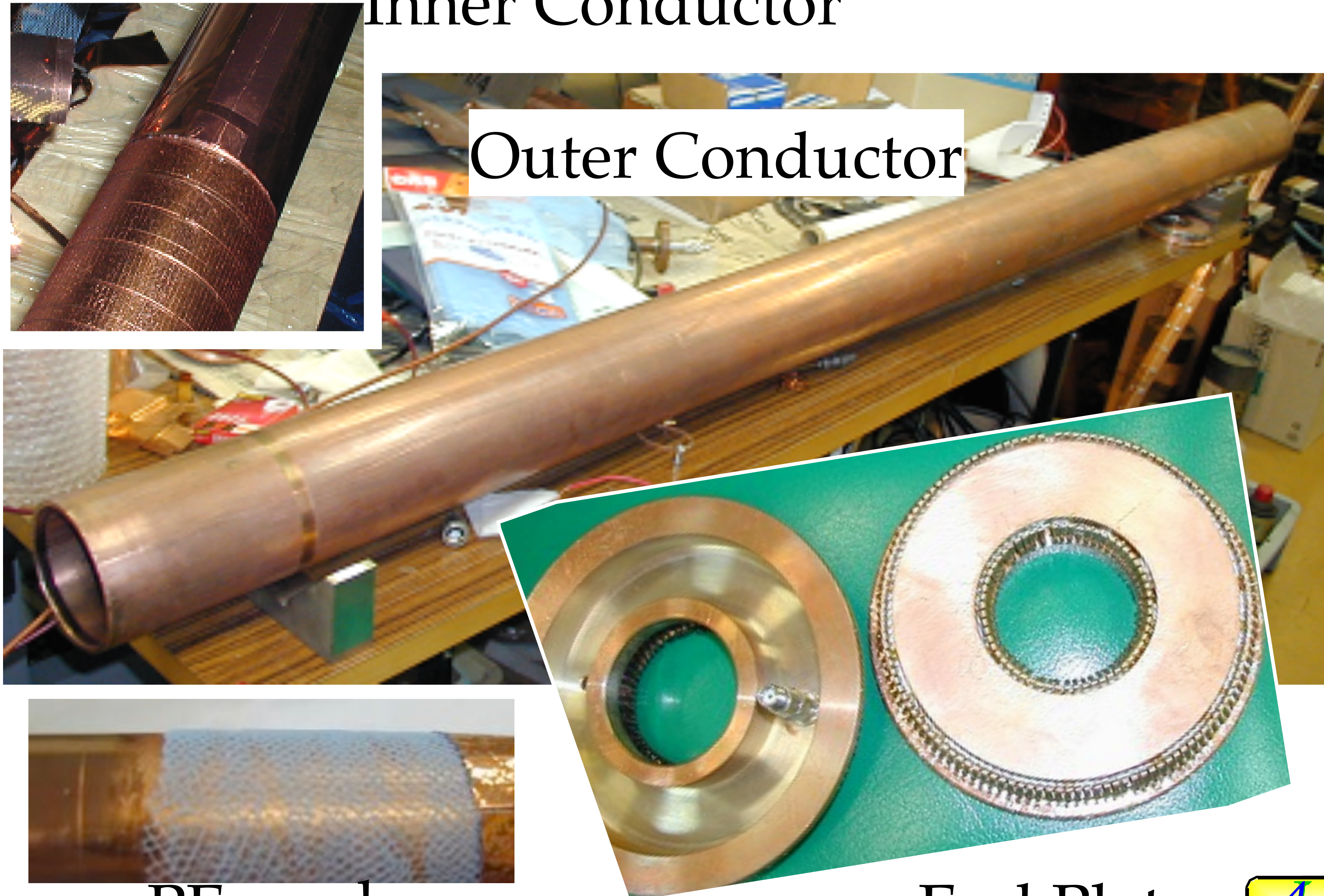
Experiment with coaxial cavity
(Simple Cavity Structure)

Second Mode has current peak at the center



Y.Tajima, Y.Iwashita, H.Fujisawa, M.Ichikawa, H.Tongu: Reduction of skin effect RF power loss by a thin conductor foil, JAPANESE JOURNAL OF APPLIED PHYSICS, 47, 4765-4768, 2008

Inner Conductor

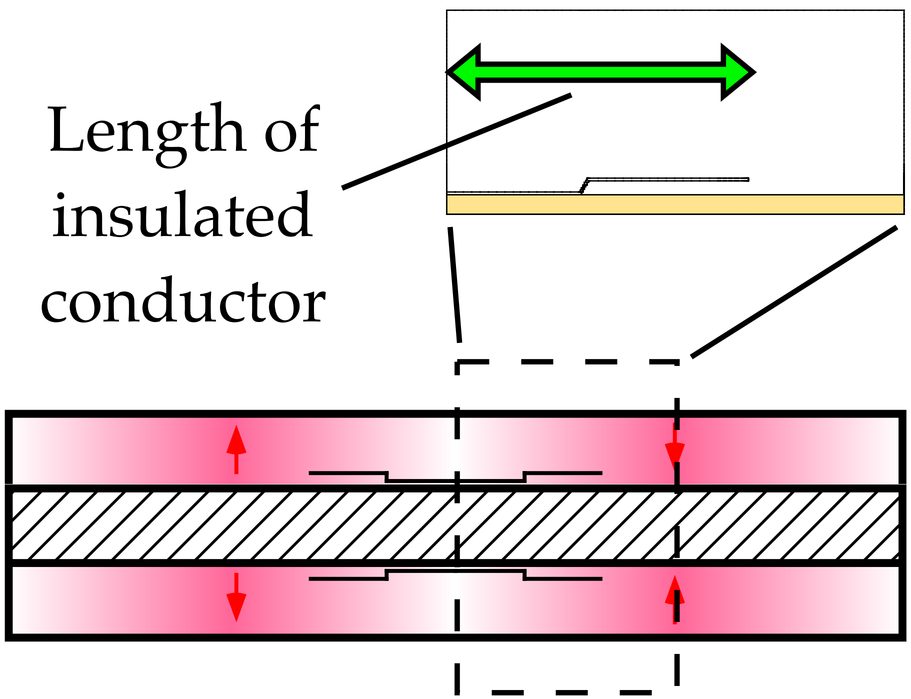


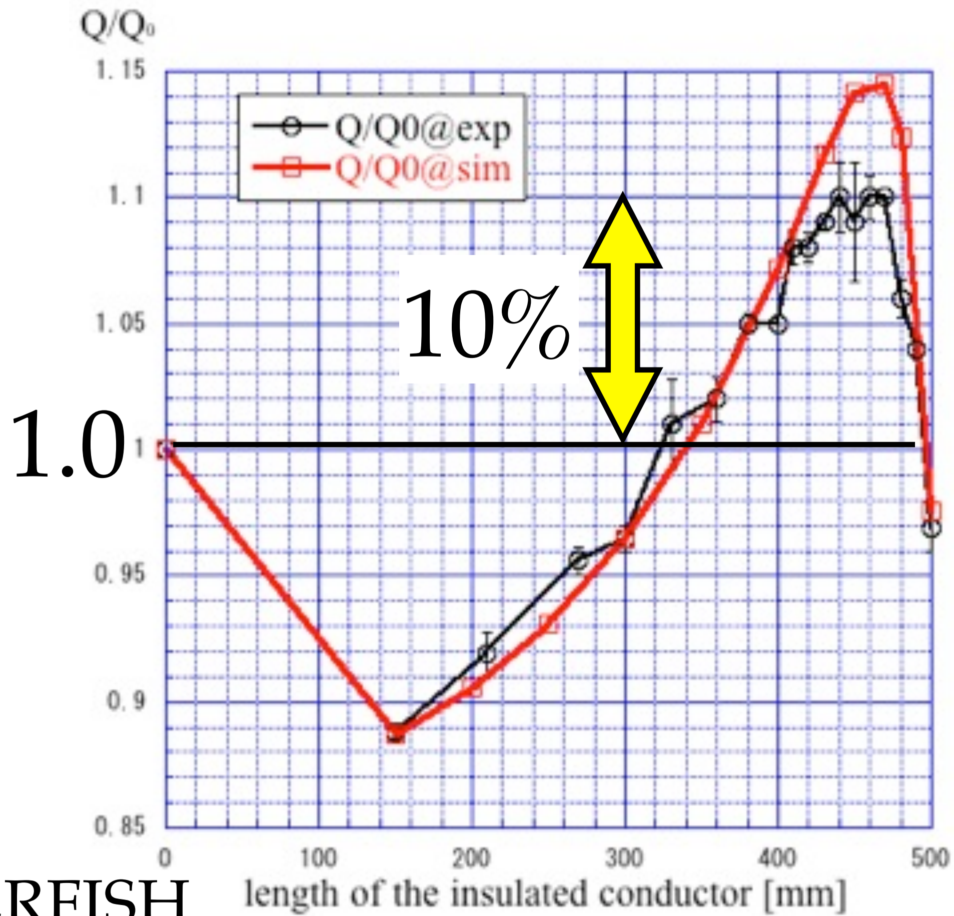
Outer Conductor

PE mesh

End Plates

Measurements and CFISH





CFISH：Complex version of SUPERFISH

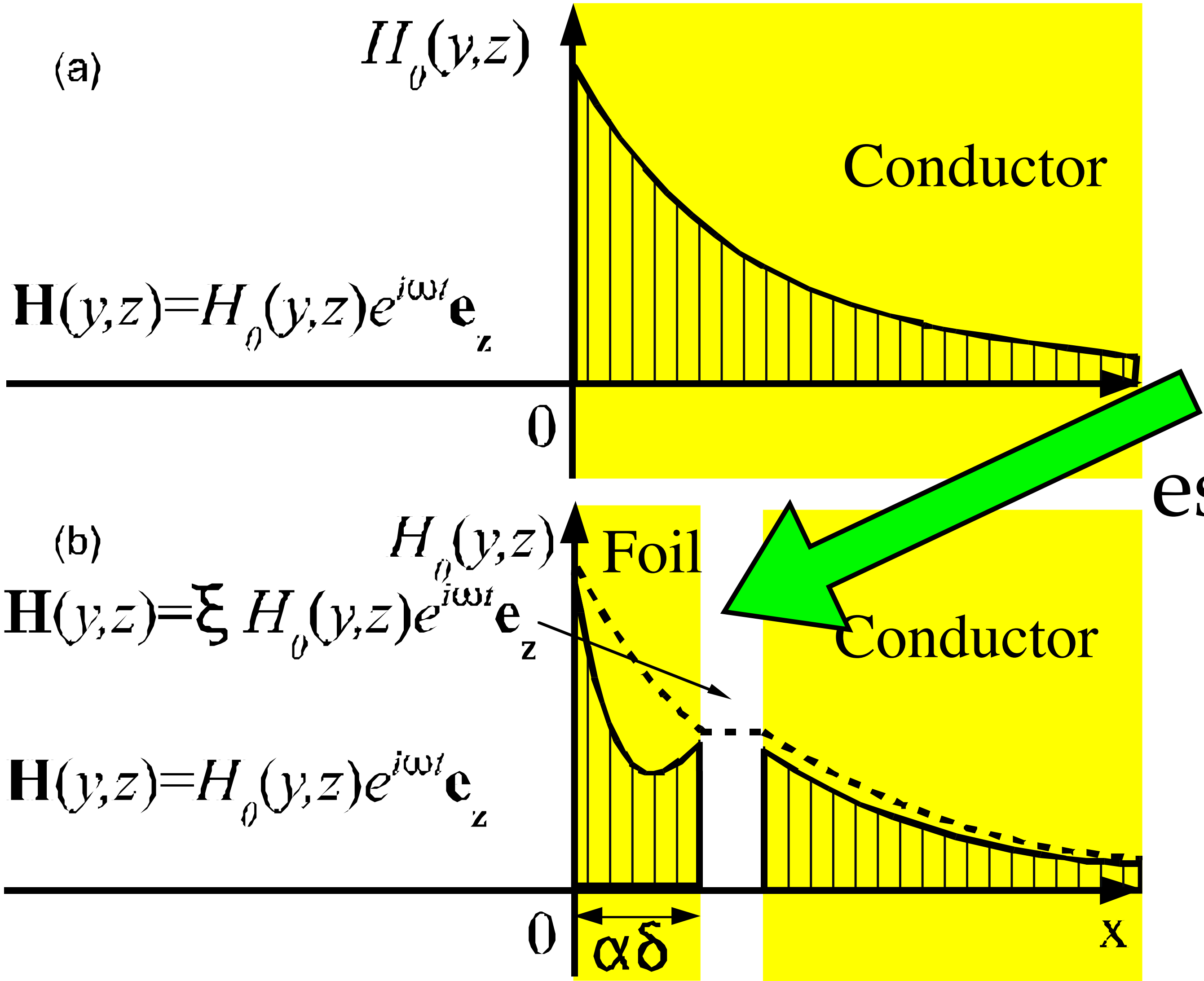
Agrees within a few %.

Only 1/4 area was covered.

corresponds to 1.4 times local effect

$\sqrt{n}$ (n=2)

Magnetic Field(current)distribution

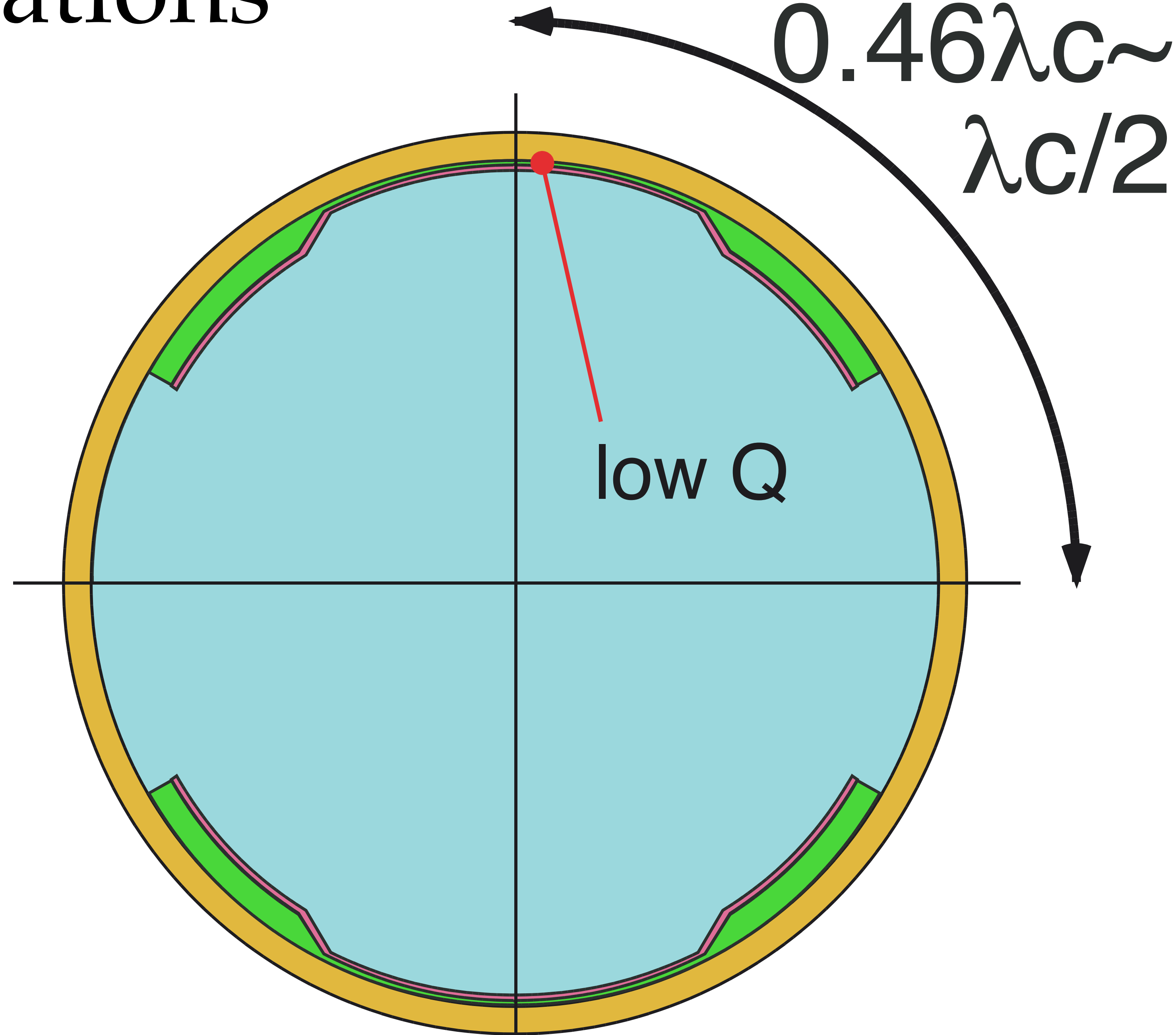


This gap is essential to bring magnetic flux.

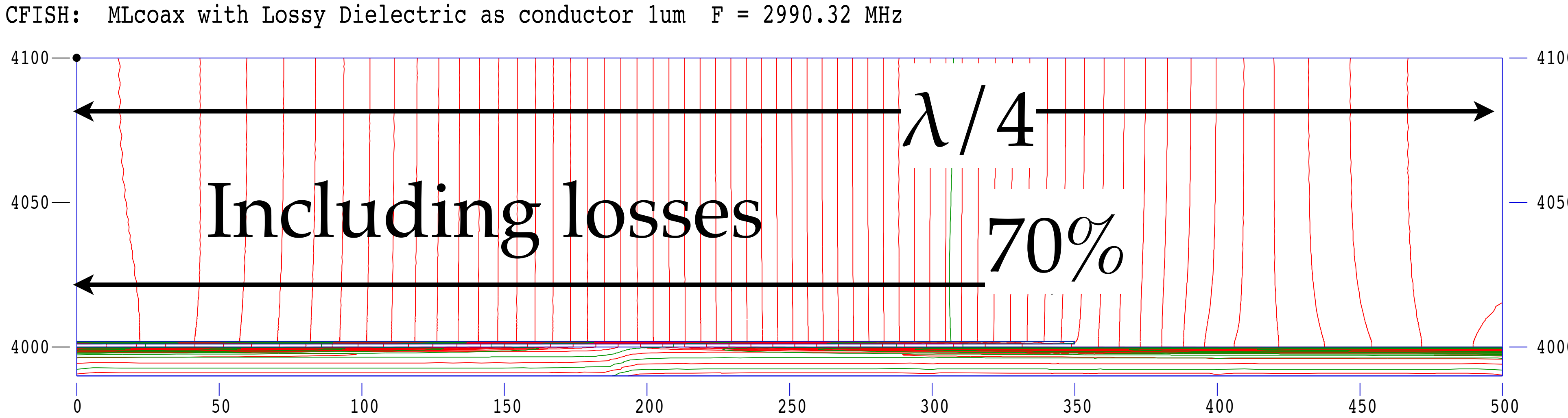
ISSUES

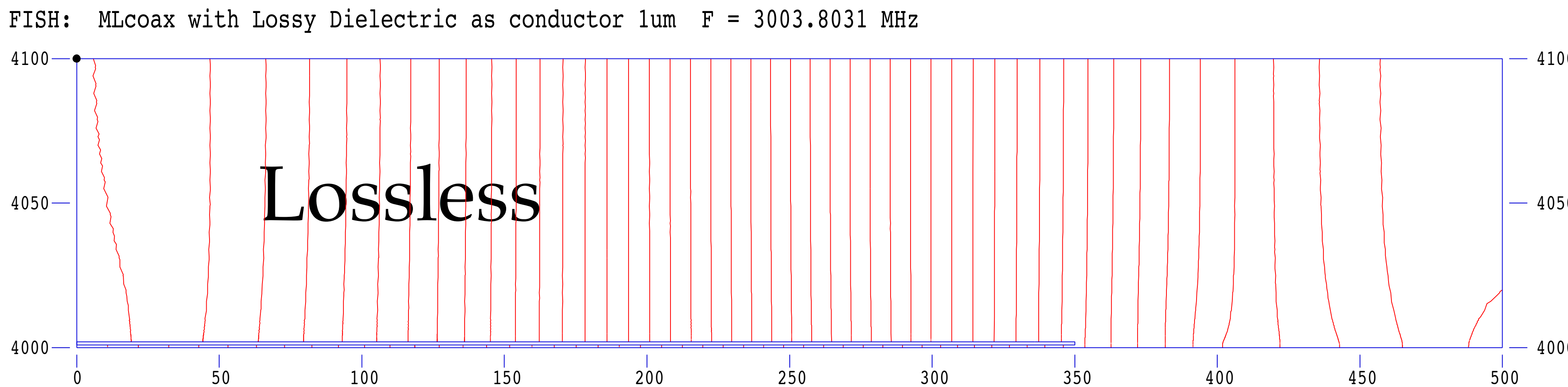
- When the insulating layers have comparable length to the wavelength, they form resonators whose resonances screw up the field distributions.
- The stepped gap structure may help it. But when the gap is too narrow, small stored energy leads to low Q.

Limitations

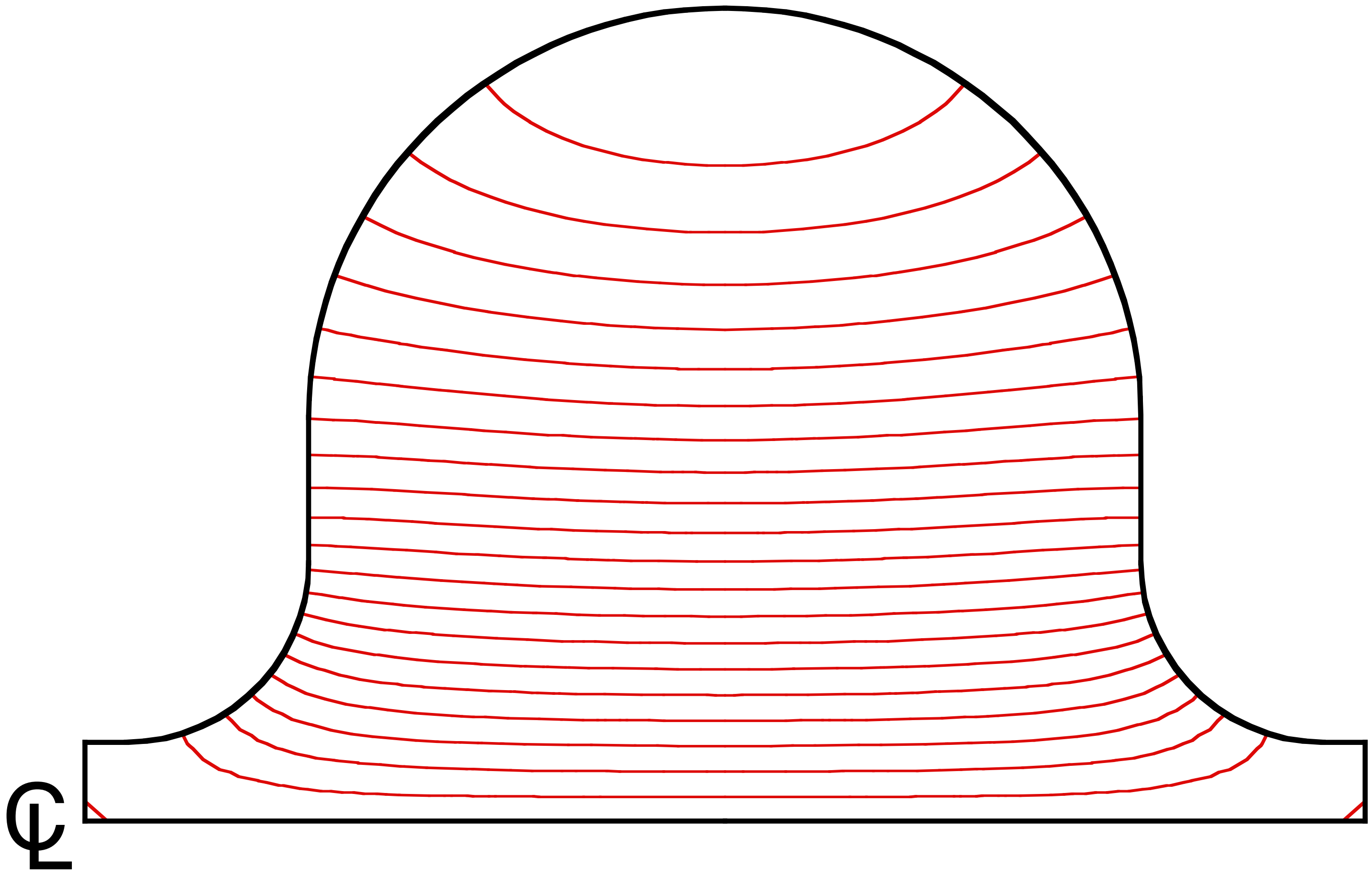


Coax Cavity Case



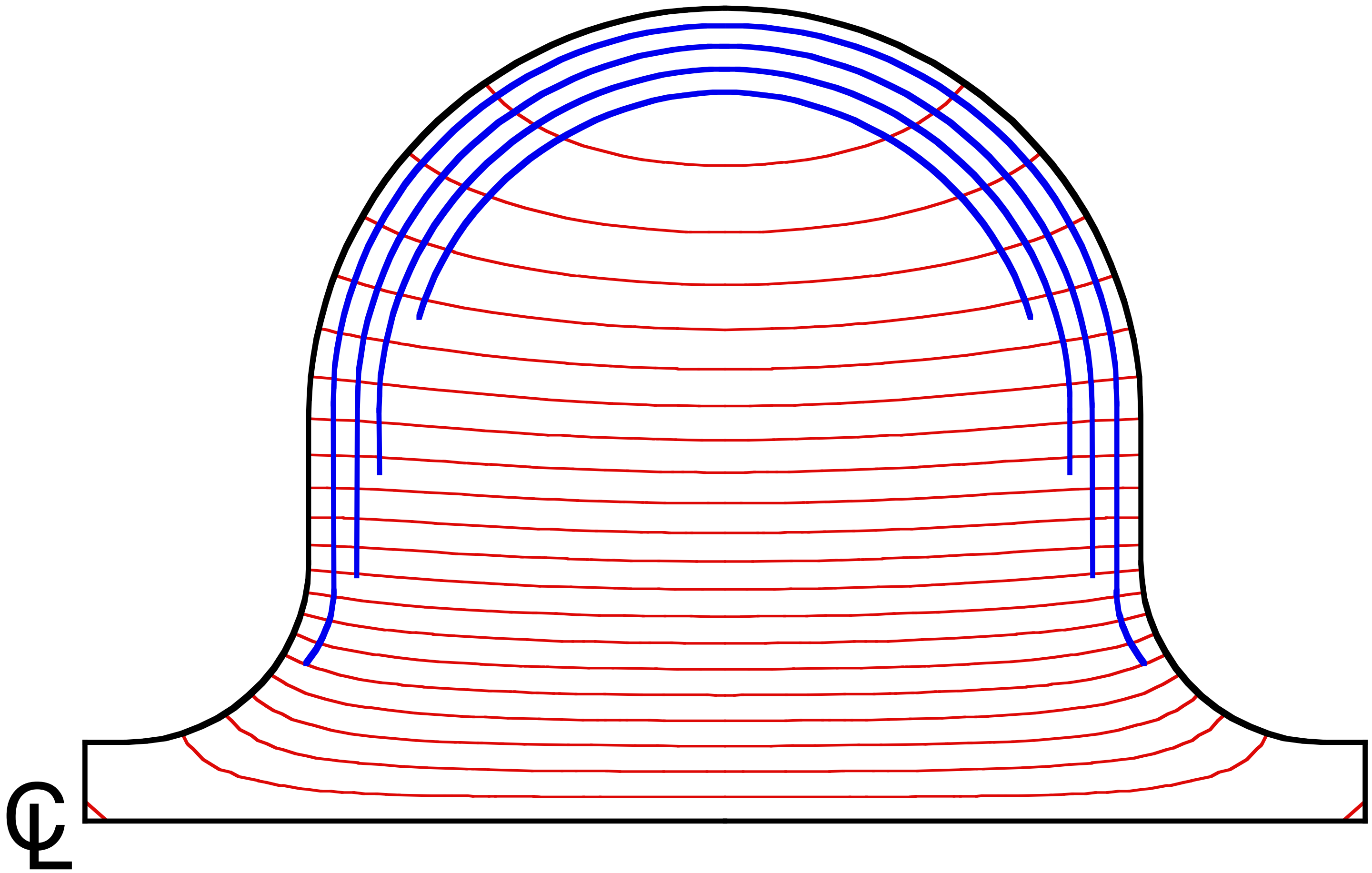


Possible Configuration

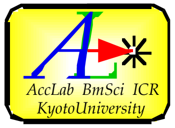


Multi-layer Coated Cavity

Possible Configuration



Multi-layer Coated Cavity



Summary for Normal Cond.

- The structure can improve Q on some structure.
- The insulating layers should have openings for magnetic flux to go through.
- An insulating layer between conducting layers forms resonator and the resonance may increase the magnetic field.
- Need more study...

Superconductor Case
— S-I-S multilayer coating —

Mainly by T.Kubo

Normal vs Super

	Nc	Sc
resistance	$Rs = \sqrt{\frac{\omega\mu_0}{2\sigma}} \quad \sim 10m\Omega$	$R_{BCS} = A\frac{\omega^2}{T}\exp(-\frac{\Delta_0}{k_B T}) \quad \sim 10n\Omega$
depth	$d = \sqrt{\frac{2}{\omega\mu_0\sigma}} \quad \sim 2\mu m$	$\lambda_L = \sqrt{\frac{\hat{m}}{\mu_0 n_s \hat{e}^2}} \quad \sim 0.1\mu m$
E Limit	Break down	Cold Emission?
H Limit	Heat (melt?)	Hc

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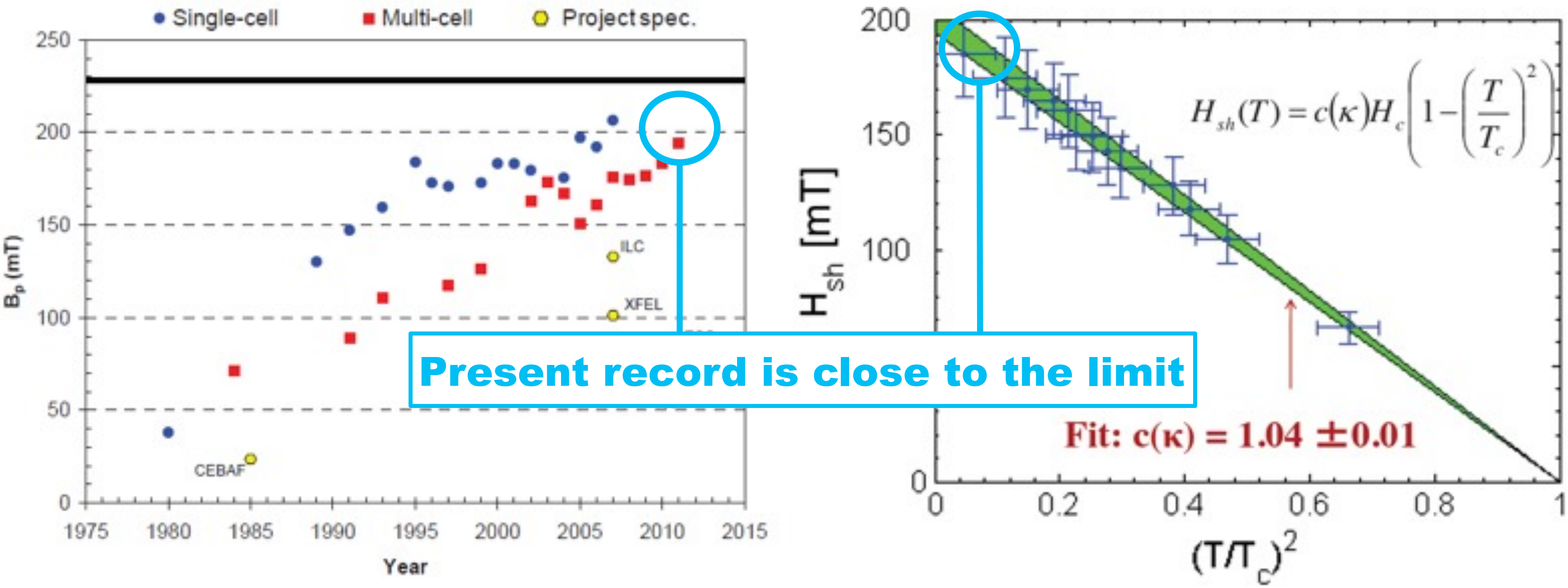
Introduction

- Technologies to fabricate the SRF cavities made of Nb have been advanced. ⇨~45MV/m (~190mT) with multi-cell cavity.
- Further high gradients, however, would not be expected because their corresponding surface magnetic fields are close to the empirical limit ~200mT of Nb.



G. Ciovati, IPAC’13

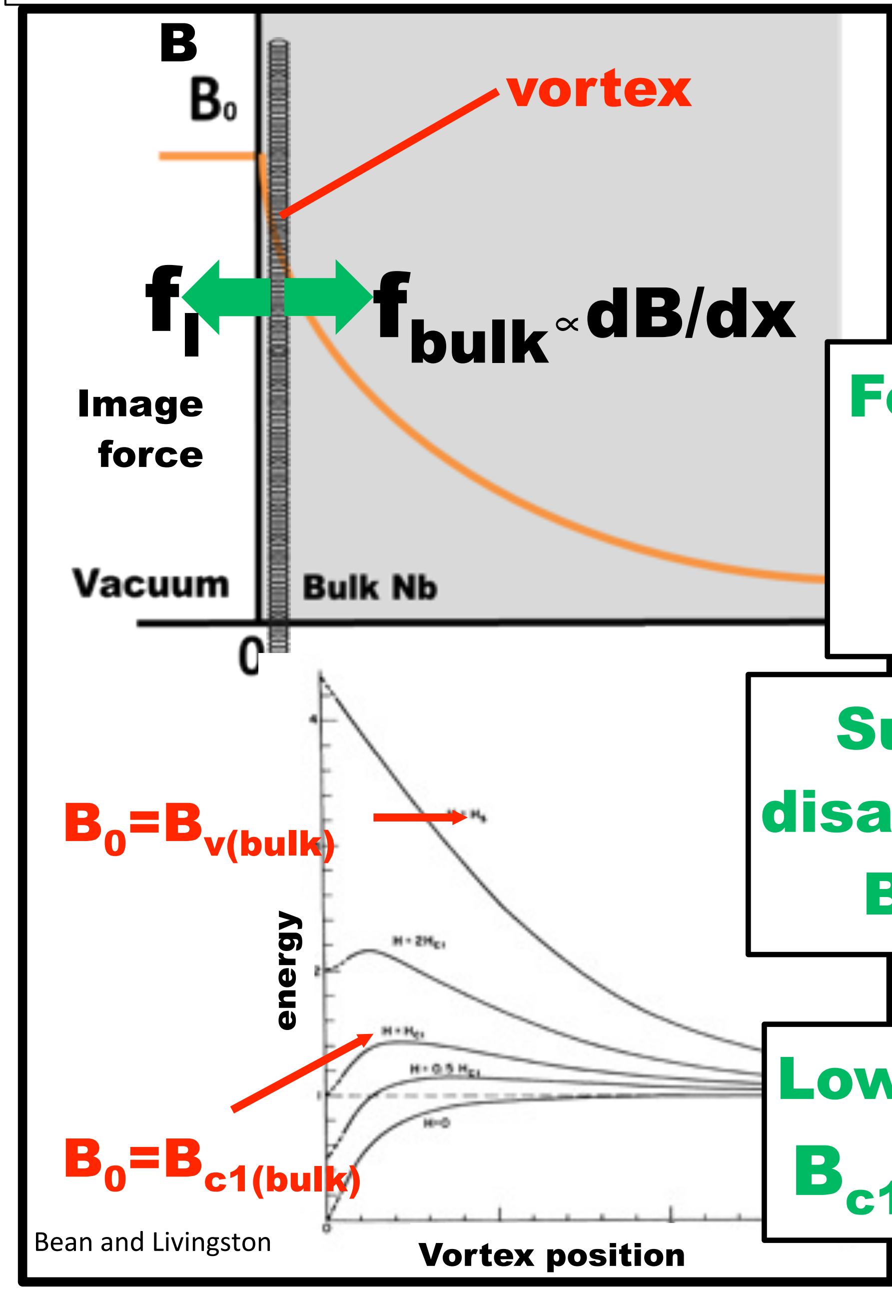
N. Valles, SRF2011

Introduction

- **Technologies to fabricate the SRF cavities made of Nb have been advanced. ⇨~45MV/m (~190mT) with multi-cell cavity.**
- **Further high gradients, however, would not be expected because their corresponding surface magnetic fields are close to the empirical limit ~200mT of Nb.**
- **A breakthrough is needed to push up the gradient limit of SRF cavity to >> 45MV/m.**

Gurevich's idea:
multilayer coating

Basic idea: bulk VS thin film



Force acting on vortex

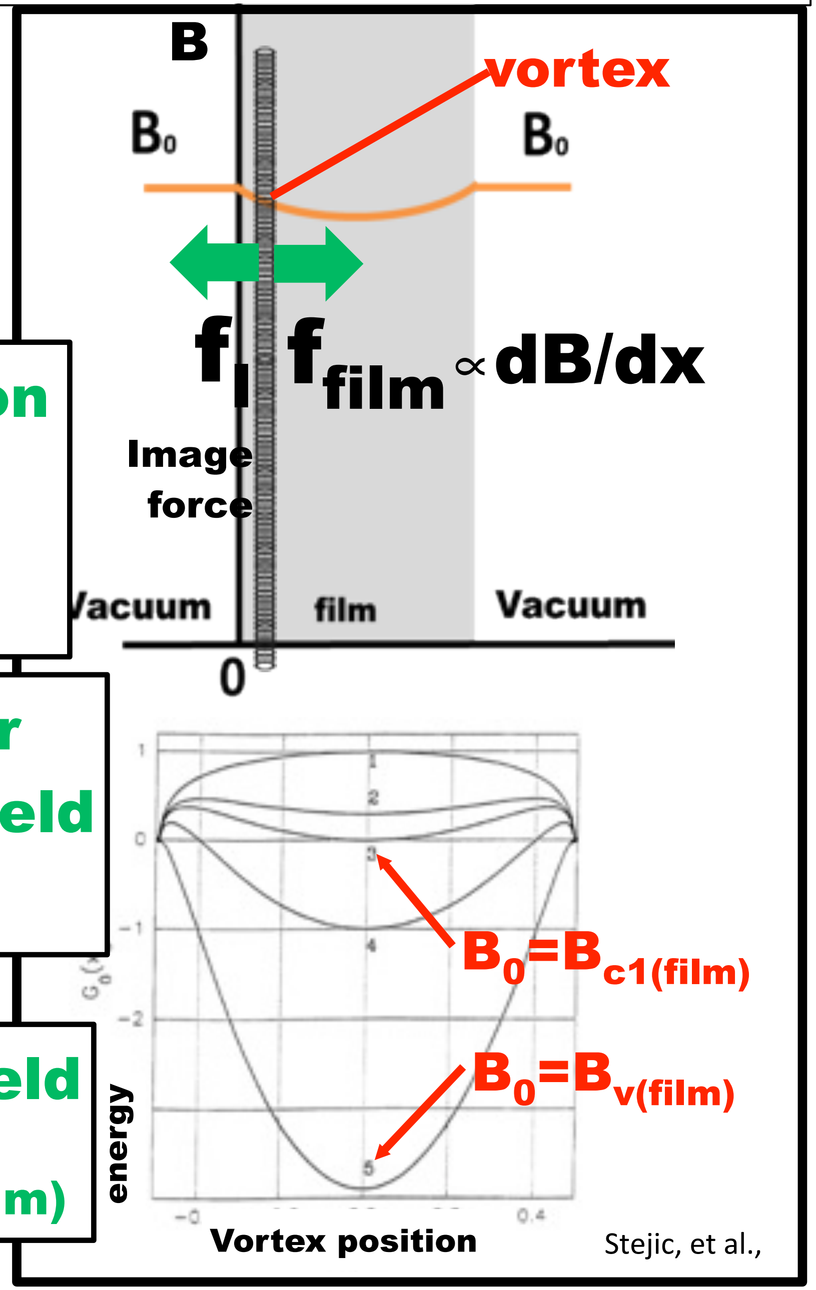
$f_{bulk} > f_{film}$

Surface-barrier disappearance-field

$B_{v(bulk)} < B_{v(film)}$

Lower critical field

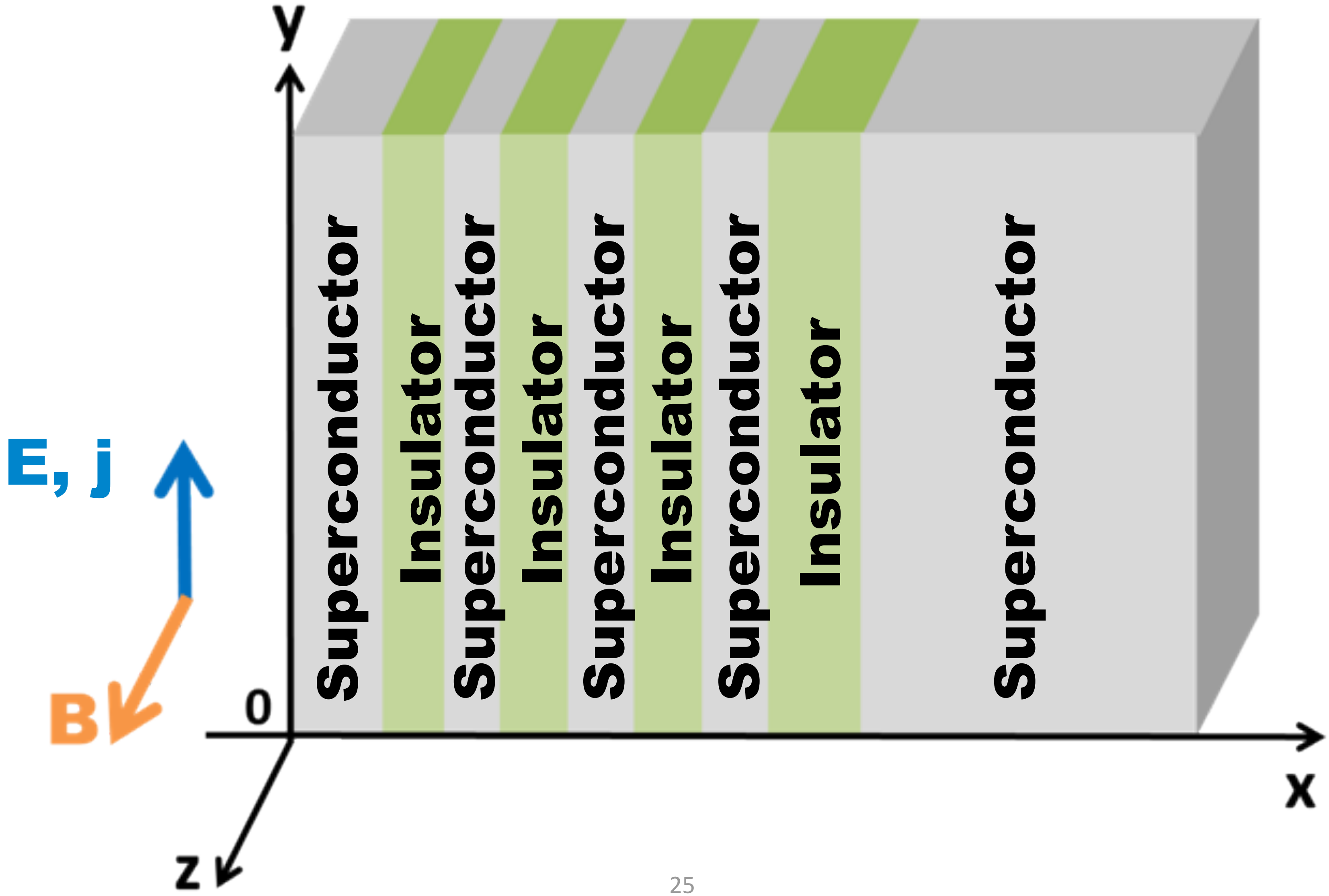
$B_{c1(bulk)} < B_{c1(film)}$



SC thin films are tolerant of vortex penetration than bulk SC.

Gurevich's Multilayer coating

- **Each superconductor layer (S layer) can withstand higher field than Nb.**
- **Layers protect bulk Nb.**

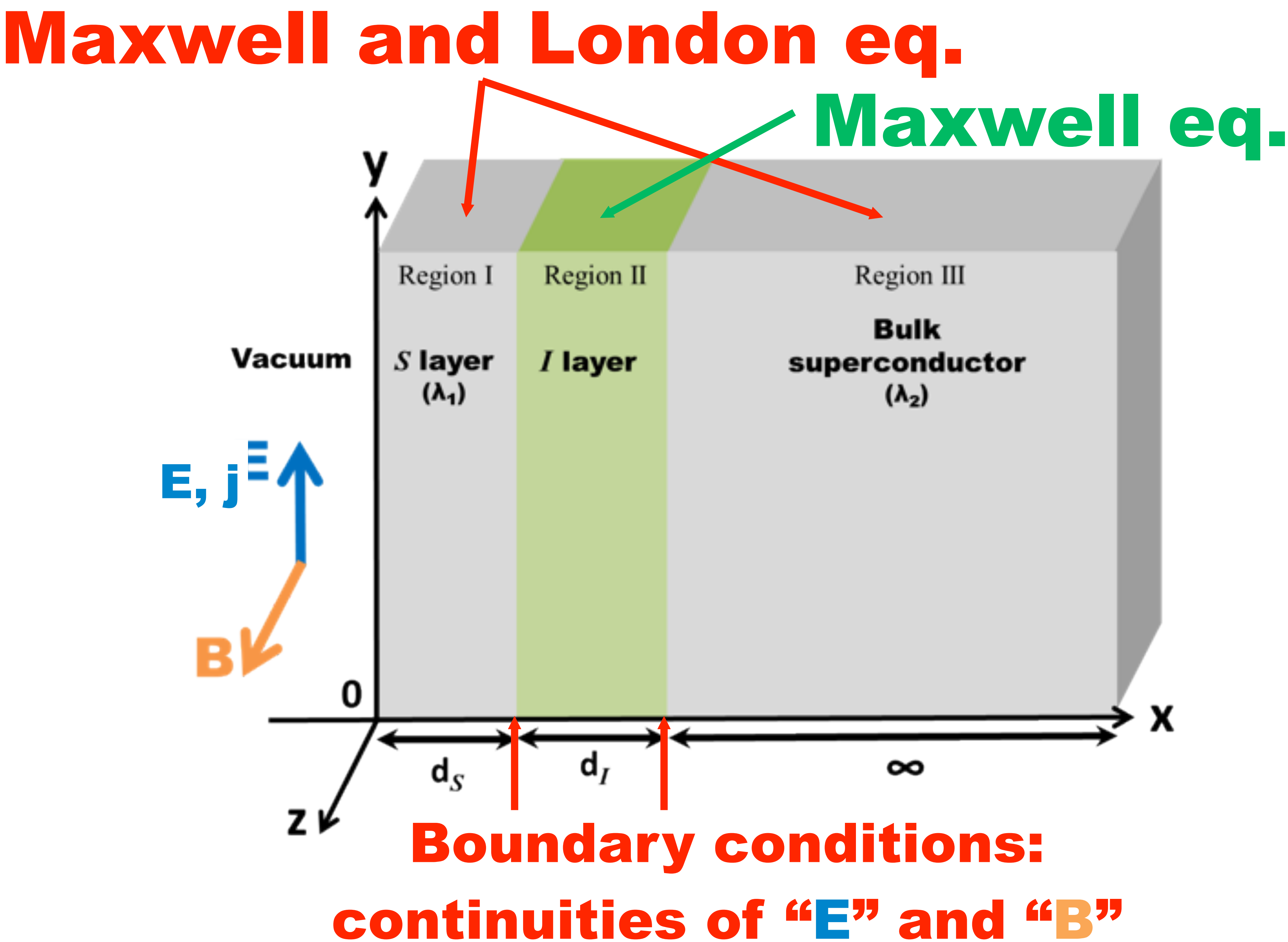


Lack of theoretical understanding

- **Experiments for the confirmation of this idea are in progress all over the world, and now it is a hot topic in the community of SRF cavity.**
- **In the present situation, however, little is known from a aspect of theory. ⇒ No guide for experiments**

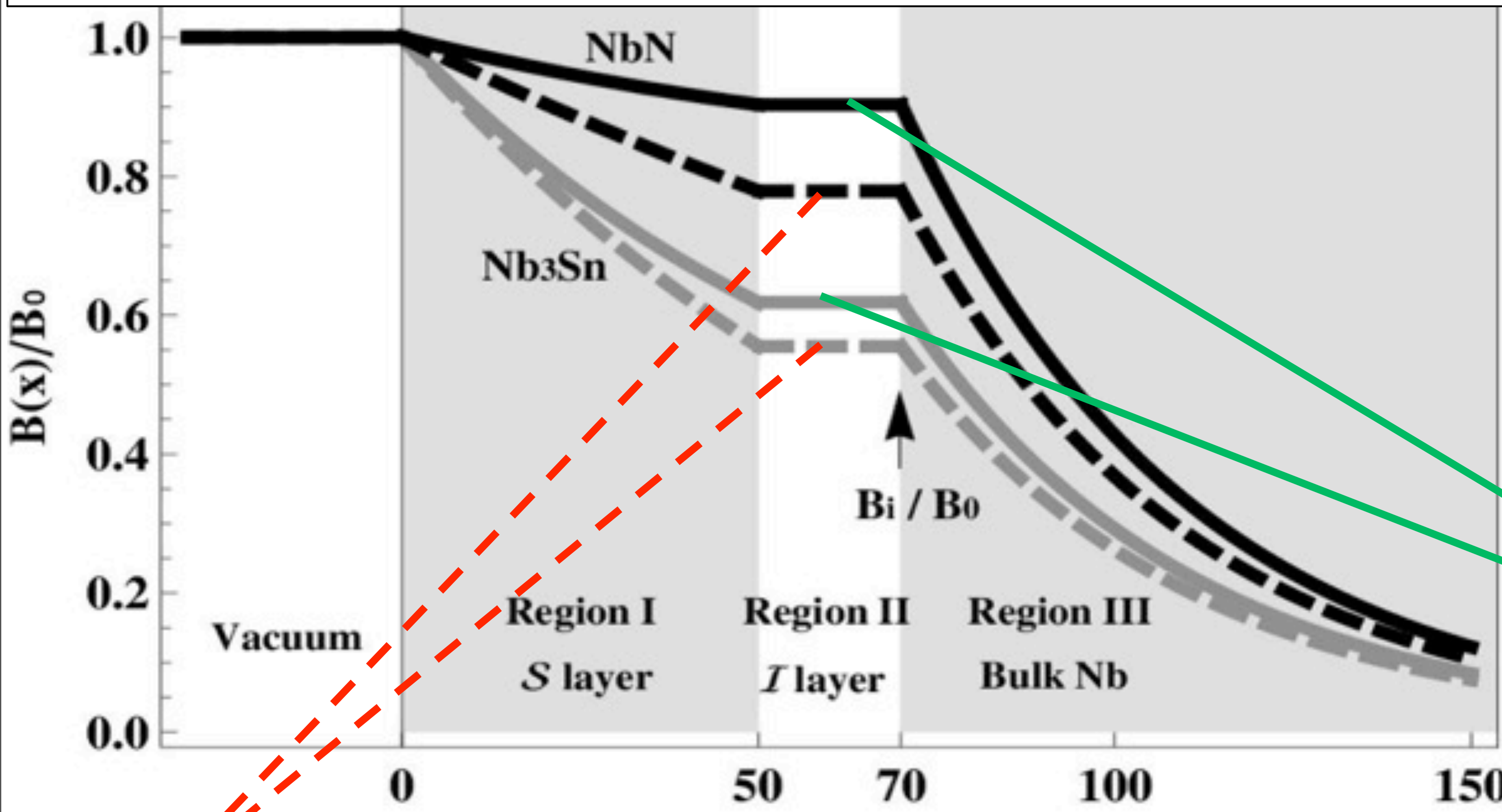
- **How does the magnetic field attenuate in a multilayer coating? Simple exponentially decay?**
- **Thicknesses of S layers? Thinner is better?**
- **Thicknesses of insulator layers (I layers) are arbitrary?**
- **Field limits of multilayer coating?**
- **...**

Re-evaluate the simplest model (S-I-S)



T. Kubo, Y. Iwashita, and T. Saeki, arXiv:1306.4823 [physics.acc-ph], in *proceedings of IPAC'13*.

Magnetic field distribution (cont.)



Rigorous calculations

Naïve estimates: Exponential decay

$$\cancel{B(x) = B_0 e^{-\frac{x}{\lambda}}}$$

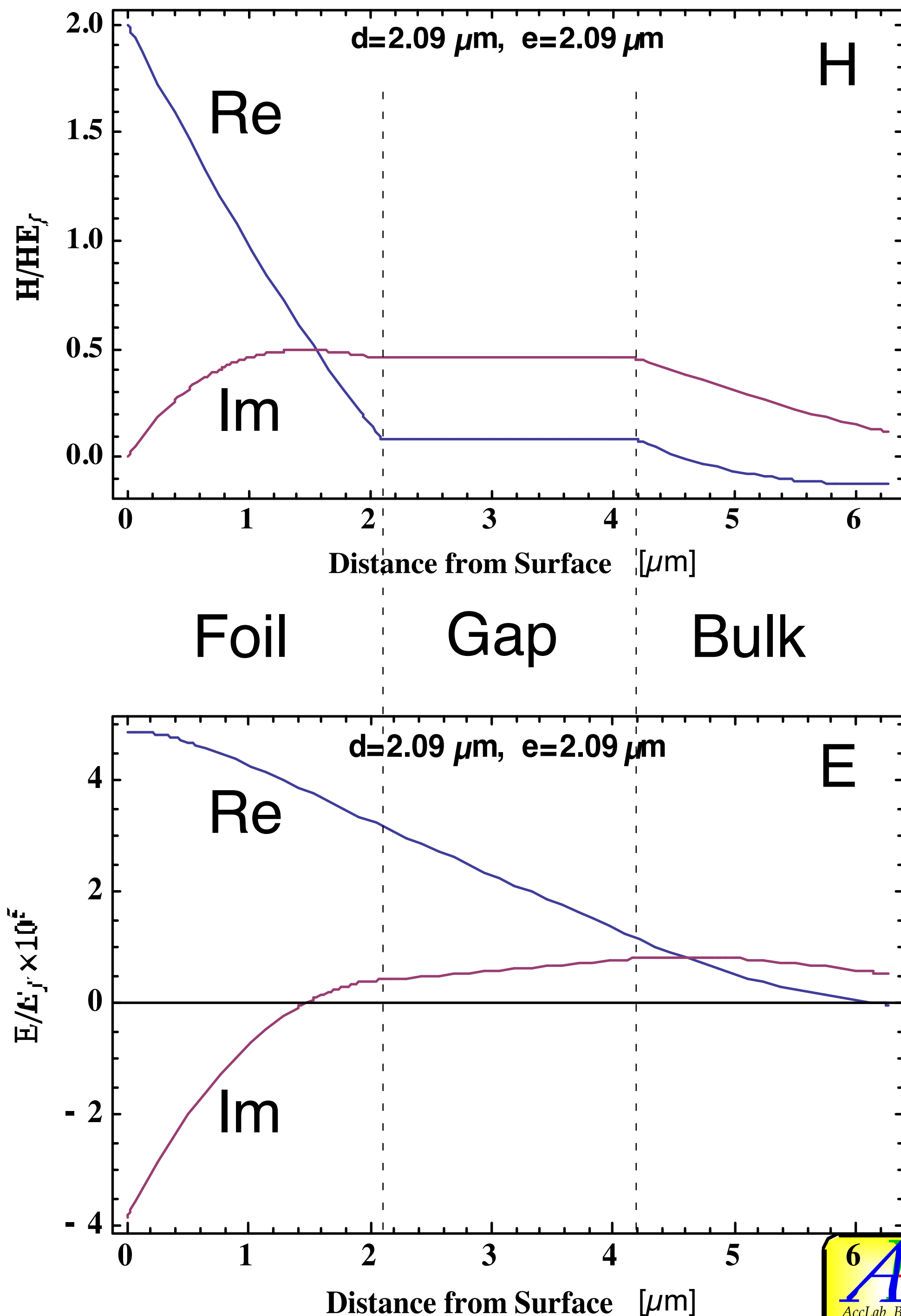
$$B_{\mathrm{I}} = B_0 \frac{\lambda_1 \cosh \frac{d_{\mathcal{S}}-x}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}}) \sinh \frac{d_{\mathcal{S}}-x}{\lambda_1}}{\lambda_1 \cosh \frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}}) \sinh \frac{d_{\mathcal{S}}}{\lambda_1}},$$

$$B_{\mathrm{II}} = B_0 \frac{\lambda_1}{\lambda_1 \cosh \frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}}) \sinh \frac{d_{\mathcal{S}}}{\lambda_1}},$$

$$B_{\mathrm{III}} = B_0 \frac{\lambda_1 e^{-\frac{x - d_{\mathcal{S}} - d_{\mathcal{I}}}{\lambda_2}}}{\lambda_1 \cosh \frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}}) \sinh \frac{d_{\mathcal{S}}}{\lambda_1}},$$

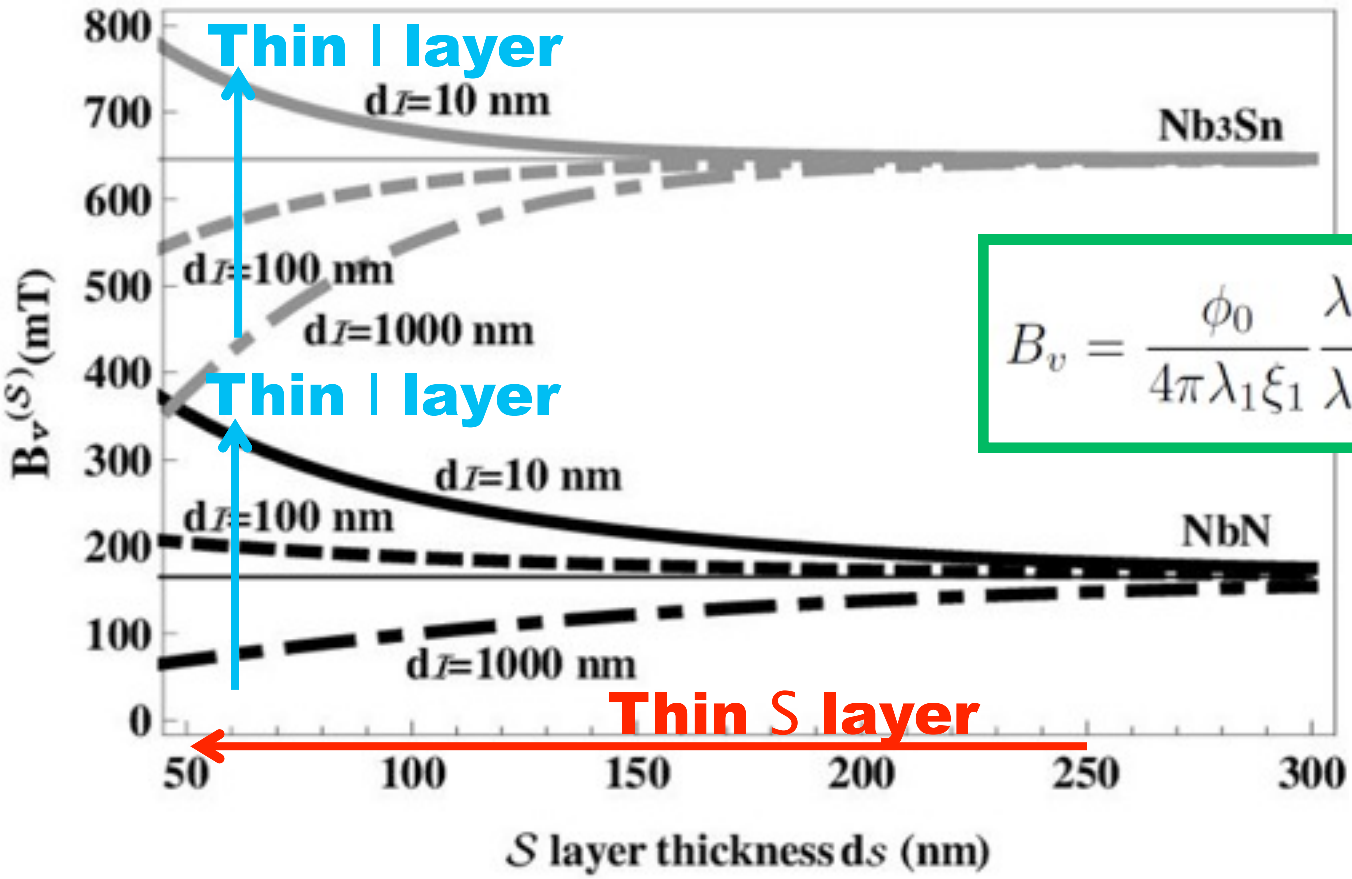
Gap Distance Dependence

Magnetic field is very flat in the insulating layer, while the electric field change is not negligible.



Impedance difference

Surface-barrier disappearance-field



Rigorous calculations

$$B_v = \frac{\phi_0}{4\pi\lambda_1\xi_1} \frac{\lambda_1 \cosh\frac{d_S}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}})\sinh\frac{d_S}{\lambda_1}}{\lambda_1 \sinh\frac{d_S}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}})\cosh\frac{d_S}{\lambda_1}},$$

- **Thin S layer with thin I layer on a bulk is tolerant of vortex penetration.**
- **It is remarkable that I layer thickness d_I also affect B_v.**

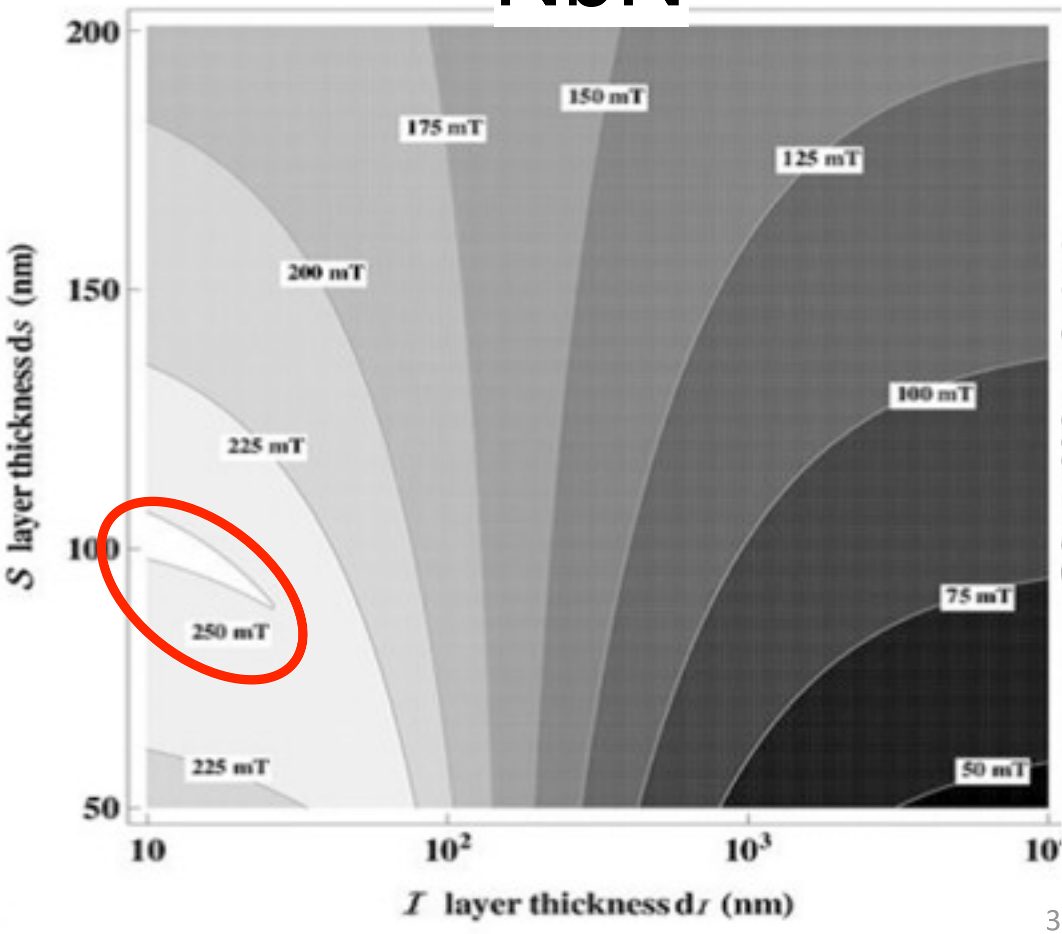
T. Kubo, Y. Iwashita, and T. Saeki, arXiv:1307.0583 [physics.acc-ph], in *proceedings of SRF 2013*.

Parameters for enhancing a gradient limit

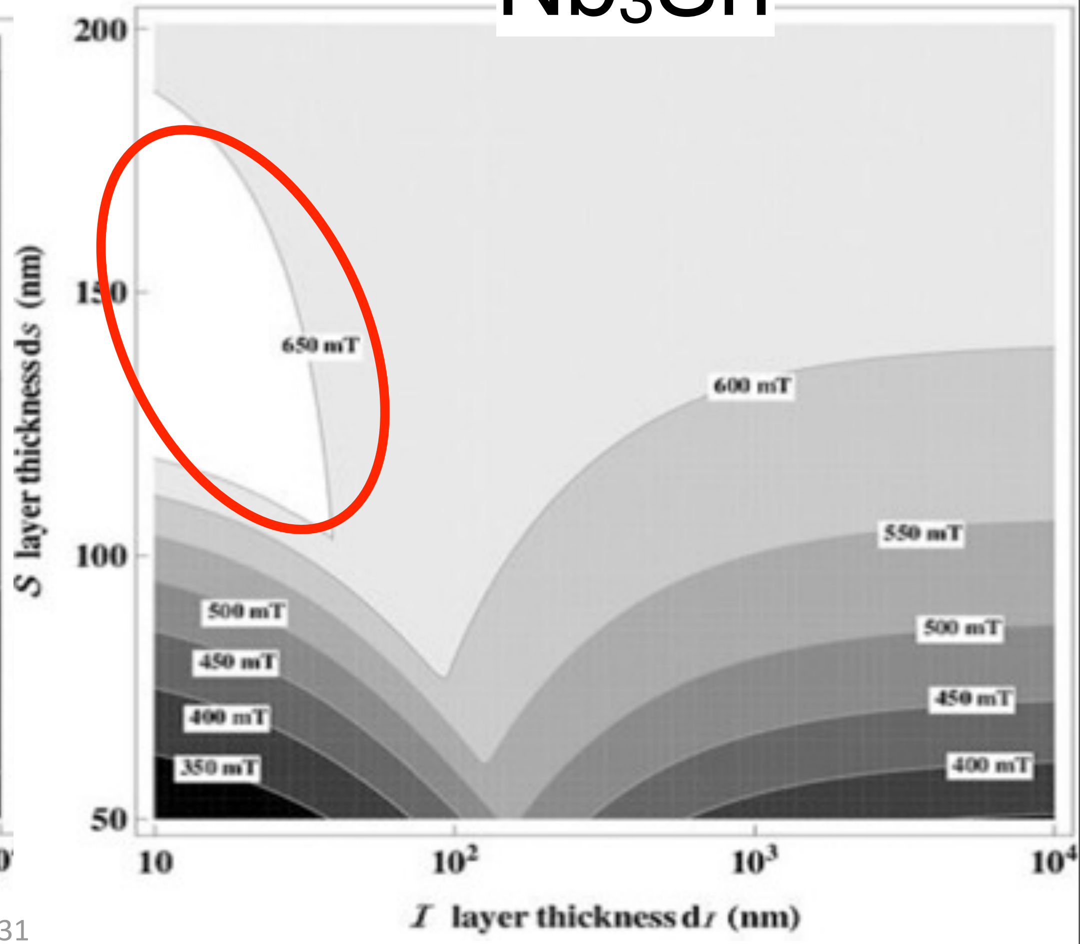
- **A thin S layer pushes up B_v , but it can not protect the bulk SC from an applied field if d_S << London depth.**
- **Not only B_v , but also the shielded magnetic field on the bulk SC must be considered simultaneously.**

$$B_{\max} = \begin{cases} B_v^{(\mathcal{S})} & (\alpha B_v^{(\mathcal{S})} < B_v^{(\mathrm{bulk})}) \\ \alpha^{-1} \times B_v^{(\mathrm{bulk})} & (\alpha B_v^{(\mathcal{S})} \geq B_v^{(\mathrm{bulk})}) \end{cases}$$

NbN



Nb3Sn



Summary

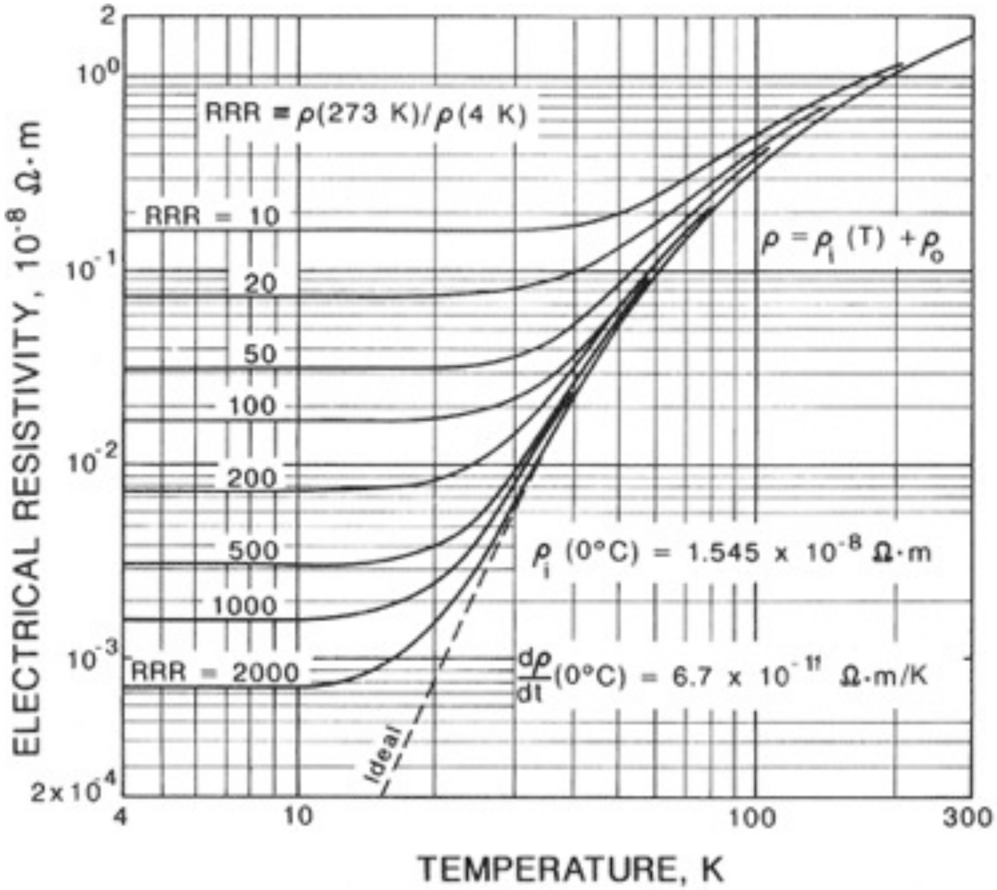
- **Multilayer coating** has a possibility of enhancing a gradient limit of the SRF cavity.
- We studied a multilayered structure with a single superconductor layer and a single insulator layer formed on a bulk superconductor.
- **General formulae for the vortex penetration field of the S layer and the magnetic field distribution** are derived.
- Using the formulae, **a combination of the thicknesses of S and I layers to enhance the field limits** can be found for any given materials.

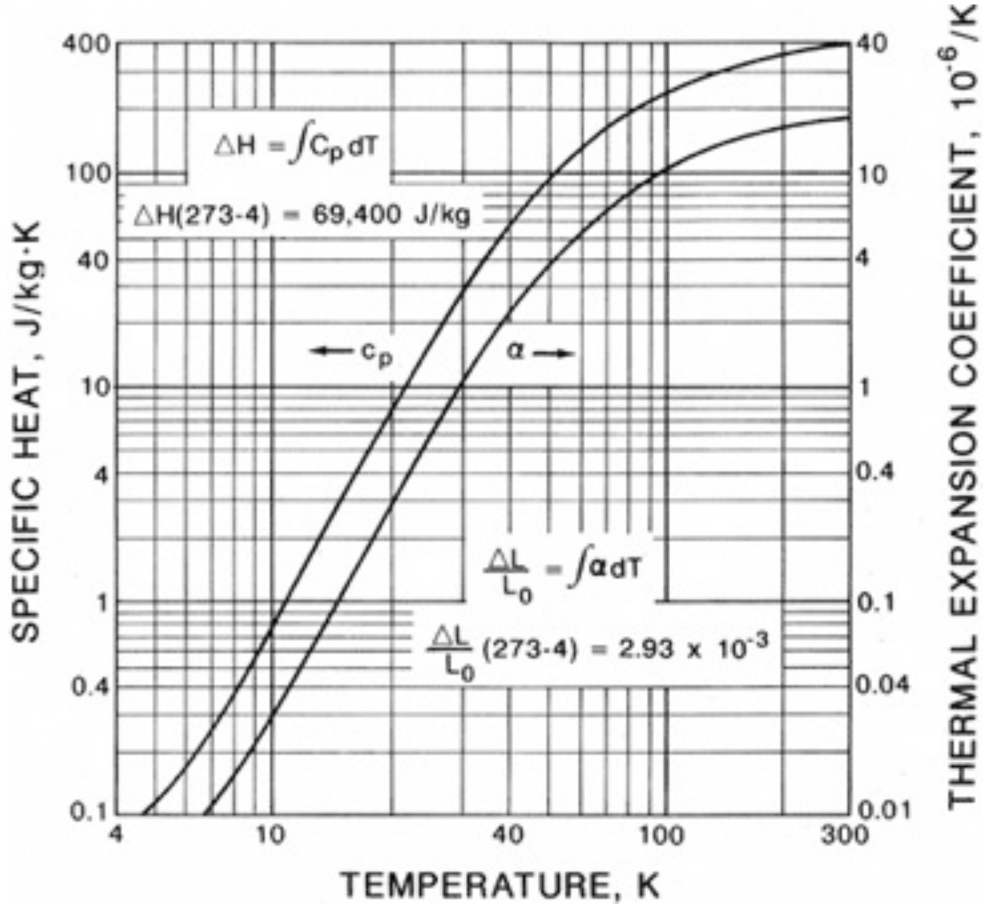
Linac 2014

- Nomination of Invited Talk is open until end of this month.
- Please let me know if any.

Backups

Electrical-Resistivity of Cu

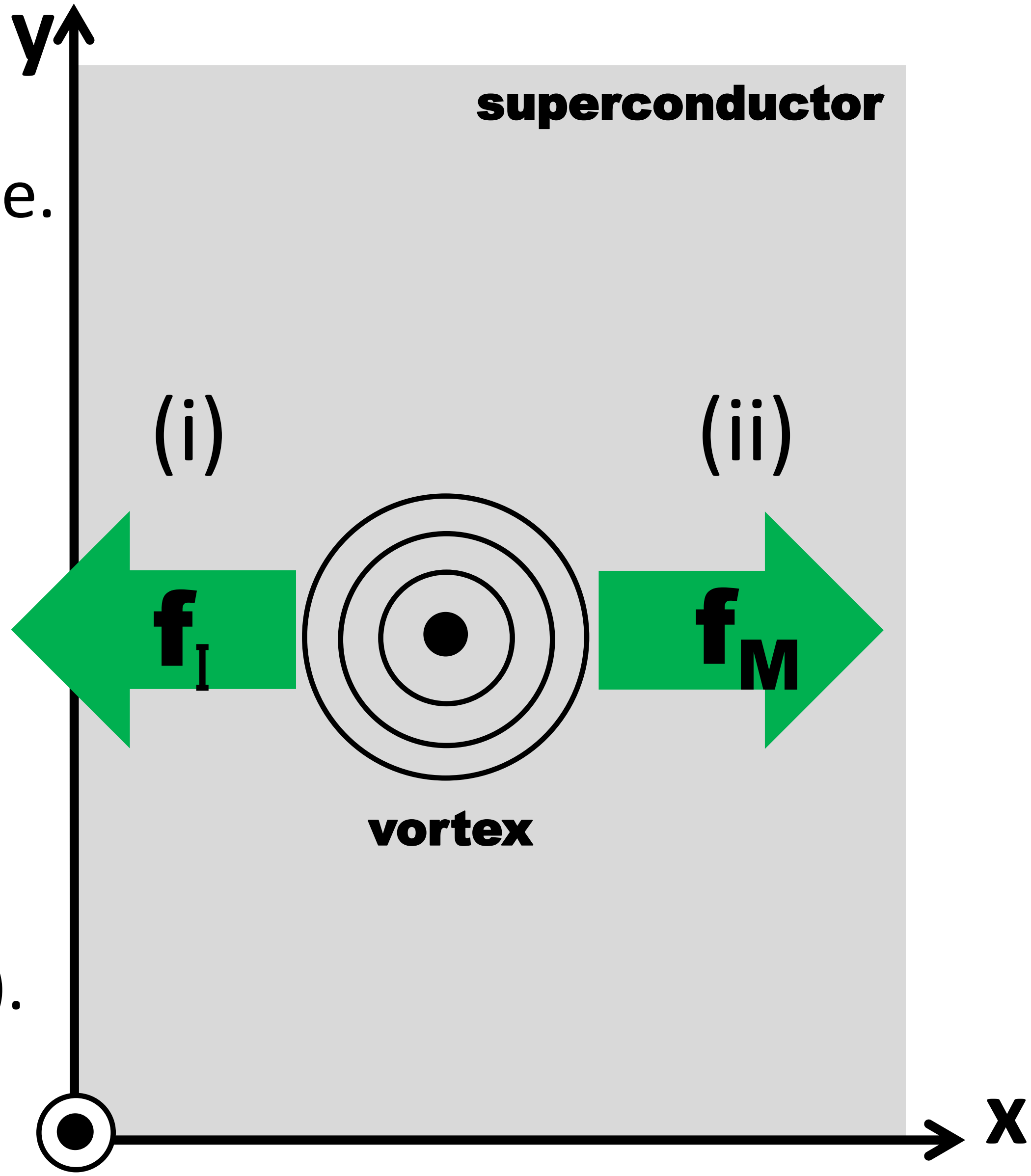




Bean-Livingston barrier

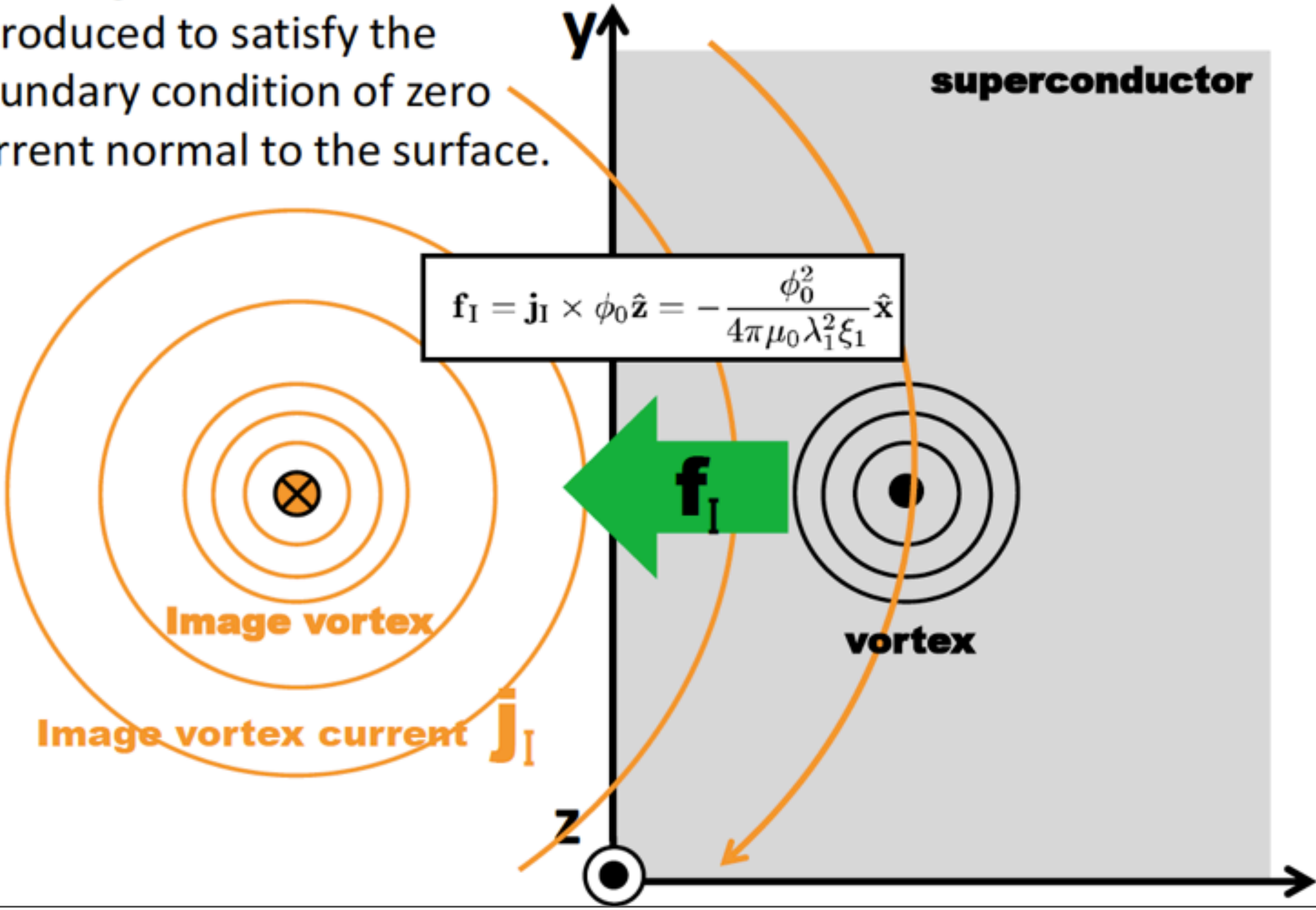
Vortex near the surface receives two different force.
(i) **Image vortex pushes the vortex outside**
and
(ii) **External field pushes the vortex inside.**

Vortex can enter the superconductor when the barrier disappears ($\mathbf{f}_I = \mathbf{f}_M$).



(i) Force from the image vortex

The image vortex is introduced to satisfy the boundary condition of zero current normal to the surface.



$$\mathbf{f}_{\mathrm{I}} = \mathbf{j}_{\mathrm{I}} \times \phi_0 \hat{\mathbf{z}} = -\frac{\phi_0^2}{4\pi\mu_0\lambda_1^2\xi_1}\hat{\mathbf{x}}$$

(ii) Force from the external field

When a magnetic field is applied to a superconductor, Meissner screening currents are induced, which pushes the vortex inside.