Transverse Emittance Exchange for Smith-Purcell Backward Wave Oscillator Free Electron Laser (BWO-FEL)

2013.11.29 @ OIST

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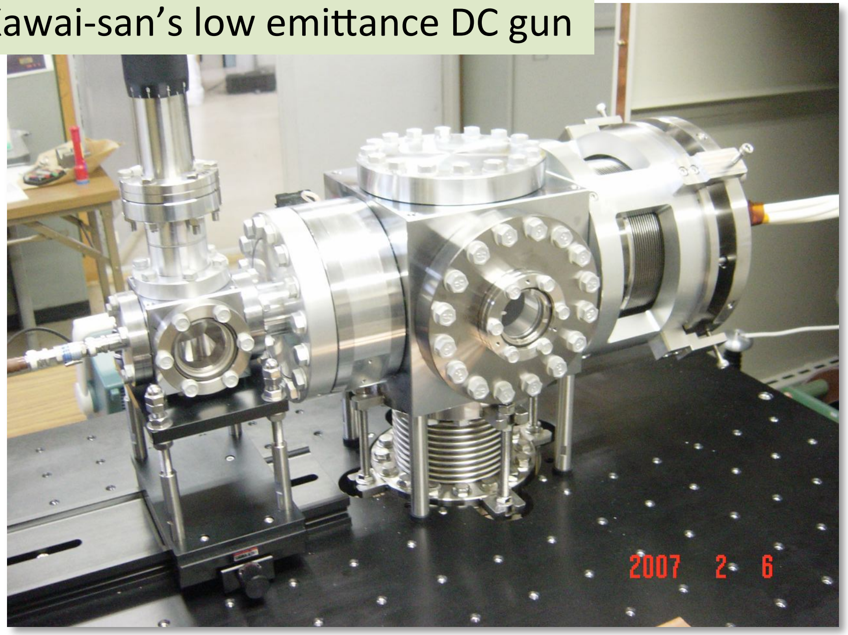
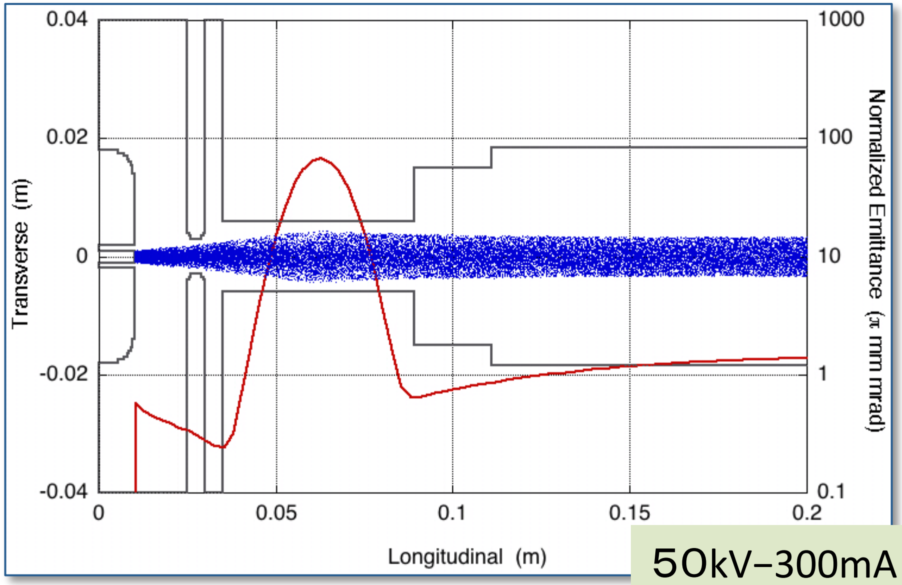
Smith-Purcell Backward Wave Oscillator FEL

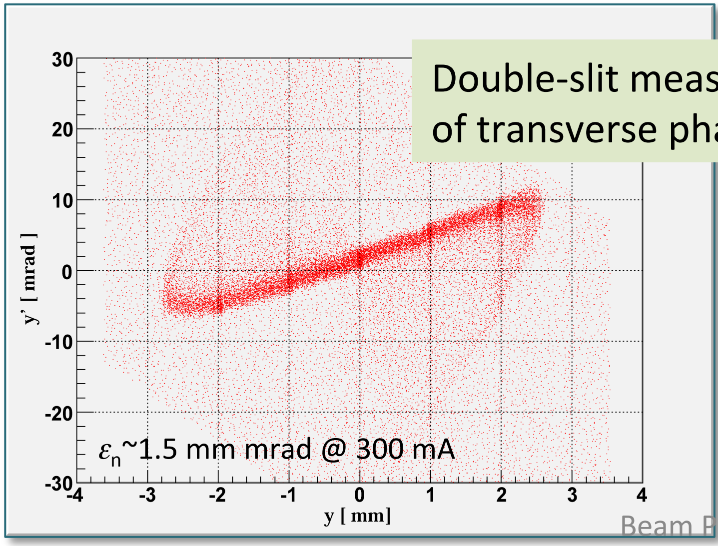
Kawai-san's low emittance DC gun





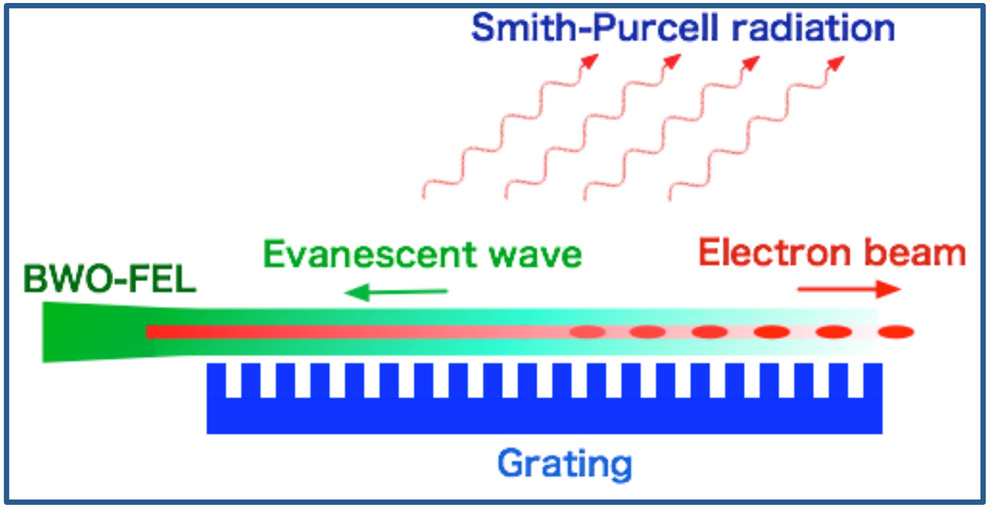
50kV–300mA

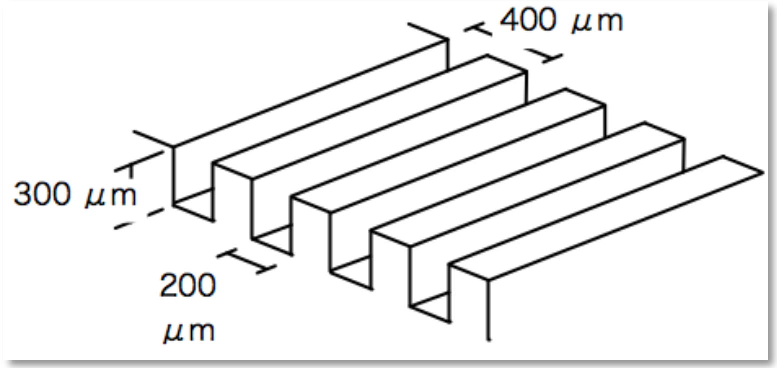
Double-slit measurement of transverse phase space

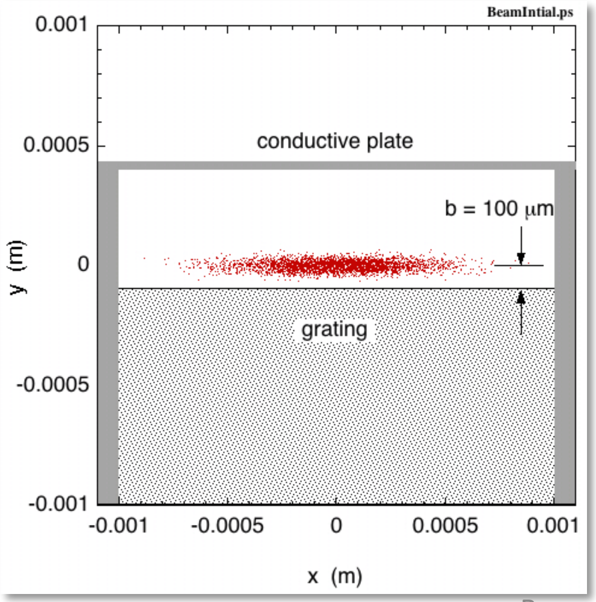


(the late) Prof. Masayuki Kawai

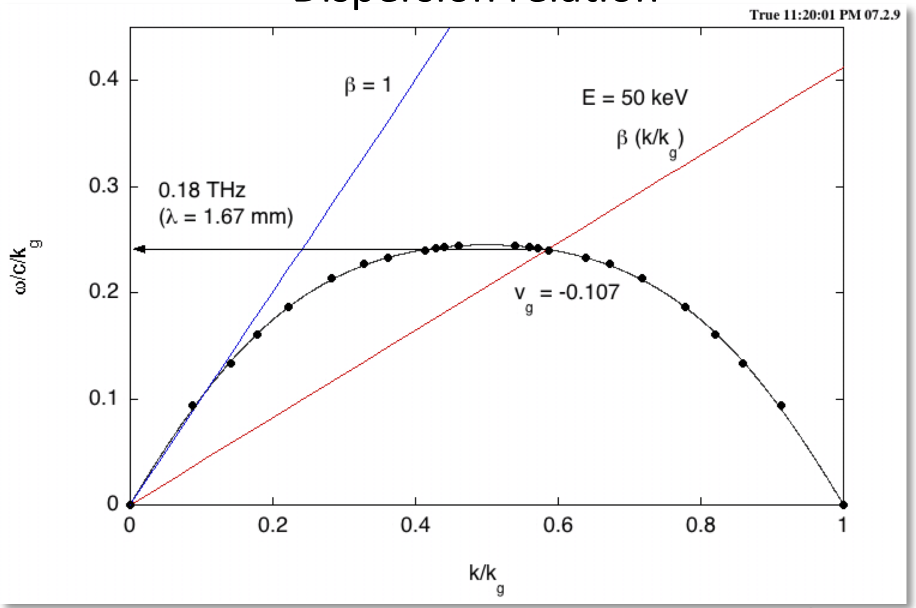
Smith-Purcell Backward Wave Oscillator FEL







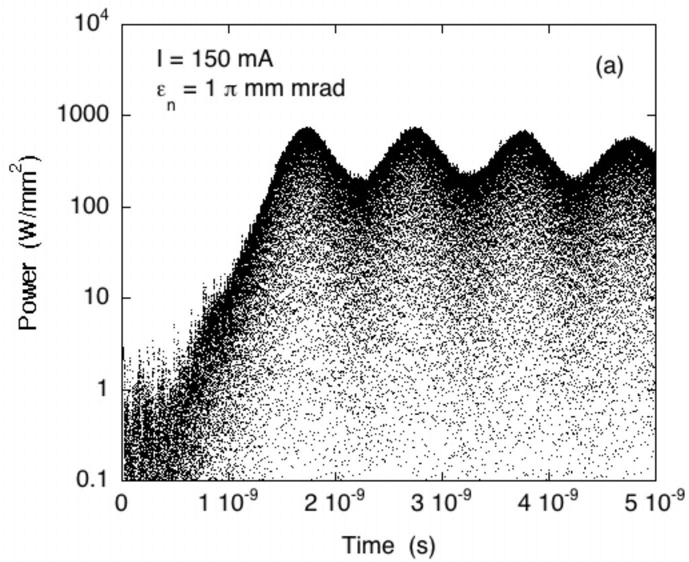
Dispersion relation



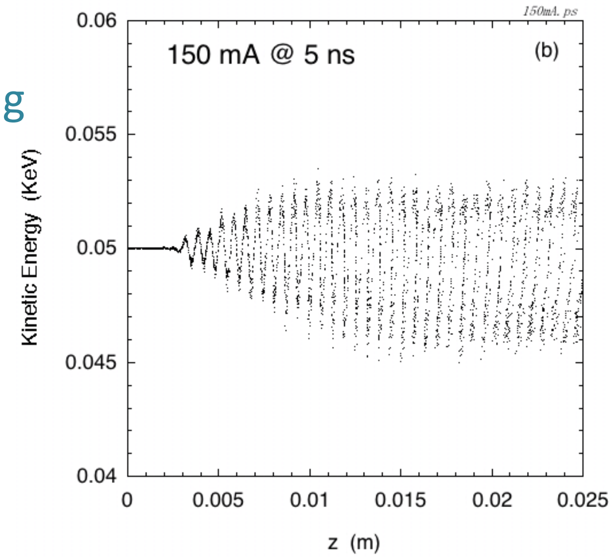
Smith-Purcell Backward Wave Oscillator FEL

3-D FDTD simulation

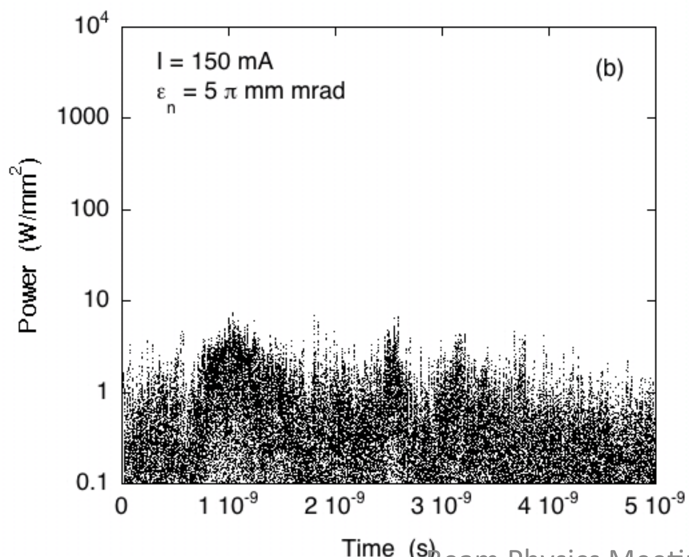
Lasing



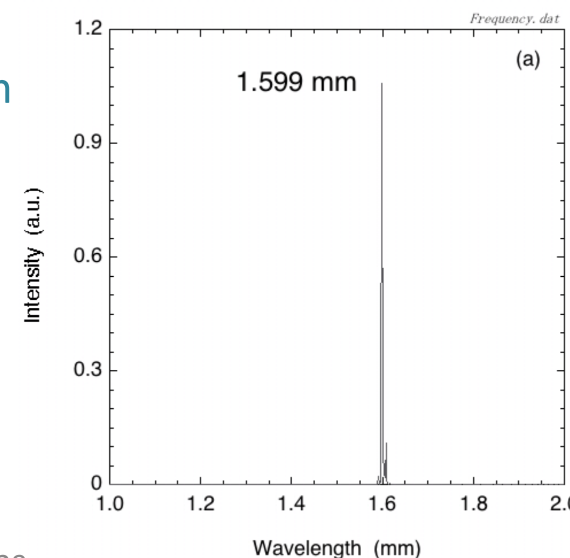
Micro-bunching



Just BW radiation



Spectrum



エミッタンス

- エミッタンスの保存は、運動がなんらかのハミルトニアンで記述される系(保存系)の、局所的な位相空間で常に成り立つ。
- 電磁場によるビームの収束、加速等はこのような系である。電磁場は一定である必要はなく、場所・時間の関数であってかまわない。
- エミッタンスの保存は各自由度毎に成り立つ。この点、Liuvilleの定理(全位相空間の体積の保存)よりも強い。
- 実は保存系では、より強い保存則(正準交換関係の保存：symlecticity)が成り立っており、エミッタンスの保存はその帰結である。

– 生出勝宣講義ノート@東大2007 –

Emittance Exchange: General

Particle motion which can be described by Hamiltonian is "SYMPLECTIC".

General transfer matrix in the real space using Twiss parameters

$$M = \left[\begin{pmatrix} 1 & 0 \\ \alpha_2 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{\beta_2} & 0 \\ 0 & \sqrt{\beta_2} \end{pmatrix}\right]^{-1}\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\left[\begin{pmatrix} 1 & 0 \\ \alpha_1 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{\beta_1} & 0 \\ 0 & \sqrt{\beta_1} \end{pmatrix}\right]$$

Transfer variables in Hamiltonian system to those in the real space at the exit

Rotation in the phase space

Transfer physical variables to those in Hamiltonian system at the entrance

$$M = \begin{pmatrix} \sqrt{\dfrac{\beta_2}{\beta_1}}(\cos\phi + \alpha_1\sin\phi) & \sqrt{\beta_1\beta_2}\sin\phi \\ -\dfrac{(1+\alpha_1\alpha_2)\sin\phi + (\alpha_2 - \alpha_1)\cos\phi}{\sqrt{\beta_1\beta_2}} & \sqrt{\dfrac{\beta_1}{\beta_2}}(\cos\phi - \alpha_2\sin\phi) \end{pmatrix}$$

Emittance Exchange: General

Emittance in many particle system
(u and p_u are canonical conjugate in Hamiltonian system)

$$\varepsilon = \sqrt{\langle u^2 \rangle \langle p_u{}^2 \rangle - \langle u p_u \rangle^2}$$

For n-th elliptical phase space $\vec{h} = {}^t(x, x', y, y', t, \delta, \ldots\ldots)$

when the beam matrix (σ-matrix) is ${}^t\vec{h}\,\sigma^{-1}\vec{h} = 1$

the volume of the n-th phase space $V_n = \dfrac{\pi^{n/2}}{\Gamma(1+n/2)}\sqrt{\det \sigma}$

The beam matrix of 2-th phase space $\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$

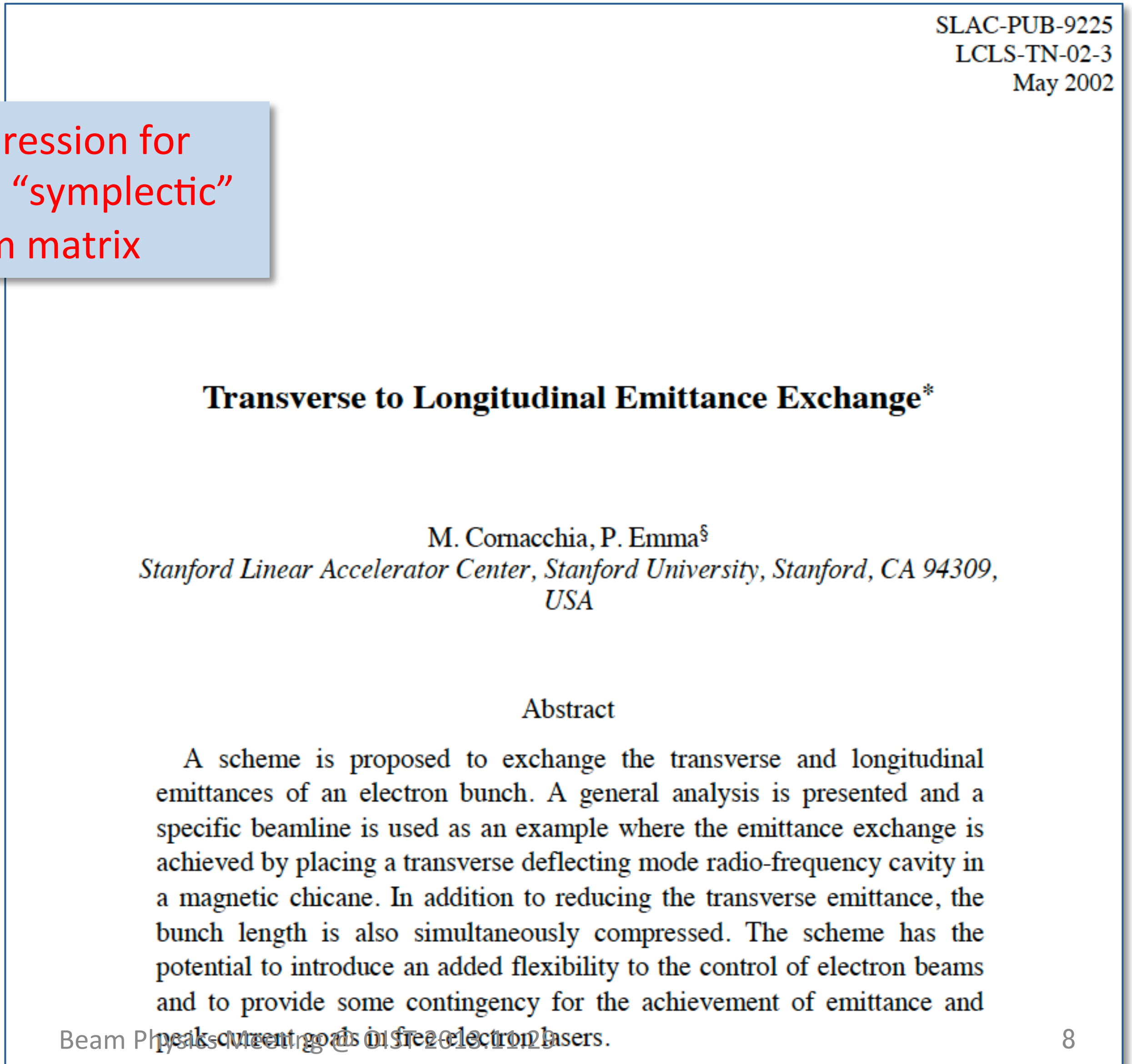
=> $$V_2 = \pi\sqrt{\det \sigma} = \pi\sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}{}^2} = \pi\varepsilon$$

Thus, for example, the horizontal emittance is

$$\varepsilon_x = \gamma x^2 + 2\alpha x x' + \beta x'^2, \quad \gamma = \left(\frac{1+\alpha^2}{\beta}\right)$$

Emittance Exchange: General

Describing general expression for emittance exchange with "symplectic" transfer of the beam matrix



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Transverse to Longitudinal Emittance Exchange*

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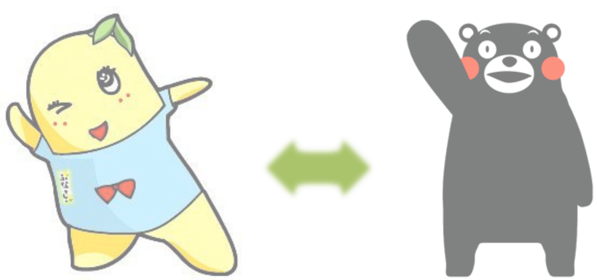
Abstract

A scheme is proposed to exchange the transverse and longitudinal emittances of an electron bunch. A general analysis is presented and a specific beamline is used as an example where the emittance exchange is achieved by placing a transverse deflecting mode radio-frequency cavity in a magnetic chicane. In addition to reducing the transverse emittance, the bunch length is also simultaneously compressed. The scheme has the potential to introduce an added flexibility to the control of electron beams and to provide some contingency for the achievement of emittance and peak-current goals in free-electron lasers.



Emittance Exchange: General

Consider a 4×4 beam convariance matrix (σ-matrix)

$$\sigma_0 = \begin{pmatrix} \varepsilon_{x0}\beta_{x0} & -\varepsilon_{x0}\alpha_{x0} & 0 & 0 \\ -\varepsilon_{x0}\alpha_{x0} & \varepsilon_{x0}\gamma_{x0} & 0 & 0 \\ 0 & 0 & \varepsilon_{y0}\beta_{y0} & -\varepsilon_{y0}\alpha_{y0} \\ 0 & 0 & -\varepsilon_{y0}\alpha_{y0} & \varepsilon_{y0}\gamma_{y0} \end{pmatrix} = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}$$

ε_{x0} and ε_{y0} are uncoupled initial beam emittance (longitudinal is also adaptable)

Introduce a 4×4 beamline transfer matrix R $\quad \sigma = R\sigma_0{}^tR$

R is symplectic, then det(R)=1 and 4-D emittance $\varepsilon_{x0}\varepsilon_{y0}$ is invariant

$$R = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \quad C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}, \quad D = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}$$

Emittance Exchange: General

After transporting beamline R
$$\sigma = \begin{pmatrix} A\sigma_x{}^tA + B\sigma_y{}^tB & A\sigma_x{}^tC + B\sigma_y{}^tD \\ C\sigma_x{}^tA + D\sigma_y{}^tB & C\sigma_x{}^tC + D\sigma_y{}^tD \end{pmatrix}$$

$$\det\sigma = \left(\frac{V_2}{\pi}\right)^2 = \sigma_{11}\sigma_{22} - \sigma_{12}{}^2 = \beta\gamma - \alpha^2 = \varepsilon^2$$

$$\Rightarrow \quad \varepsilon_x{}^2 = \left|A\sigma_x{}^tA + B\sigma_y{}^tB\right| \quad \left(= \det\left(A\sigma_x{}^tA + B\sigma_y{}^tB\right)\right)$$
$$\varepsilon_y{}^2 = \left|C\sigma_x{}^tC + D\sigma_y{}^tD\right|$$

X^a is adjoint of X （随伴行列；行列要素の全てが実数ならば、$X^a={}^tX$）

$$|X+Y| = |X| + |Y| + tr\left(X^aY\right)$$

$$X^a = |X|X^{-1}, \quad |X| \neq 0, \quad X^a = J^{-1\,t}XJ \quad \text{with} \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad J^2 = -I$$

Emittance Exchange: General

$$\varepsilon_x{}^2 = |A|^2 \varepsilon_{x0}{}^2 + |B|^2 \varepsilon_{y0}{}^2 + tr\left\{ \left(A\sigma_x{}^t A \right)^a B\sigma_y{}^t B \right\}$$

$$\varepsilon_y{}^2 = |C|^2 \varepsilon_{x0}{}^2 + |D|^2 \varepsilon_{y0}{}^2 + tr\left\{ \left(C\sigma_x{}^t C \right)^a D\sigma_y{}^t D \right\}$$

On the other hand

$$\sigma_x = \varepsilon_{x0} Q_x{}^t Q_x, \quad Q_x = \frac{1}{\sqrt{\beta_x}} \begin{pmatrix} \beta_x & 0 \\ -\alpha_x & 1 \end{pmatrix}$$

$$\sigma_y = \varepsilon_{y0} Q_y{}^t Q_y, \quad Q_y = \frac{1}{\sqrt{\beta_y}} \begin{pmatrix} \beta_y & 0 \\ -\alpha_y & 1 \end{pmatrix}$$

$$Q^t Q = \frac{1}{\sqrt{\beta}} \begin{pmatrix} \beta & 0 \\ -\alpha & 1 \end{pmatrix} \frac{1}{\sqrt{\beta}} \begin{pmatrix} \beta & -\alpha \\ 0 & 1 \end{pmatrix} = \frac{1}{\beta} \begin{pmatrix} \beta^2 & -\beta\alpha \\ -\beta\alpha & \alpha^2 + 1 \end{pmatrix} = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \dfrac{\alpha^2 + 1}{\beta} = \gamma \end{pmatrix}$$

Emittance Exchange: General

Some vector manipulation using $tr\{XYZ\} = tr\{YZX\} = tr\{ZXY\}$

$$\varepsilon_x{}^2 = |A|^2 \varepsilon_{x0}{}^2 + |B|^2 \varepsilon_{y0}{}^2 + \varepsilon_{x0}\varepsilon_{y0} tr\{U^t U\}$$

$$\varepsilon_y{}^2 = |C|^2 \varepsilon_{x0}{}^2 + |D|^2 \varepsilon_{y0}{}^2 + \varepsilon_{x0}\varepsilon_{y0} tr\{V^t V\}$$

where

$$U \equiv Q_x{}^{-1} A^a B Q_y$$

$$V = Q_x{}^{-1} C^a D Q_y$$

Symplectic condition for R

$$^t RSR = RS^t R = S = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}, \quad J \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad |R| = 1$$

$$\left|^t AJA + {}^t CJC\right| = \left|AJ^t A + BJ^t B\right| = 1$$

$$\left|^t BJB + {}^t DJD\right| = \left|CJ^t C + DJ^t D\right| = 1$$

$$|A| + |C| = 1, \quad |A| = |D|, \quad |B| = |C|$$

Emittance Exchange: General

We obtain $V = Q_x^{-1} C^a D Q_y = Q_x^{-1}\left(J^{-1}\,{}^t C J\right) D Q_y$

Using ${}^t CJD = -{}^t AJB$, V becomes

$$V = Q_x^{-1} C^a D Q_y = Q_x^{-1} J^{-1}\left({}^t CJD\right) Q_y = Q_x^{-1} J^{-1}\left(-{}^t AJB\right) Q_y$$

$$= -Q_x^{-1} A^a B Q_y = -U$$

$\Rightarrow$ $tr\left\{U\,{}^t U\right\} = tr\left\{V\,{}^t V\right\}$ $\quad tr\left\{U\,{}^t U\right\} = U_{11}^{\ 2} + U_{12}^{\ 2} + U_{21}^{\ 2} + U_{22}^{\ 2} \equiv \lambda^2 \geq 0$

---

finally

$$\boxed{\begin{aligned} \varepsilon_x^{\ 2} &= |A|^2 \varepsilon_{x0}^{\ 2} + \left(1 - |A|\right)^2 \varepsilon_{y0}^{\ 2} + \varepsilon_{x0}\varepsilon_{y0}\lambda^2 \\ \varepsilon_y^{\ 2} &= \left(1 - |A|\right)^2 \varepsilon_{x0}^{\ 2} + |A|^2 \varepsilon_{y0}^{\ 2} + \varepsilon_{x0}\varepsilon_{y0}\lambda^2 \end{aligned}}$$

No emittance exchange allowed for round beam !!

$$\lambda^2 \approx 0 \ \text{ and } \ |A| = 0 \Rightarrow \varepsilon_x = \varepsilon_{y0} \ \text{ and } \ \varepsilon_y = \varepsilon_{x0}$$

$$\text{if } \varepsilon_{x0} = \varepsilon_{y0} \Rightarrow \varepsilon_x = \varepsilon_y$$

Emittance Exchange: Transverse

PHYSICAL REVIEW E **66**, 016503 (2002)

Circular modes, beam adapters, and their applications in beam optics

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(Received 13 June 2001; revised manuscript received 18 March 2002; published 30 July 2002)

In the optics of charged particle beams, circular transverse modes can be introduced; they provide an adequate basis for rotation-invariant transformations. A group of these transformations is shown to be identical to a group of the canonical angular momentum preserving mappings. These mappings and the circular modes are parametrized similar to the Courant-Snyder forms for the conventional uncoupled, or planar, case. The planar-to-circular and reverse transformers (beam adapters) are introduced in terms of the circular and planar modes; their implementation on the basis of skew quadrupole blocks is described. Various kinds of matching for beams, adapters and solenoids are considered. Applications of the planar-to-circular, circular-to-planar and circular-to-circular transformers are discussed. A range of applications includes round beams at the interaction region of circular colliders, flat beams for linear colliders, relativistic electron cooling, and ionization cooling.

DOI: 10.1103/PhysRevE.66.016503 PACS number(s): 29.27.Eg

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **6**, 104002 (2003)

Round-to-flat transformation of angular-momentum-dominated beams

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(Received 13 June 2003; published 30 October 2003)

A study of round-to-flat configurations, and vice versa, of angular-momentum-dominated beams is presented. The beam propagation in an axial magnetic field is described in terms of the familiar Courant-Snyder formalism by using a rotating coordinate system. The discussion of the beam transformation is based on the general properties of a cylindrically symmetric beam matrix and the existence of two invariants for a symplectic transformation in 4D phase space.

DOI: 10.1103/PhysRevSTAB.6.104002 PACS numbers: 29.27.–a, 41.75.Lx, 41.85.–p

Emittance Exchange: Transverse

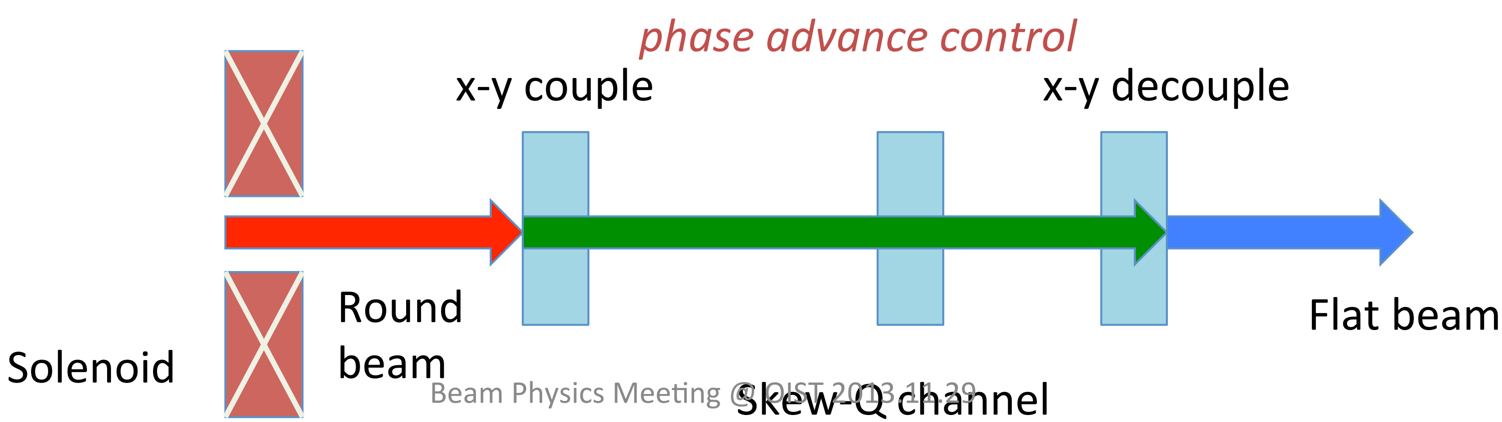
Solenoid

$$S\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ -\kappa & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} x \\ x'+\kappa y \\ y \\ -\kappa x + y' \end{pmatrix} \qquad \kappa \equiv \frac{eB_s}{2p_s}$$

SkewQchannel

$$\begin{pmatrix} X \\ X' \\ Y \\ Y' \end{pmatrix} = R(-45^\circ)QR(45^\circ)\begin{pmatrix} x \\ x'+\kappa y \\ y \\ -\kappa x + y' \end{pmatrix}$$

Non-symplectic !



Emittance Exchange: Transverse

Rotate coordinate by 45 deg

$$R(45°) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \qquad R(-45°) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Quadrupole channel in the rotated coordinate

$$Q = \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix}$$

$$M = \begin{bmatrix} \cos\mu & \beta\sin\mu \\ -\dfrac{\sin\mu}{\beta} & \cos\mu \end{bmatrix}$$

90° difference between x and y phase advances

$$N = FM = \begin{bmatrix} 0 & \beta \\ -\dfrac{1}{\beta} & 0 \end{bmatrix}\begin{bmatrix} \cos\mu & \beta\sin\mu \\ -\dfrac{\sin\mu}{\beta} & \cos\mu \end{bmatrix} = \begin{bmatrix} -\sin\mu & \beta\cos\mu \\ -\dfrac{\cos\mu}{\beta} & -\sin\mu \end{bmatrix}$$

Emittance Exchange: Transverse

Matching condition

Initial beam (α = 0)

$$\langle x_0{}^2\rangle = \langle y_0{}^2\rangle = \sigma_0{}^2$$

$$\langle x'_0{}^2\rangle = \langle y'_0{}^2\rangle = \sigma'_0{}^2$$

$$\varepsilon_0 = \sigma_0 \sigma'_0$$

Exit of the solenoid end-field = entrance of the skew Q channel

$$\langle x^2\rangle = \sigma_0{}^2$$

$$\langle x'^2\rangle = \langle (x'_0 + k y_0)^2\rangle = \langle x'_0{}^2\rangle + \kappa^2\langle y_0{}^2\rangle + 2\langle x'_0 y_0\rangle = \sigma'_0{}^2 + \kappa^2\sigma_0{}^2$$

$$\varepsilon = \sqrt{\sigma_0{}^2\sigma'_0{}^2 + \kappa^2\sigma_0{}^4}$$

$$\sigma_0 = \sqrt{\beta\varepsilon_0} = \sqrt{\beta\varepsilon}$$

<= Matched beta function to skew system

$$\Rightarrow \beta = \frac{\sigma_0{}^2}{\sqrt{\varepsilon_0{}^2 + \kappa^2\sigma_0{}^4}} = \frac{1}{\sqrt{\dfrac{\sigma'_0{}^2}{\sigma_0{}^2} + \kappa^2}}$$

Adapted emittances

Skew Q channnel $R(-45°)QR(45°)$

$$\varepsilon_x = \sqrt{\varepsilon_0{}^2 + \kappa^2\sigma_0{}^4} + k\sigma_0{}^2 = \sigma_0{}^2\left(\sqrt{\frac{\sigma'_0{}^2}{\sigma_0{}^2} + \kappa^2} + \kappa\right) \approx 2\kappa\sigma_0{}^2$$

$$\varepsilon_y = \sqrt{\varepsilon_0{}^2 + \kappa^2\sigma_0{}^4} - k\sigma_0{}^2 = \sigma_0{}^2\left(\sqrt{\frac{\sigma'_0{}^2}{\sigma_0{}^2} + \kappa^2} - \kappa\right) \approx \frac{\varepsilon_0{}^2}{2\kappa\sigma_0{}^2} = \frac{\sigma'_0{}^2}{2\kappa}$$

$$\varepsilon_x\varepsilon_y = \varepsilon_0{}^2$$

If we want $\varepsilon_y = \chi\varepsilon_0$ for the flat beam, $\beta = \dfrac{\sigma^2}{\varepsilon_0\sqrt{1 + \dfrac{1}{4\chi^2}}}$

$$Bz = \frac{\varepsilon_0}{\chi\sigma_0{}^2}(B\rho), \quad (B\rho) = \frac{p}{c} = \frac{\sqrt{\gamma^2 - 1}}{c}mc^2[eV], \quad \gamma = \frac{mc^2 + K}{mc^2}$$

Flat Beam Generation for S-P BWO-FEL

$K = 50\ \ keV\ \left(\sqrt{\gamma^2 - 1} = 0.453\right)$ **-> For Smith-Purcell radiation**

$\varepsilon_{n0} = \varepsilon_0 / \sqrt{\gamma^2 - 1} = 1 \times 10^{-6}\ \ m \cdot rad$ **<- Our experimental result**

$\chi = 0.1\ \left(\varepsilon_{ny} = 0.1 \times 10^{-6}\ \ m \cdot rad\right)$ **-> For Smith-Purcell BWO-FEL**

$\kappa = 5\ \left(B_z = 0.00772256\mathrm{T}\right)$

$\beta_0 = 1\ \ m\ \left(\beta = 0.2\ \ m\right)$ **-> Should not be small for transporting beam with reasonable skew-Q channel length**

$\Rightarrow$

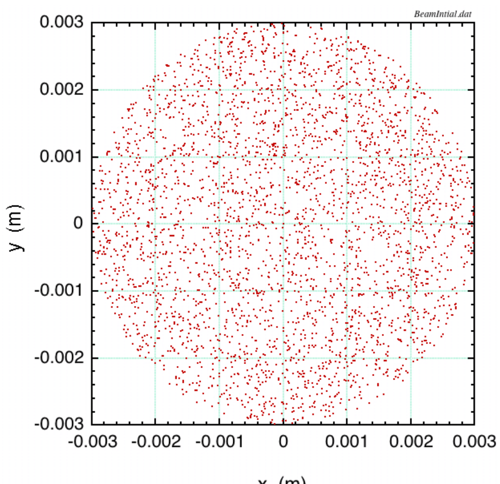
$\sigma_0{}^2 = 2.25 \times 10^{-6}$ uniform distribution $r_0 = 3\ \ mm$, Gaussian $\sigma_0 = 1.5\ \ mm$

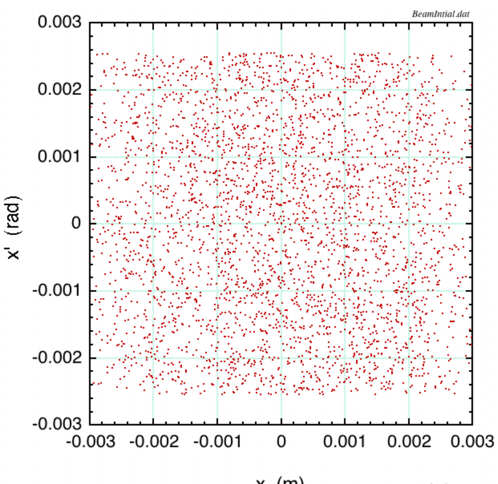
$\sigma'_0{}^2 = 2.15 \times 10^{-6}$

$\varepsilon_0 = 2.2 \times 10^{-6}\ \ m \cdot rad$

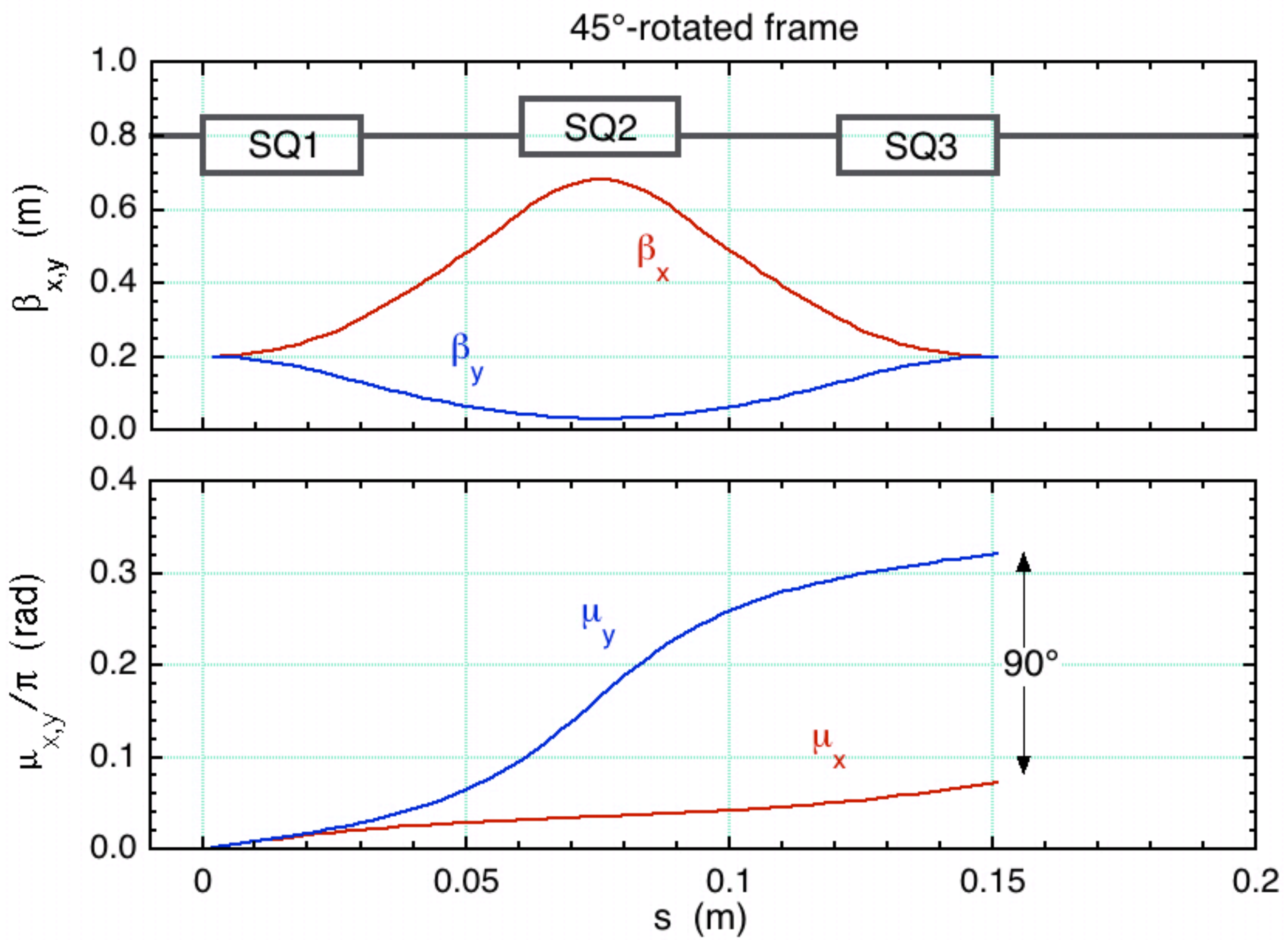
$\varepsilon_x \approx 22.5 \times 10^{-6}\ \ m \cdot rad$

$\varepsilon_y \approx 0.221 \times 10^{-6}\ \ m \cdot rad$

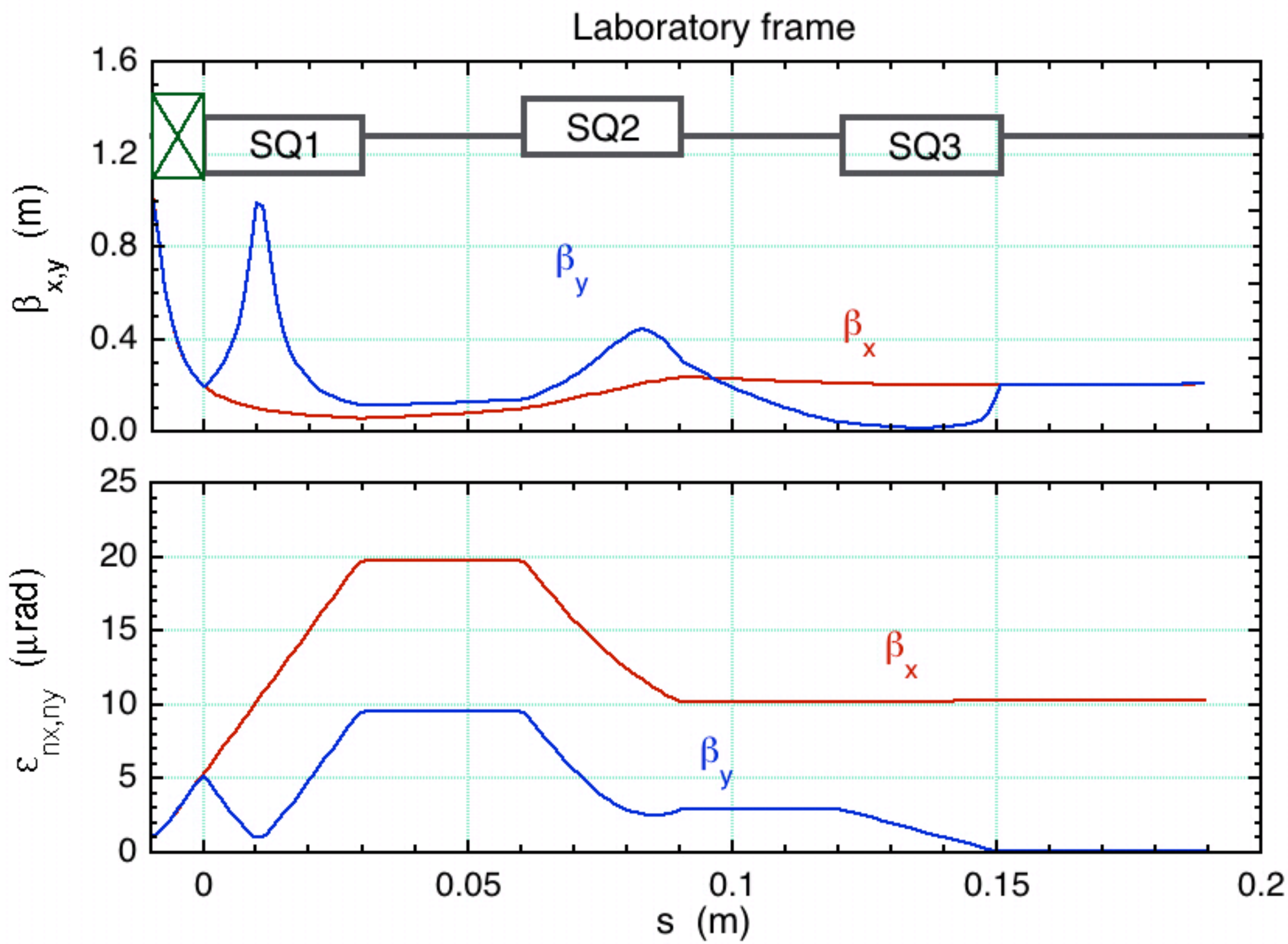


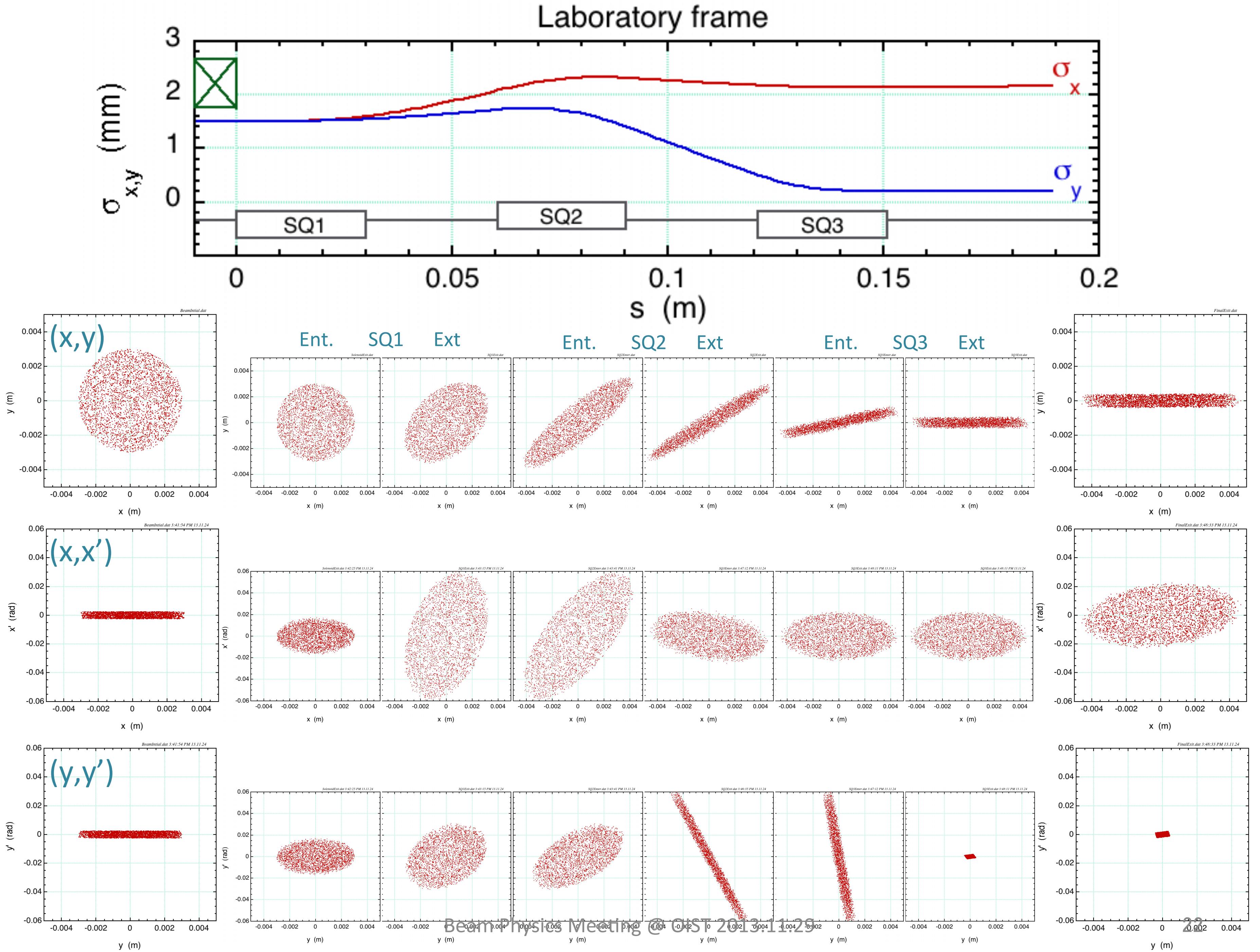


Flat Beam Generation for S-P BWO-FEL



Flat Beam Generation for S-P BWO-FEL





Laboratory frame
$\sigma_{x,y}$ (mm)
s (m)
σ_x
σ_y
SQ1
SQ2
SQ3
(x,y)
(x,x')
(y,y')
Ent.
SQ1
Ext
Ent.
SQ2
Ext
Ent.
SQ3
Ext

Flat Beam Generation for S-P BWO-FEL

	Initial	After solenoid end-field	Final
$\sigma_x{}^2$ ($\times 10^{-6}$ m^2)	2.25	2.25	4.40
$\sigma'_x{}^2$ ($\times 10^{-6}$ rad^2)	2.14	57.0	115.
$\sigma_y{}^2$ ($\times 10^{-6}$ m^2)	2.27	2.27	0.00436
$\sigma'_y{}^2$ ($\times 10^{-6}$ rad^2)	2.18	58.1	1.08
β_x (m)	1.02	0.199	0.196
β_y (m)	1.02	0.198	0.201
ε_x ($\times 10^{-6}$ m·rad)	2.19	11.3	22.5
ε_y ($\times 10^{-6}$ m·rad)	2.22	11.5	0.217
$\varepsilon_x^{norm.}$ ($\times 10^{-6}$ m·rad)	0.994	5.13	10.2
$\varepsilon_y^{norm.}$ ($\times 10^{-6}$ m·rad)	1.01	5.21	0.0984

Yes, we did (just on the paper) !

Conclusion

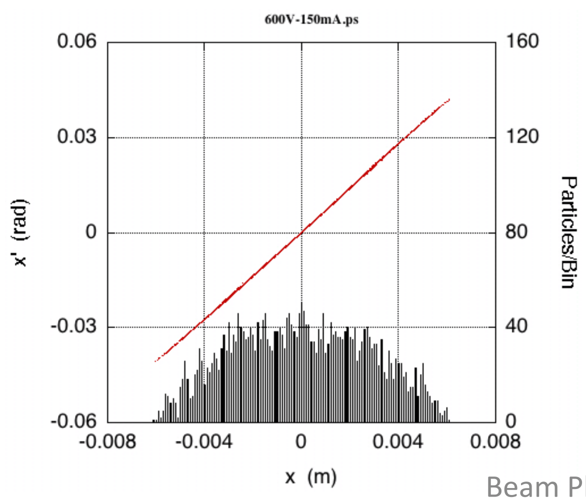
1 ε_{yn} ~ 0.1 μmrad may be possible

2 Precise phase advance control is very much crucial

3 Space charge effect may be significant for high intensity beam

4 How will 2-D K-V beam work to suppress space charge effect ?

(still the beam is K-V after the skew-Q channel ?)



The additional bias applied between the wehnelt and the cathode can manipulate equi-potential surface around the cathode, then Kapchinskij-Vladimirskij (KV) beam is obtained.

$$\varepsilon_{norm} \sim \varepsilon_{thermal} = 0.2\times10^{-6}\ \mathrm{mrad}$$