Study of Longitudinal Phase Space Distribution Measurement via a Linear Focal Cherenkov Ring Camera

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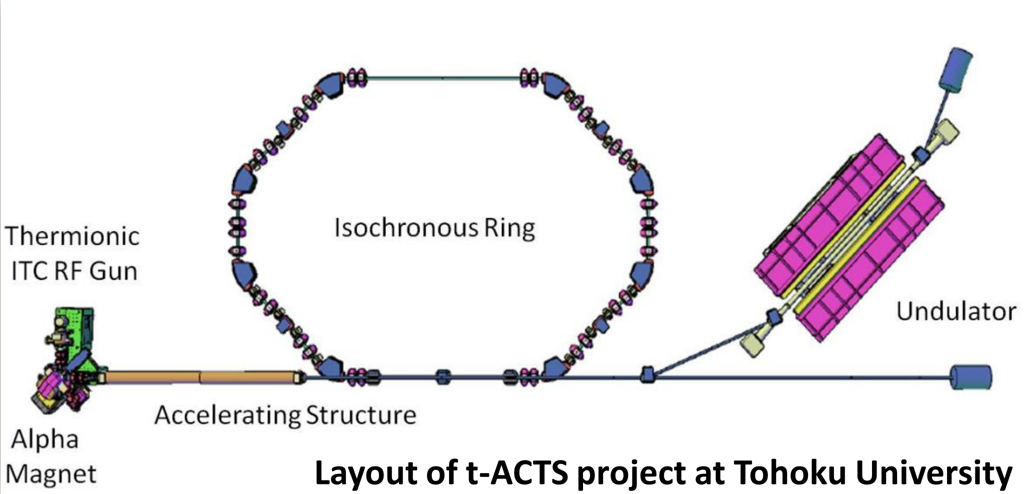
Outline

- Introduction
- Method for longitudinal phase space distribution measurement
  - Cherenkov radiation
  - Reflective optics
- Energy resolution enhancement
- Proposed experimental setup
- Numerical results
- Conclusion

Introduction

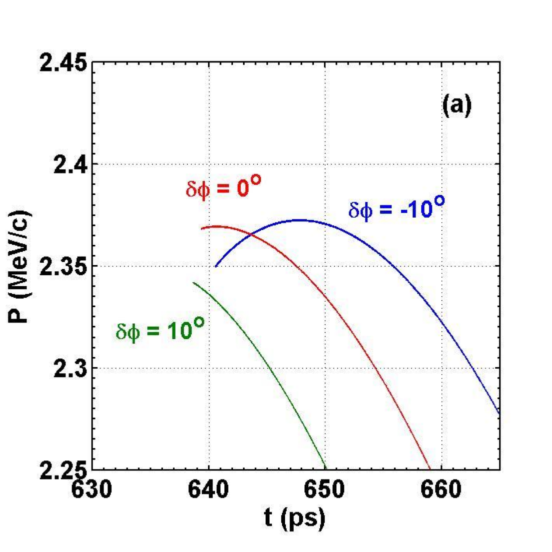
- A test accelerator for the coherent terahertz source (t-ACTS) at Tohoku University has been constructed
  - to generate intense coherent terahertz (THz) radiation from sub-picosecond electron bunches
  - an advanced independently tunable cells (ITC) thermionic RF gun consisting of two uncoupled cavities was proposed



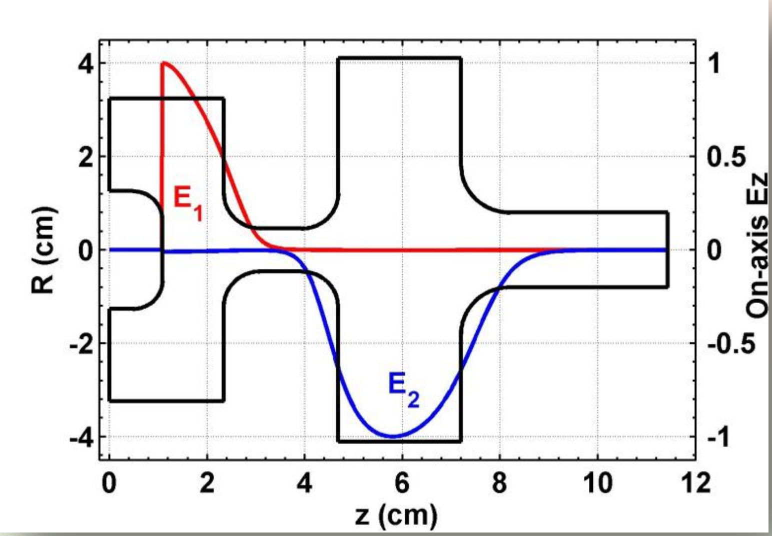
Layout of t-ACTS project at Tohoku University

Introduction

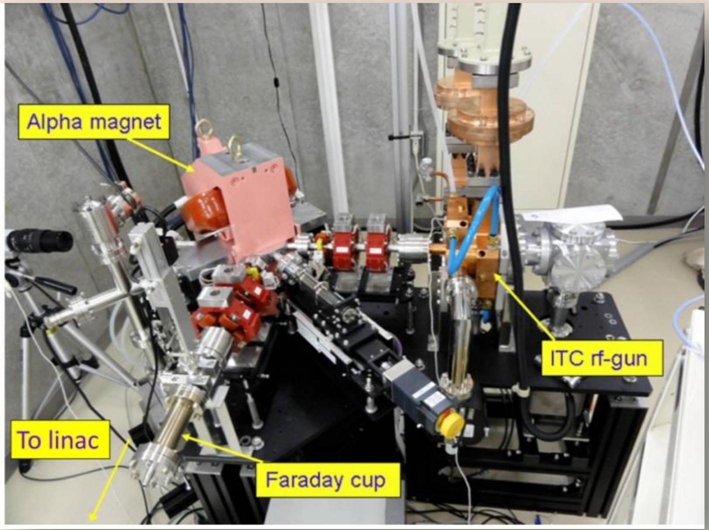
- electron beam will be introduced from the RF-gun into the bunch compression system
- To obtain extremely short electron bunch production
  - proper longitudinal phase space distribution by the ITC RF-gun adjusted relative RF phases and field strengths of the two cavities



longitudinal phase space (phase dependence)



Cross-section view of the ITC RF gun



Injector part for t-ACTS

Cherenkov Radiation

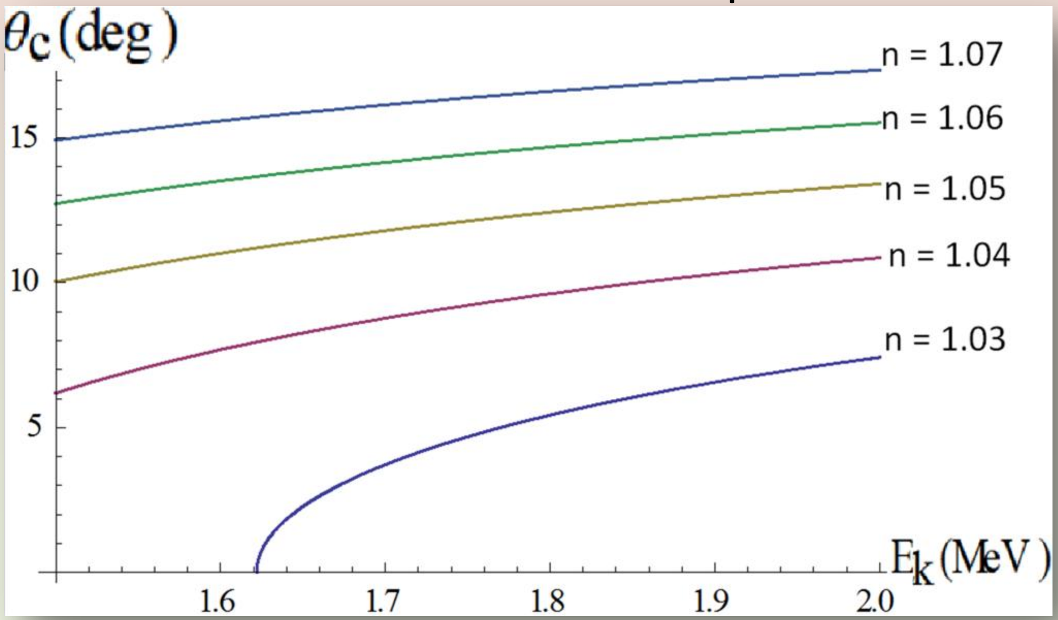
- one of diagnostic tools to measure electron energy
  (electron velocity corresponds to opening angle of Cherenkov light)
- **Cherenkov angle contains information of the particle energy** $\beta > 1/n(\omega)$

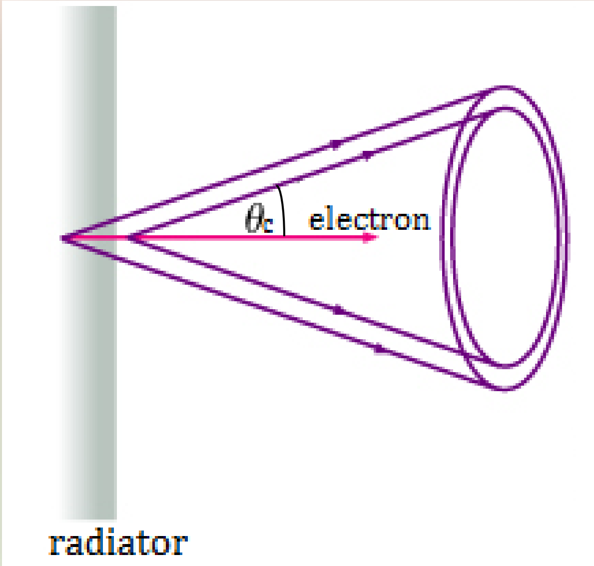
$$\cos\theta_c = 1/n(\omega)\beta$$

- aerogel (refractive index *n* = 1.05) = radiator

$$N = 2\pi\alpha z(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})\sin^2\theta_c$$

- number of the Cherenkov photons is enough to detect





Linear Focal Cherenkov Ring Camera

novel method for longitudinal phase space distribution measurement

1
- e^- with same Energy -> photon with same Cherenkov angle

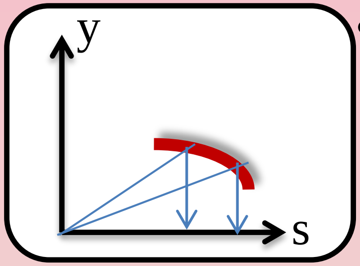
2
- Special Mirror : “Turtle-back” mirror

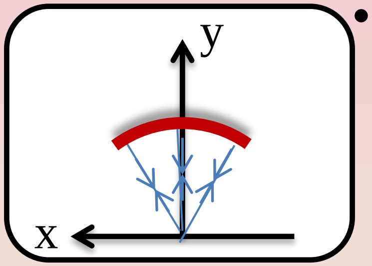
3
- Focus “same-Cherenkov-Angle photon” onto one certain Position
- “different-Cherenkov-Angle photon” gives Linear Position (focal line)

4
- Streak Camera

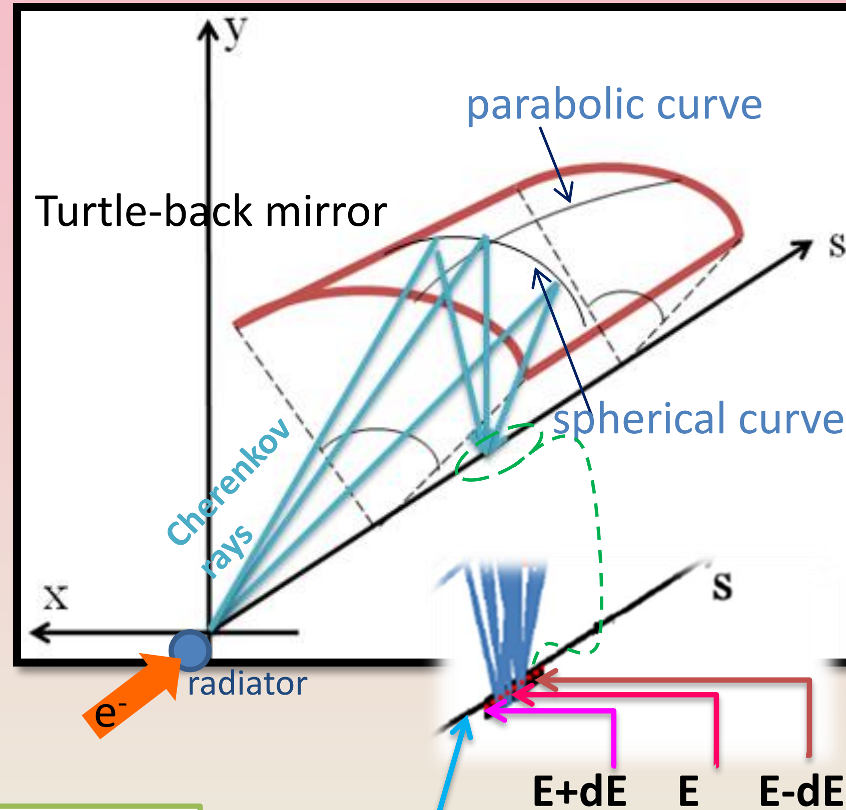
5
- directly observe longitudinal phase space distribution

Special Mirror : "Turtle-back" mirror





- Parabolic curve : reflect the photons having the same Cherenkov angle on a certain position
- Spherical curve : designed for symmetry due to Cherenkov cone (to focus Cherenkov light on the beam axis)



$$x^2 + y^2 - (-\frac{1}{2A}s^2 + \frac{A}{2})^2 = 0$$

A = 2x(focal length of parabolic curve)
higher A => lower energy dependence

- Surface Equation:

  e.g. A = 35 cm;

  mirror azimuthal size = 18 deg

focal position
(corresponds to electron energy)

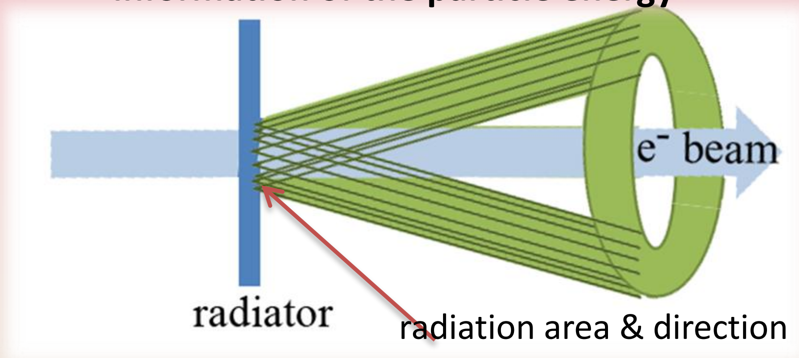
$$s_f(\beta) = An\beta\left(1 - \sqrt{1 - \left(\frac{1}{n\beta}\right)^2}\right)$$

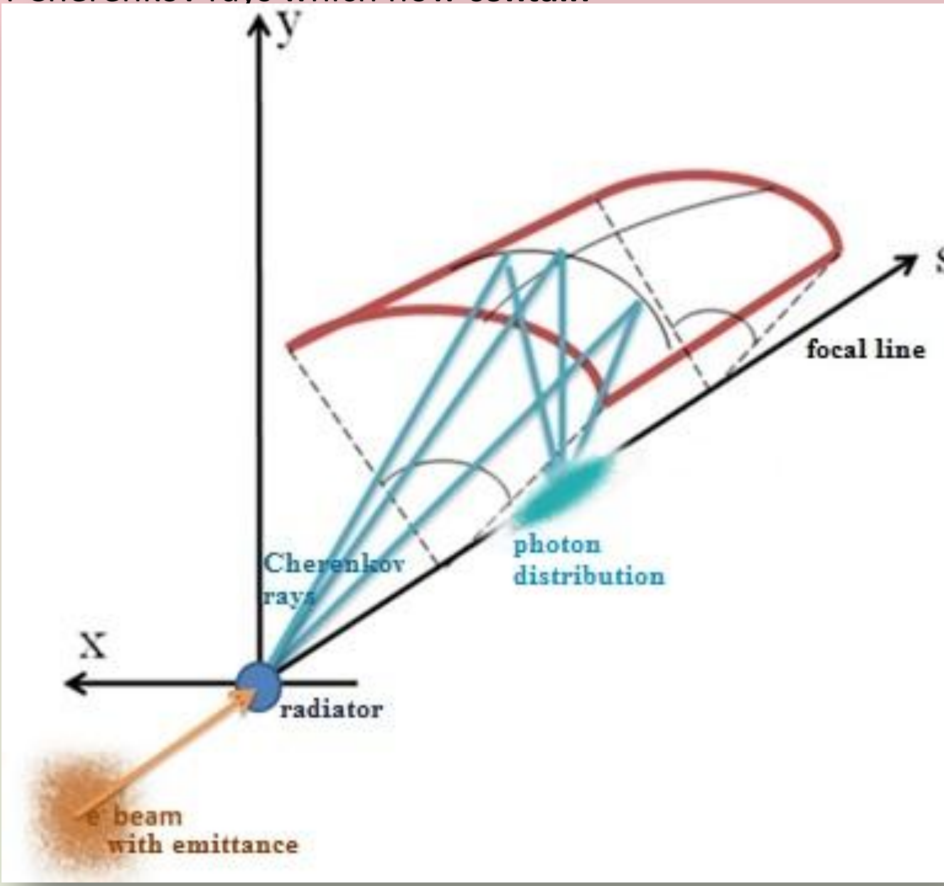
Energy Resolution Factors

- **Transverse emittance**
  - Beam size -> radiation area -> Cherenkov ring

    "turtle-back" mirror **cannot** focus Cherenkov **Ring** from same energy of electron to one point
  - Beam divergence -> change direction of Cherenkov rad.

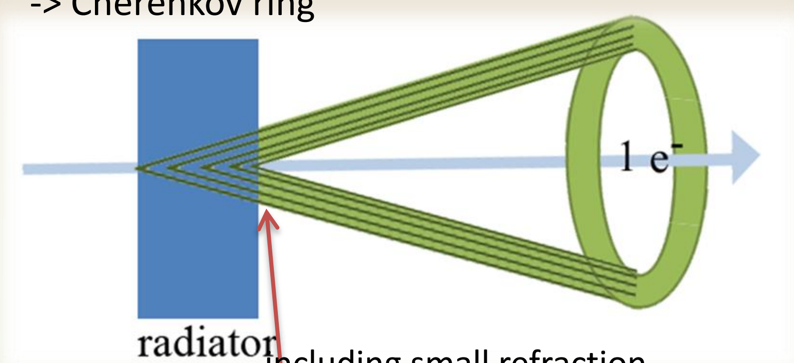
    Direction of each electron dictates direction of Cherenkov rays which now **contain information of the particle energy**





- **Thickness of radiator**

  -> Cherenkov ring



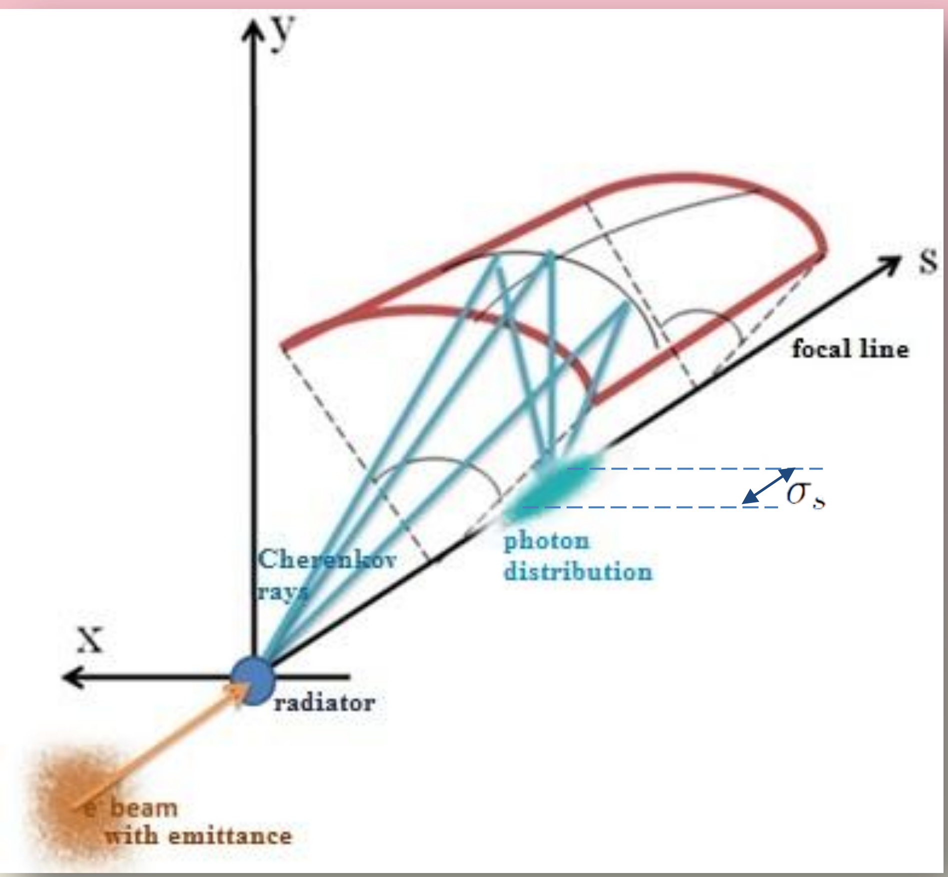
Momentum and Time Resolution

- Momentum resolution
  - defined as $\sigma_s\left(\frac{\mathrm{d}p}{\mathrm{d}s_f}\right)$

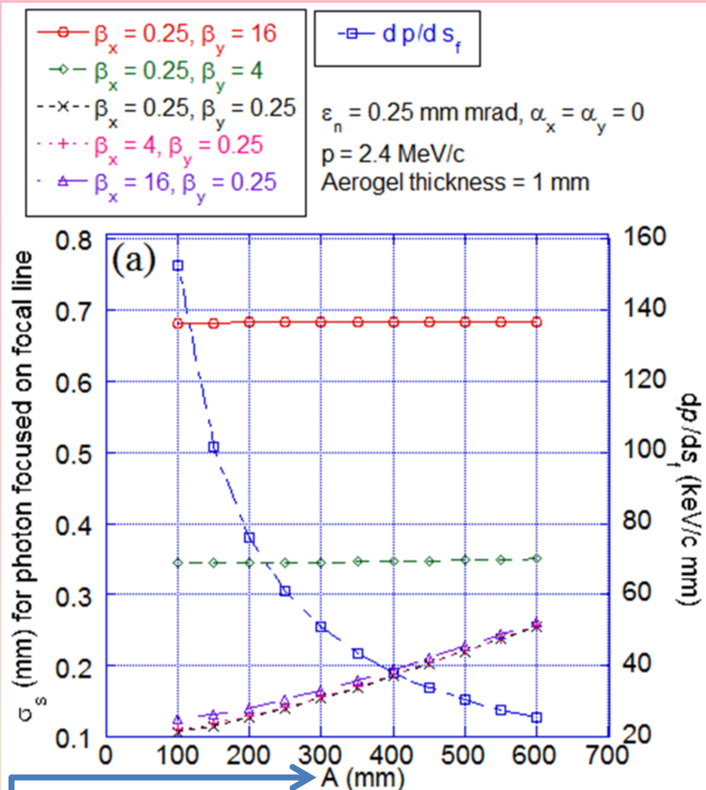
standard deviation of photon distribution in s direction

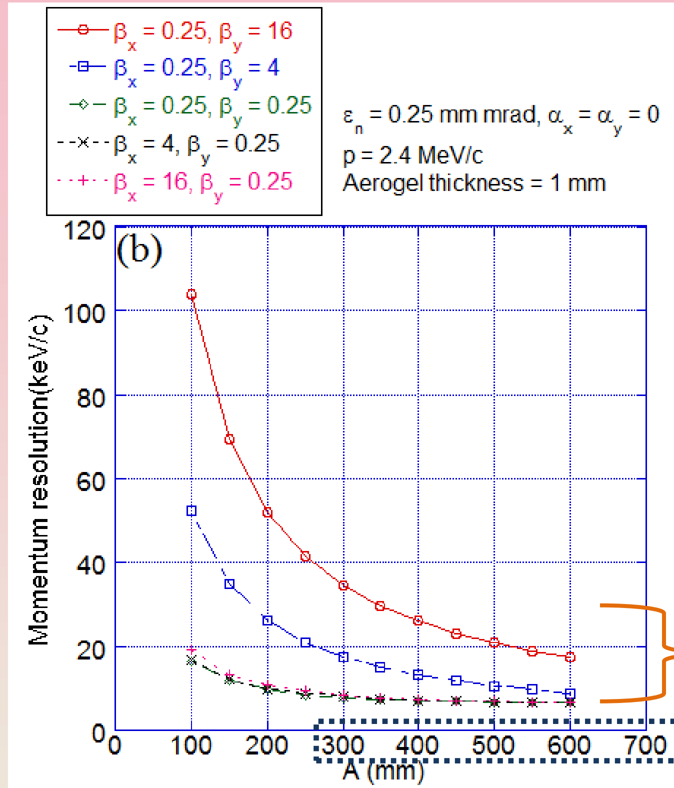
momentum dependence at focal position

- Time resolution
  - defined as the standard deviation of the arrival time of Cherenkov rays



A Dependence of Momentum Resolution



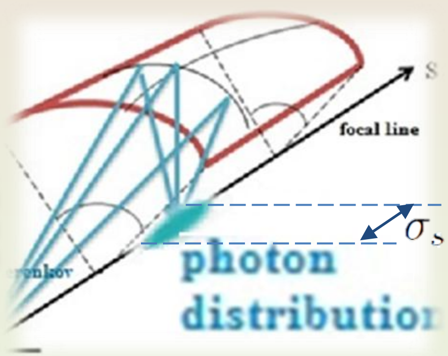


- (time resolution is nearly constant for different A)

momentum resolution is about 10-40 keV/c

turtle-back mirror's parameter A

$$x^2 + y^2 - \left(-\frac{1}{2A}s^2 + \frac{A}{2}\right)^2 = 0$$



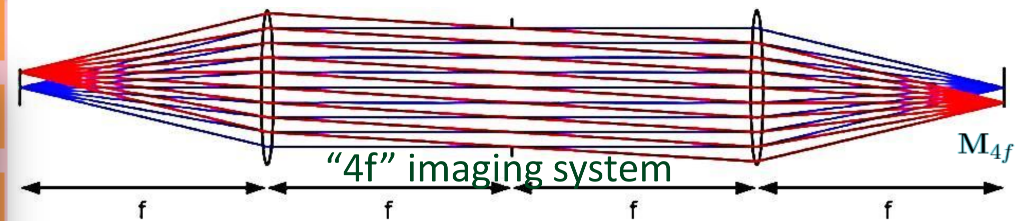
- momentum resolution is getting better, due to higher A (bigger mirror)
- proper beam focusing (proper twiss parameter beta) can enhance momentum resolution

Energy Resolution Enhancement

$$\Delta s \approx -y_i/\sqrt{\tan^2\theta_c + 1} \qquad \text{for } y_i \ll f = A/2,$$

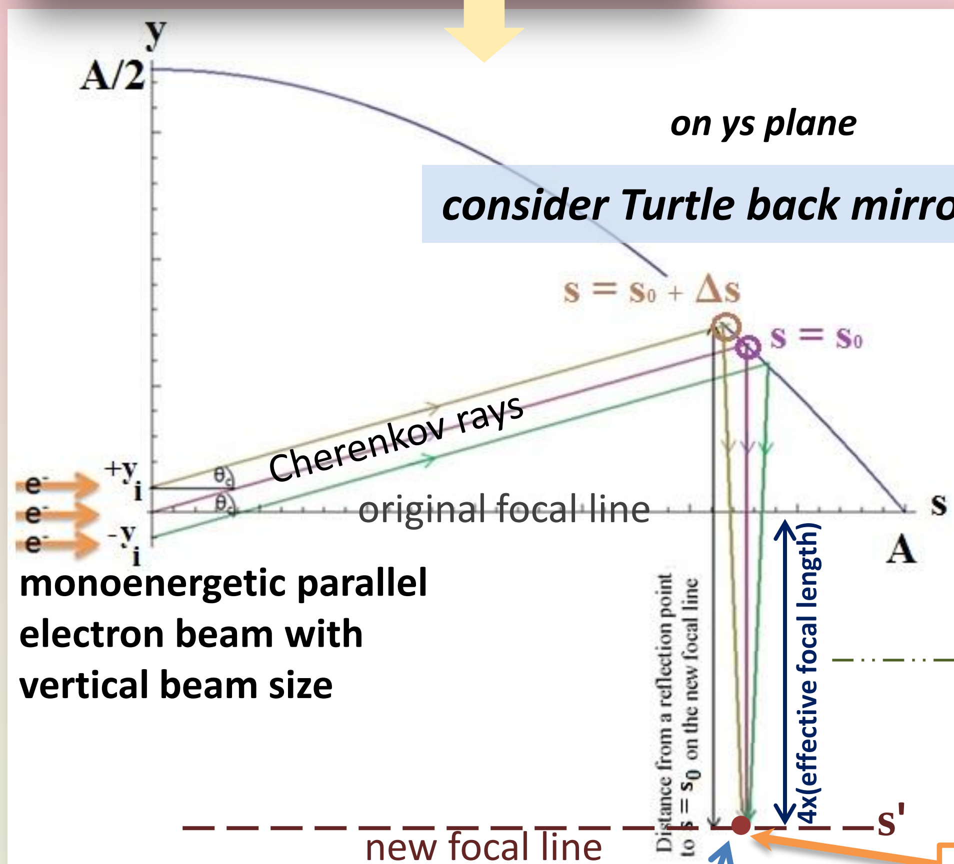
$$\tan\theta_s = s' = \frac{\mathrm{d}s}{\mathrm{d}y} = \frac{1 - 4(s_0+\Delta s)\left(\frac{2(s_0+\Delta s)+4f\tan\theta_c}{4(s_0+\Delta s)^2+16f^2}\right)}{\tan\theta_c - 8f\left(\frac{2(s_0+\Delta s)+4f\tan\theta_c}{4(s_0+\Delta s)^2+16f^2}\right)},$$

$$\text{distance from a reflection point to } s = s_0 \text{ on the new focal line} = \frac{\Delta s}{s'}$$

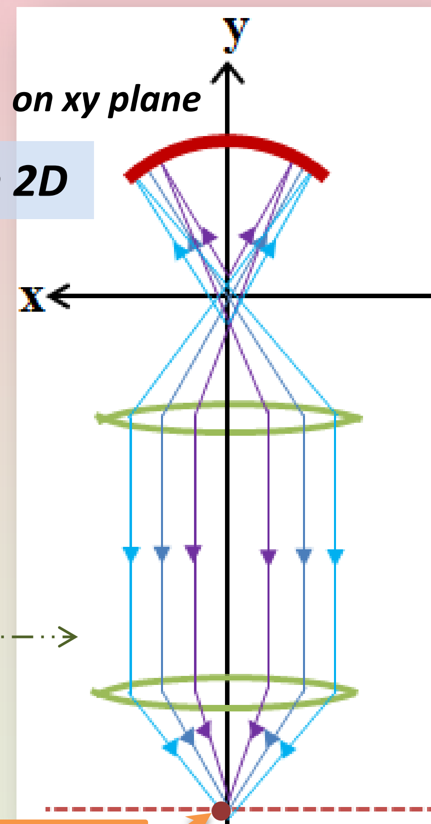


$$\mathbf{M}_{4f} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

to transport horizontal (x) image to the new focal line



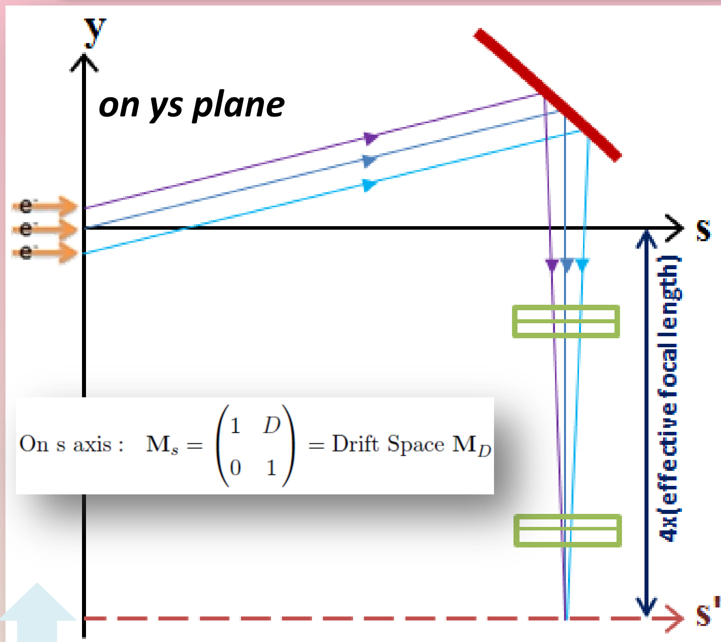
monoenergetic parallel electron beam with vertical beam size



focal point

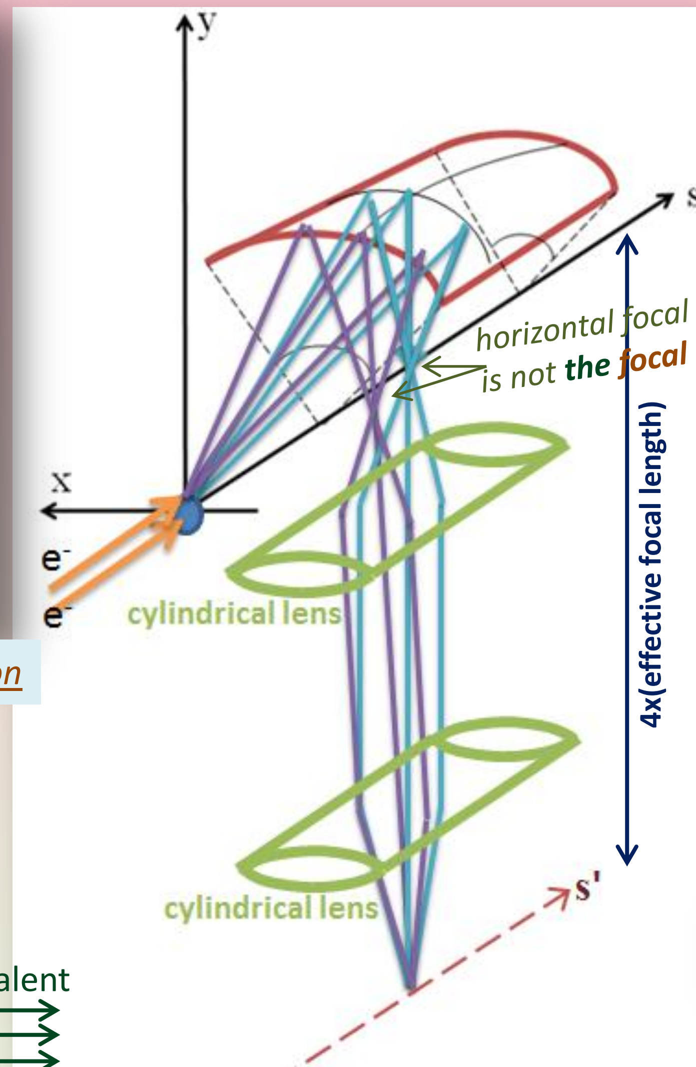
practically smaller photon distribution in s direction, or better energy resolution

Energy Resolution Enhancement

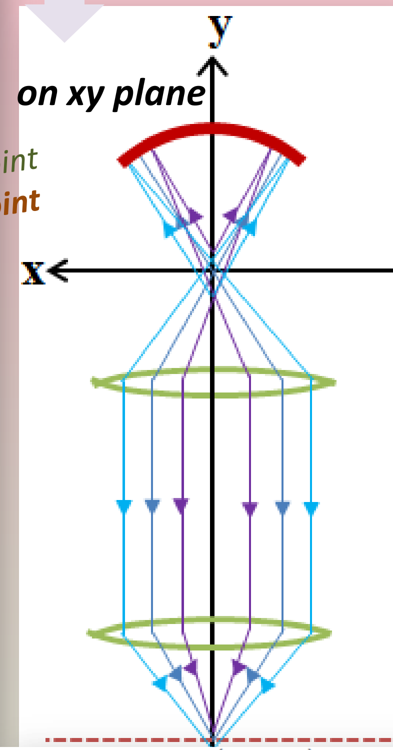


On s axis : $\mathrm{M}_s = \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix} = \text{Drift Space } \mathrm{M}_D$

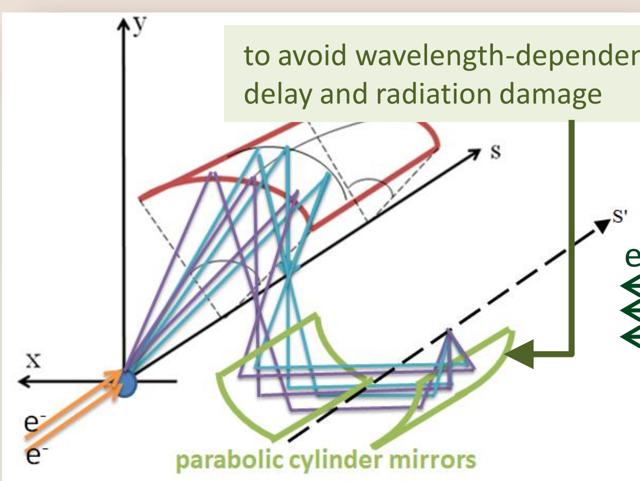
allow turtle-back mirror's focusing in s-direction



transport horizontal image to the new focal line



On x axis : $\mathrm{M}_x = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \mathrm{M}_{4f}$

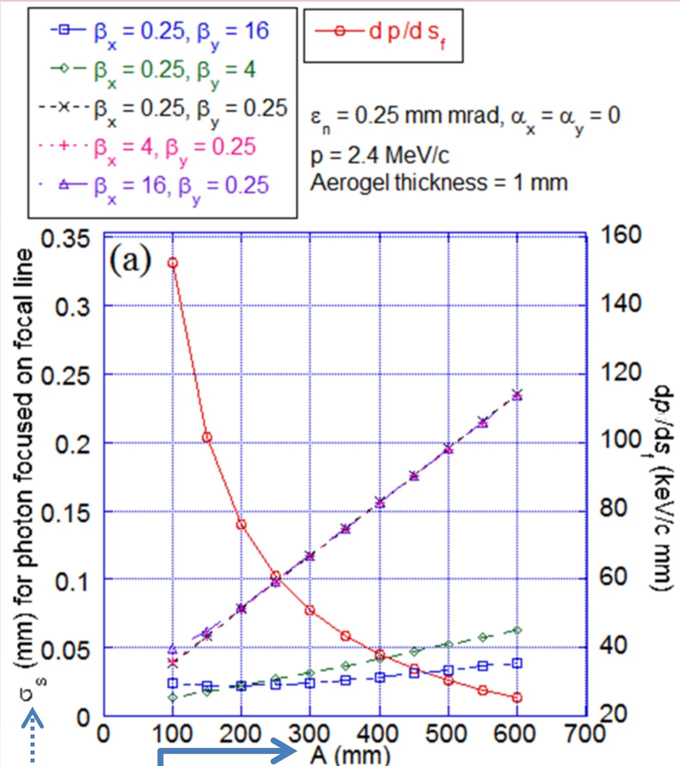


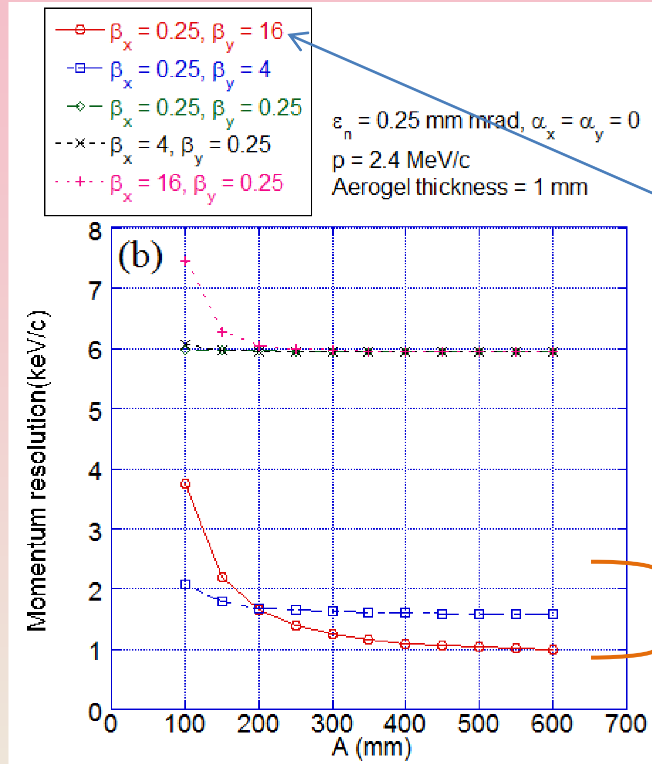
to avoid wavelength-dependent delay and radiation damage

equivalent

- "4f" imaging system consisted of 2 off-axis parabolic cylinder mirrors can be used to enhance energy resolution

A Dependence of Momentum Resolution

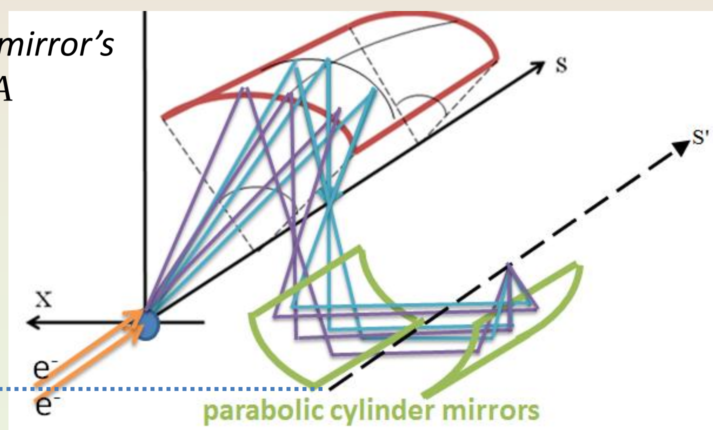




- (time resolution is nearly constant for different A)
- enhancement by off-axis parabolic cylinder mirrors requires small beam divergence y'

momentum resolution is about 1-3 keV/c

turtle-back mirror's parameter A



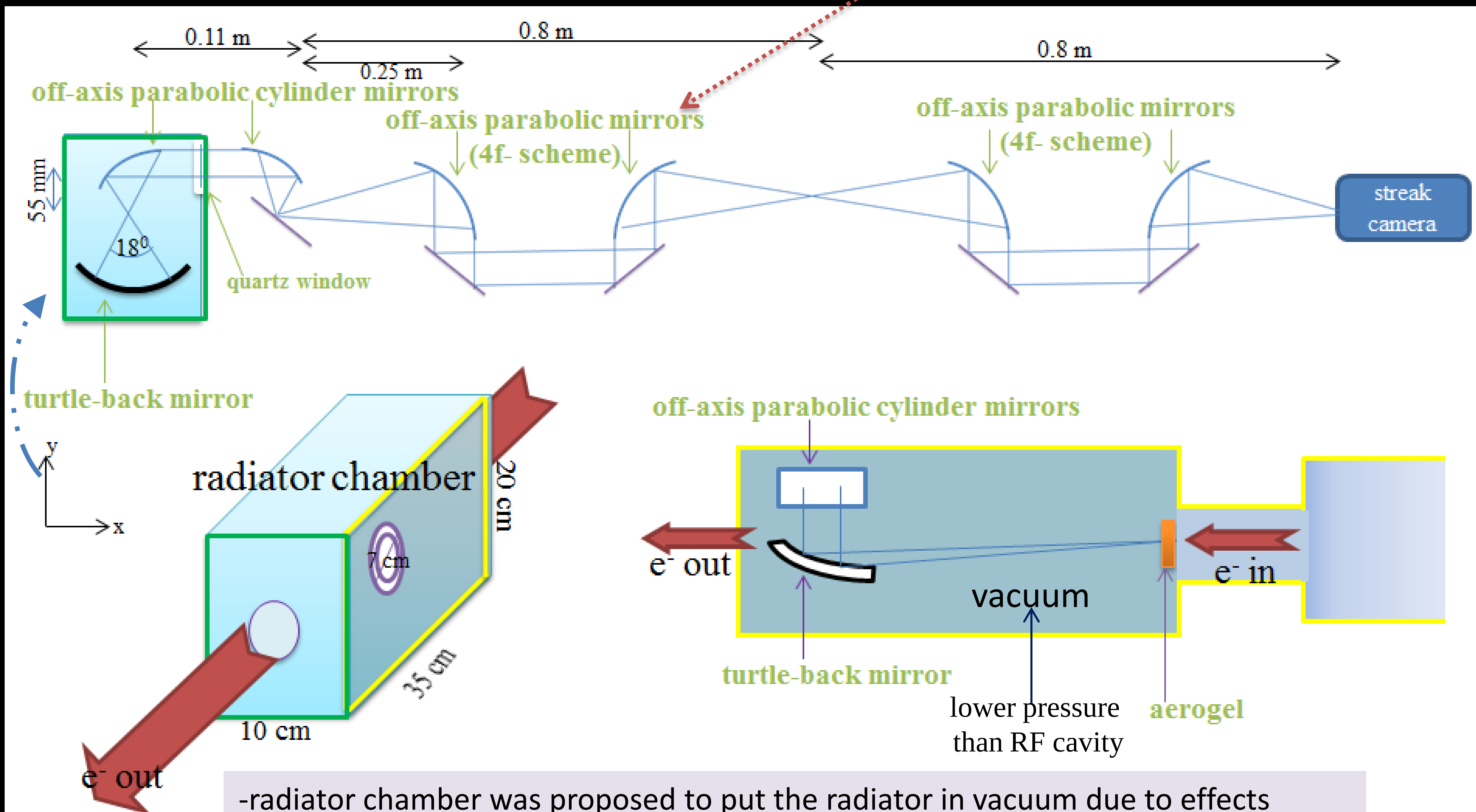
photon distribution at new focal line s'

enhanced by off-axis parabolic cylinder mirrors

effective focal length $f_{OAPC} = 0.1571A$

Proposed Experimental Setup

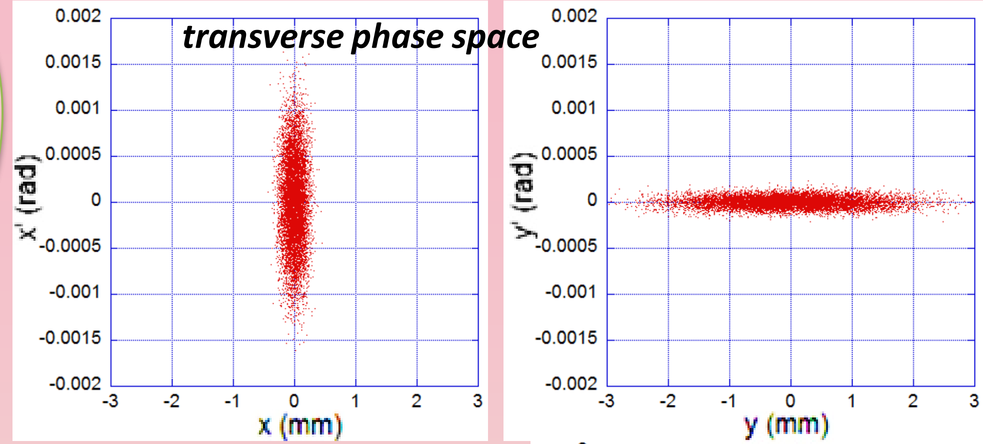
Recommended in
K. Rosbach et al. Reective optical system for time-resolved electron bunch measurements at pitz. NIM A, 654, 2011.

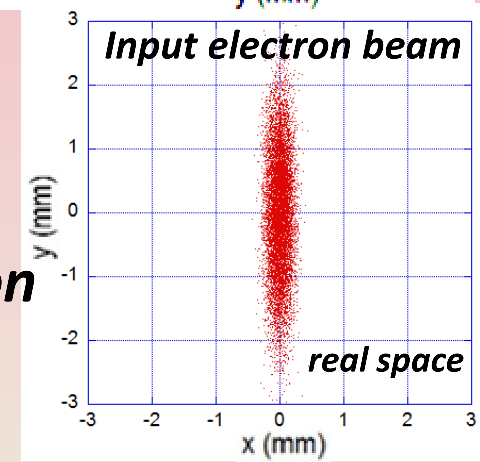


-radiator chamber was proposed to put the radiator in vacuum due to effects of multiple scatterings of electrons when extracted through beam window
-OAPC mirror used to transport light to outside of vacuum through quartz window

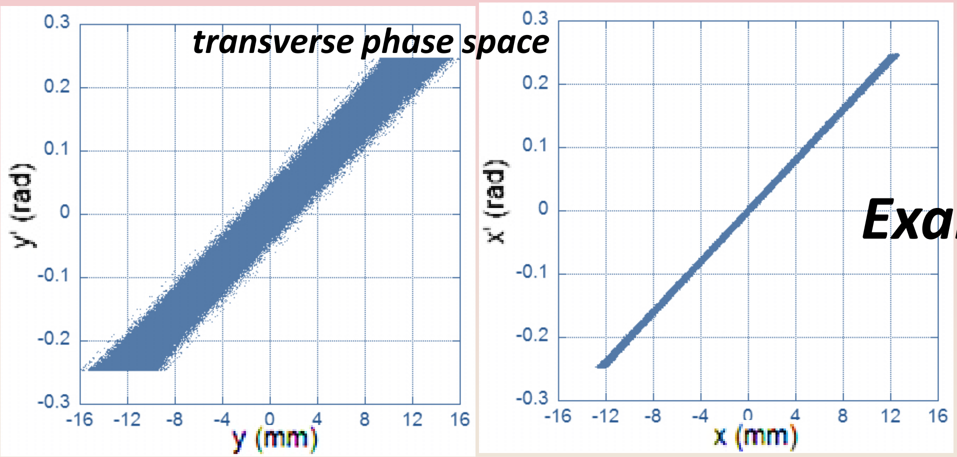
Generating Cherenkov Rays

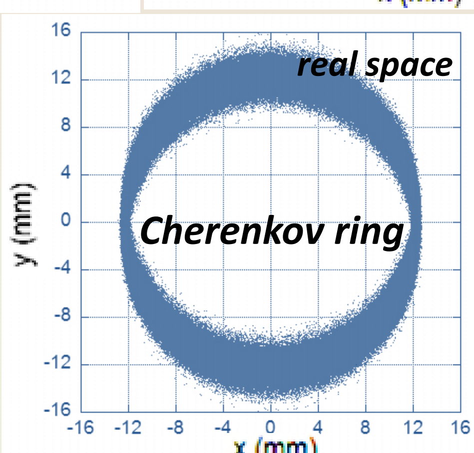
The experimental setup with reflective optics is examined by numerical ray tracing.

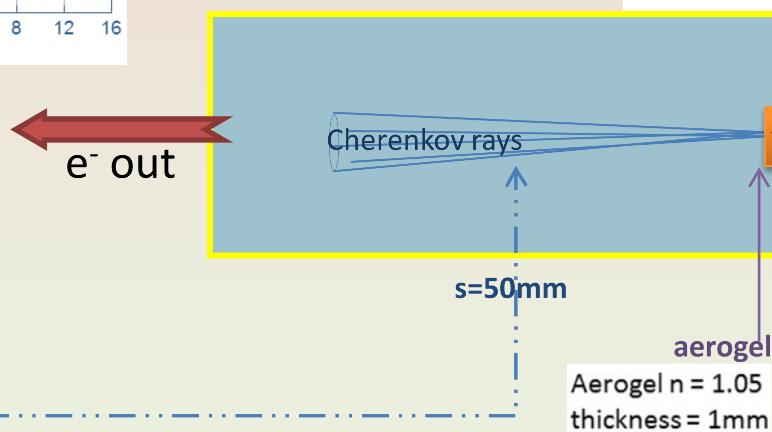




Example of Calculation







p = 2.4 MeV/c
ϵ_{nx} = 0.25 mm mrad
twiss parameter
$\alpha_x = \alpha_y = 0$
$\beta_x = 0.25$ m
$\beta_y = 16$ m

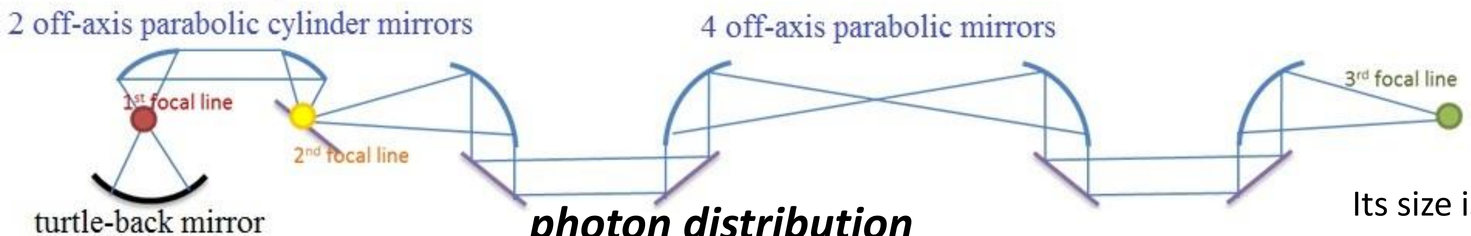
Ray Tracing Results

p = 2.4 MeV/c
ϵ_{nx} =0.25 mm mrad
twiss parameter
$\alpha_x = \alpha_y = 0$
$\beta_x = 0.25$ m
$\beta_y = 16$ m

Aerogel n = 1.05
thickness = 1 mm

center of parabolic mirror can be set by matching p =2.35MeV/c

p =2.294 -2.412 MeV/c
can be observed at once
(3mm steak camera entrance)

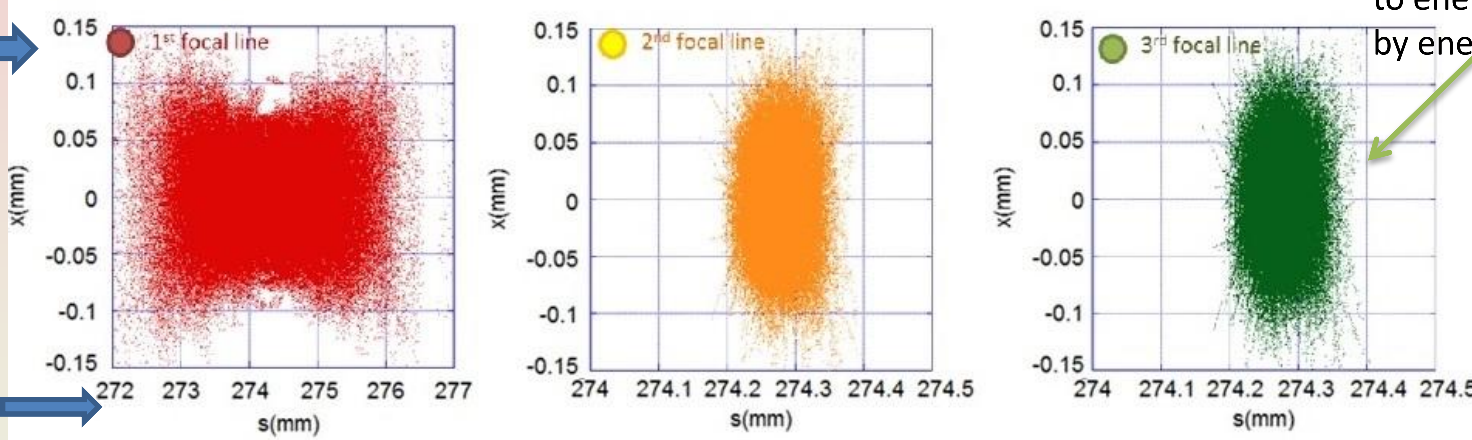


photon distribution

Detector screen

focal line

Its size is proportional to energy resolution by energy dependence



mean_x (mm)	+0.0001
mean_s (mm)	274.2690
σ_x (mm)	0.0340
σ_s (mm)	0.6829
p resolution(keV/c)	29.72
mean_time (ps)	1169.252
$\sigma_{arrival\ time}$ (ps)	0.7406

mean_x (mm)	+0.0001
mean_s (mm)	274.2750
σ_x (mm)	0.0358
σ_s (mm)	0.0241
p resolution(keV/c)	1.049
mean_time (ps)	1910.431
$\sigma_{arrival\ time}$ (ps)	0.7420

mean_x (mm)	+0.0001
mean_s (mm)	274.27500
σ_x (mm)	0.0361
σ_s (mm)	0.0241
p resolution(keV/c)	1.049
mean_time (ps)	8583.384
$\sigma_{arrival\ time}$ (ps)	0.7423

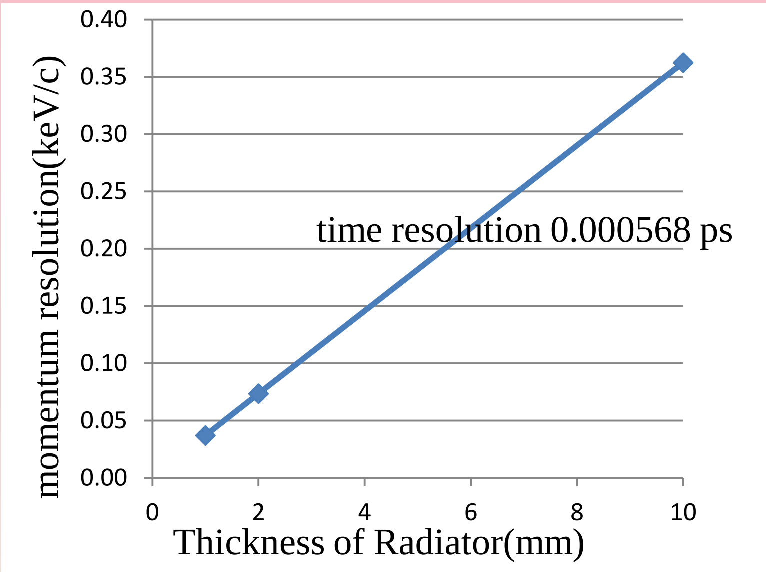
seems satisfied

Ray Tracing Results

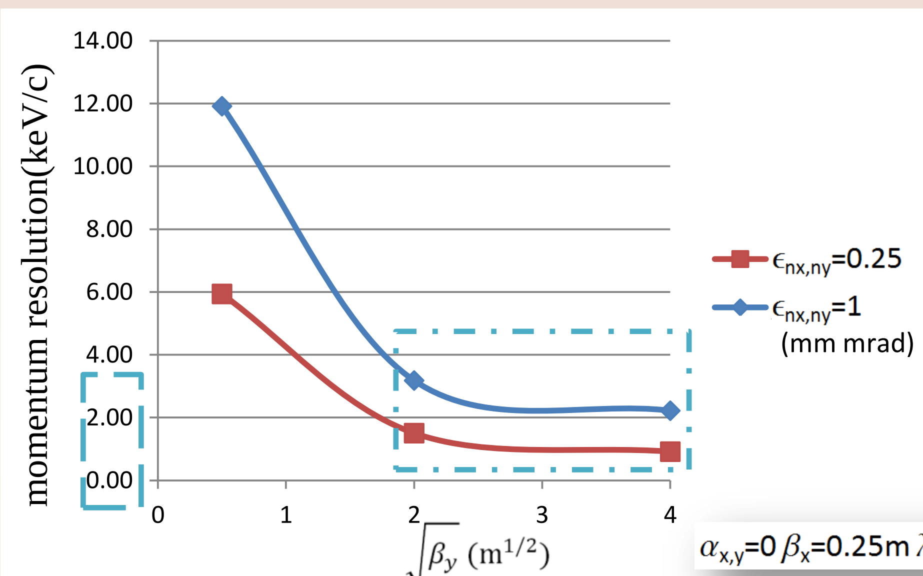
wavelength-dependence delay(552.225-557.775nm)
1%bandwidth of 555nm
- momentum resolution 0.019 keV/c
- time resolution 0.0014 ps

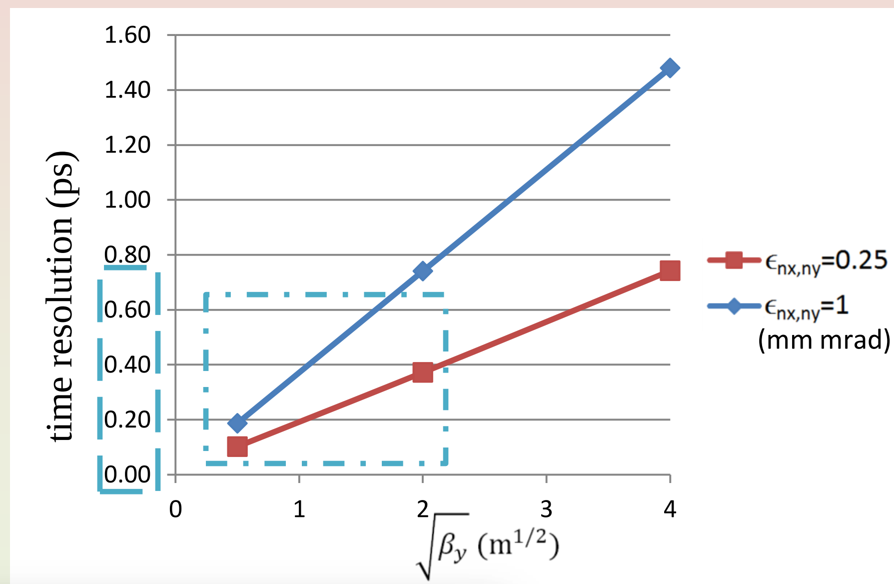
thickness of radiator

} small effect



y-emittance mainly gives significant resolution
- smaller beam divergence (y')
  - degrades time resolution
  - better momentum resolution



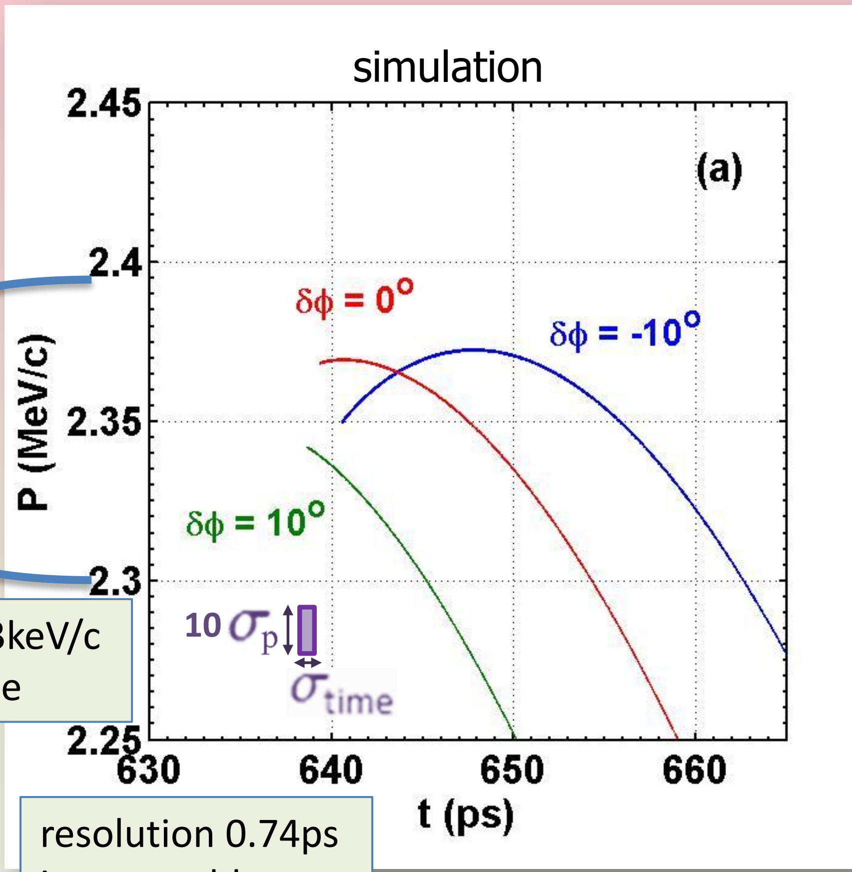


$\alpha_{x,y}$=0 β_x=0.25m λ1%bandwidth of 555nm aerogel1mm

Discussion

- with the example setup
- for normalized emittance of 1 mm mrad with selected twiss parameters

Range in one shot

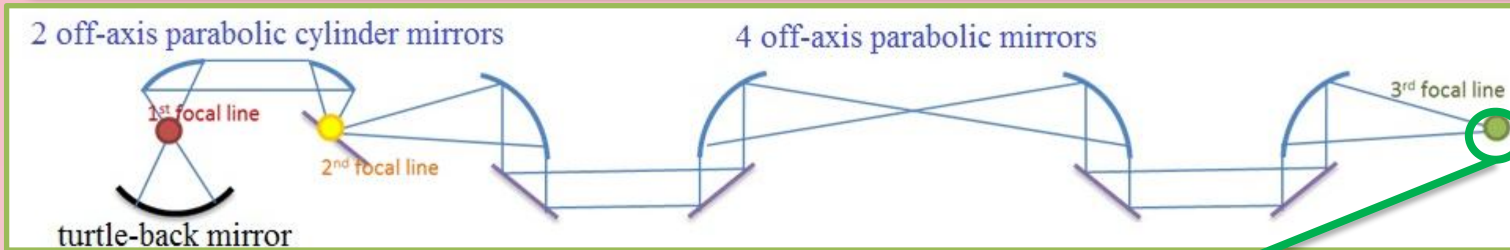


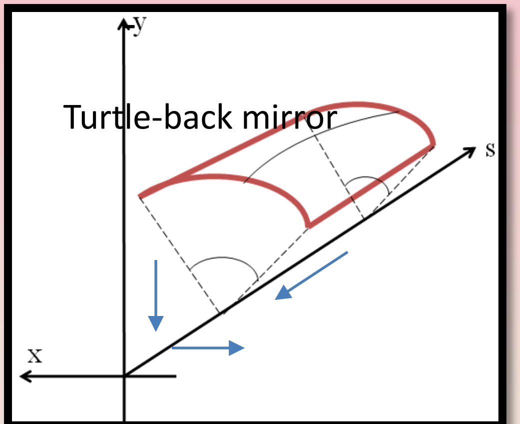
resolution 3keV/c is acceptable

resolution 0.74ps is acceptable

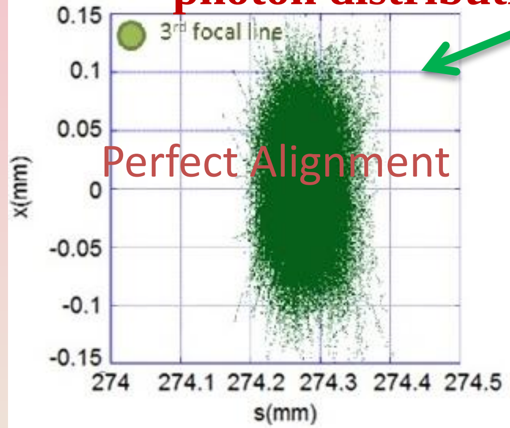
- Sufficient momentum & time resolutions were derived

Effects of Simple Misalignment

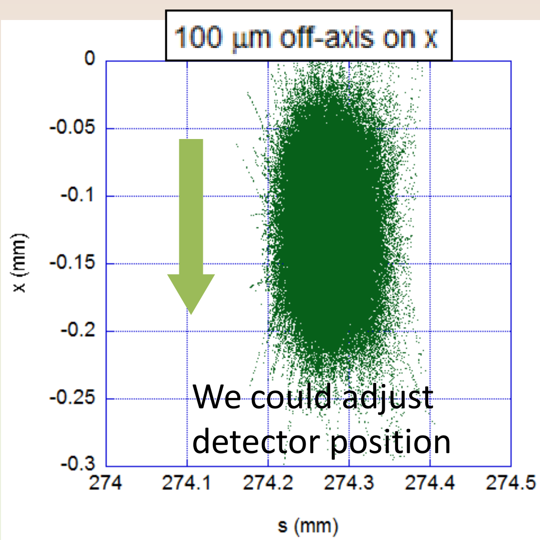


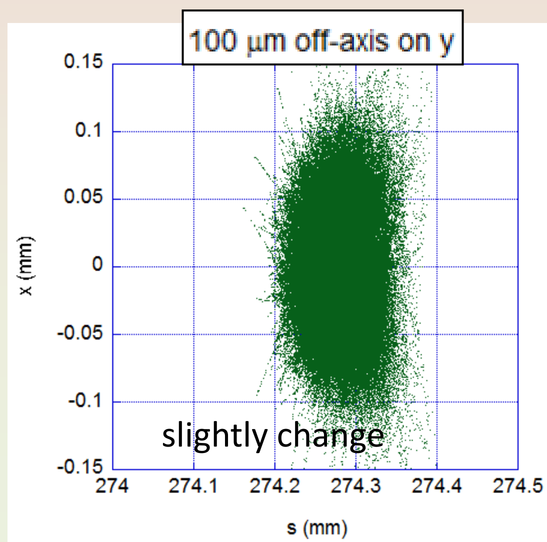


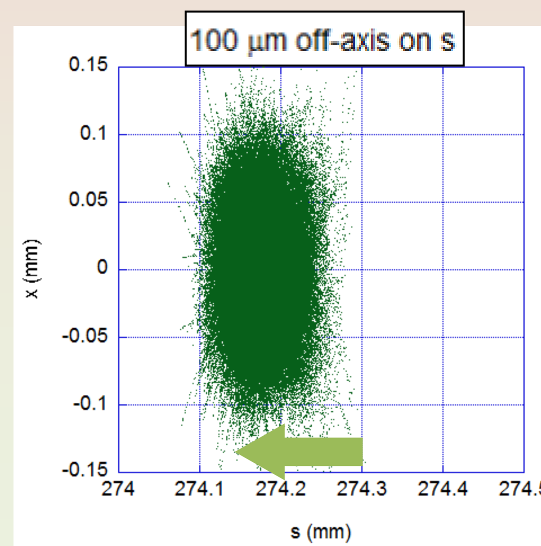
photon distribution on detector screen



off-axis on x and y
: won't become a problem
off-axis on s
: give error in energy measurement
(position on focal line <-> energy)







Conclusion

- Longitudinal phase space distribution measurement via a linear focal Cherenkov ring camera has been studied
- The "4f" imaging system consisted of two off-axis parabolic cylinder mirrors can transport the Cherenkov light to outside of the vacuum system through a quartz window and enhance energy resolution of the system
- Numerical ray tracing
  - optimize the optical elements' parameters and configuration
  - Sufficient energy (*or* momentum) and time resolutions were derived

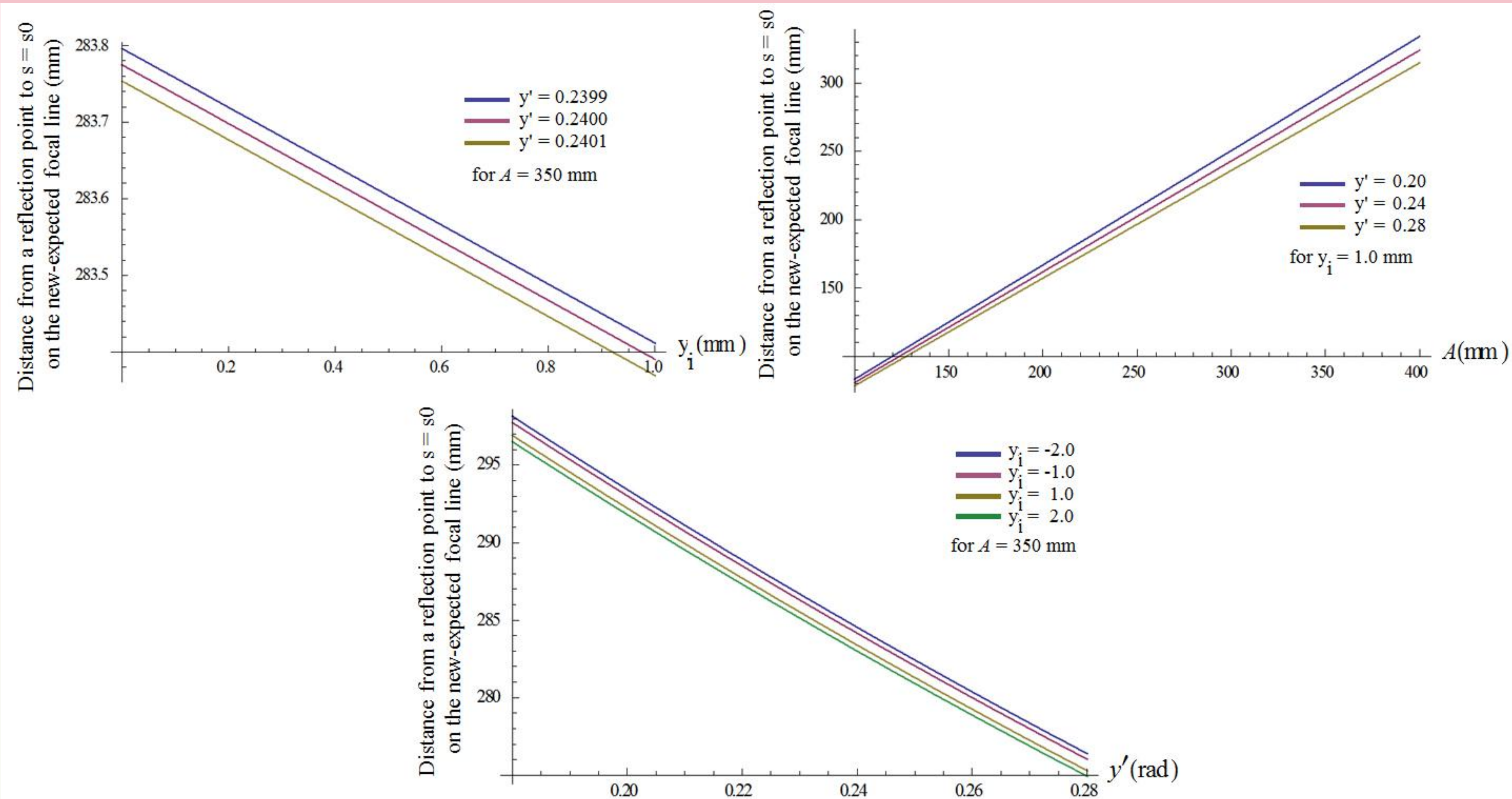
Thank you for your kind attention

Back up : Number of Photon

$$N = 2\pi\alpha z\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\sin^2\theta_c$$

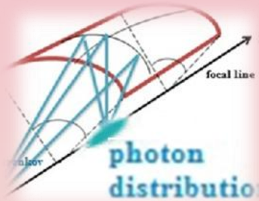
- number of the Cherenkov photons is enough to detect
- Example (N = 5 million)
  - At t-ACTS
    - electron momentum extracted from ITC-RF gun 2.3 MeV/c with energy spread of about 2%
  - Cherenkov radiator = aerogel
    - refractive index n = 1.05, thickness = 1.0 mm
    - by 20 pC bunch (in micro-pulse) , p=2.3MeV/c
    - 1% bandwidth of a wavelength around 555 nm
      - ignore frequency dependence in refractive index

Back up : Energy Resolution Enhancement

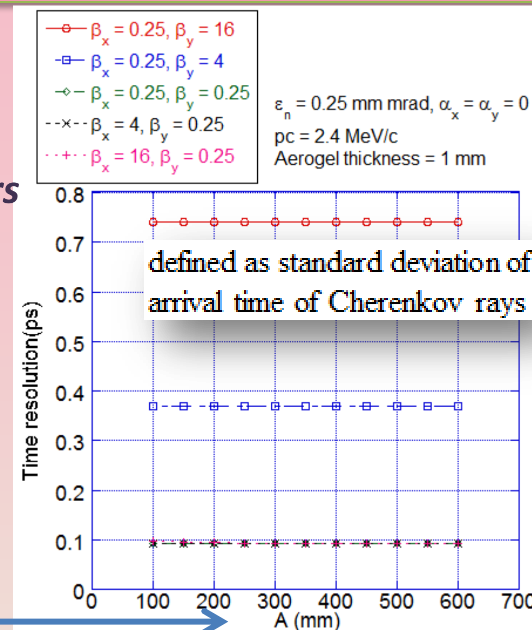


Back up : Calculation Results

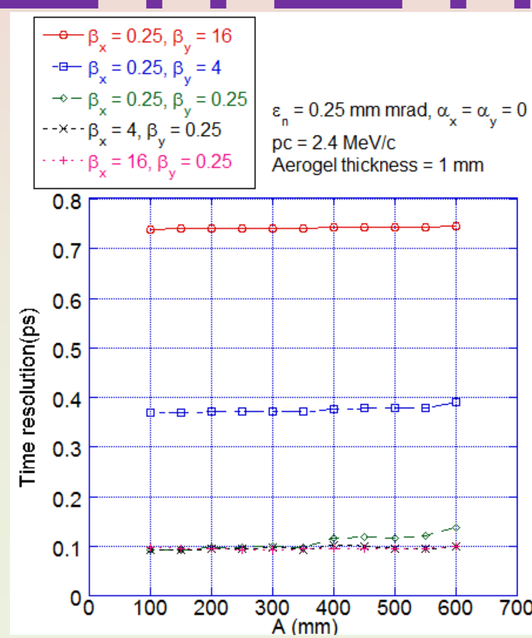
not enhanced by off-axis parabolic cylinder mirrors

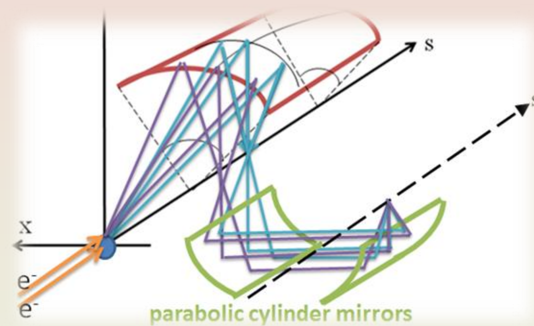


*turtle-back mirror's parameter **A***



defined as standard deviation of arrival time of Cherenkov rays





enhanced by off-axis parabolic cylinder mirrors

effective focal length

$$f_{OAPC} = 0.1571A$$

Back up: Calculation Results

y-emittance mainly gives significant resolution

electron which hits the radiator at $(x, y) = (0, 0)$

smaller beam divergence (y')
- degrades time resolution
- better momentum resolution

case		Time Resolution (ps) defined as standard deviation of arrival time of Cherenkov rays			Momentum Resolution (keV/c) defined as $\sigma_s(\frac{dpc}{ds_f})$		
		1st	2nd	3rd	1st	2nd	3rd
1	ref e- +wavelengthλ(552.225-557.775) 1%bandwidth of 555nm	0.0000000	0.0010273	0.0013551	0.0634745	0.0634745	0.0190349
2	ref e- +radiator thickness 1.0 mm aerogel n=1.05	0.0000176	0.0000003	0.0005687	2.1712830	0.0369714	0.0368912
3	ref e- +radiator thickness 2.0 mm aerogel n=1.05	0.0000705	0.0000012	0.0005687	4.3427108	0.0734926	0.0734262
4	ref e- +radiator thickness 10.0 mm aerogel n=1.05	0.0017629	0.0000304	0.0005683	21.7191306	0.3623170	0.3622650
5	ϵ_{nx}=0 ϵ_{ny}=0.25 α_y=0 β_y=0.25m	0.0925872	0.2667220	0.2674832	7.0184796	5.9360204	5.9360207
6	ϵ_{nx}=0 ϵ_{ny}=0.25 α_y=0 β_y=4m	0.3702555	0.3935371	0.3941017	14.8886487	1.5027686	1.5027639
7	ϵ_{nx}=0 ϵ_{ny}=0.25 α_y=0 β_y=16m	0.7405188	0.7463854	0.7467152	29.6208227	0.9133822	0.9133230
8	ϵ_{nx}=0 ϵ_{ny}=0.25 α_y=0 β_y=0.25m λ1%bandwidth of 555nm aerogel 1mm	0.0925796	0.2580870	0.2588665	7.3443100	5.9364653	5.9364650
9	ϵ_{nx}=0 ϵ_{ny}=0.25 α_y=0 β_y=4m λ1%bandwidth of 555nm aerogel 1mm	0.3702333	0.3920253	0.3925992	15.0520362	1.5032023	1.5031899
10	ϵ_{nx}=0 ϵ_{ny}=0.25 α_y=0 β_y=16m λ1%bandwidth of 555nm aerogel 1mm	0.7404706	0.7463548	0.7466844	29.7072245	0.9134198	0.9133740
11	$\epsilon_{nx,ny}$=0.25 $\alpha_{x,y}$=0 β_x=0.25m β_y=0.25m λ1%bandwidth of 555nm aerogel1mm	0.0925860	0.0998227	0.1019841	7.3690126	5.9583790	5.9583776
12	$\epsilon_{nx,ny}$=0.25 $\alpha_{x,y}$=0 β_x=0.25m β_y=4m λ1%bandwidth of 555nm aerogel1mm	0.3702872	0.3715240	0.3721355	15.0668293	1.5887432	1.5887368
13	$\epsilon_{nx,ny}$=0.25 $\alpha_{x,y}$=0 β_x=0.25m β_y=16m λ1%bandwidth of 555nm aerogel1mm	0.7406017	0.7419743	0.7423214	29.7182403	1.0487822	1.0487348
14	$\epsilon_{nx,ny}$=1 $\alpha_{x,y}$=0 β_x=0.25m β_y=0.25m λ1%bandwidth of 555nm aerogel1mm	0.1852009	0.1868234	0.1868273	14.2446314	11.9152394	11.9152381
15	$\epsilon_{nx,ny}$=1 $\alpha_{x,y}$=0 β_x=0.25m β_y=4m λ1%bandwidth of 555nm aerogel1mm	0.7405793	0.7409596	0.7409756	29.8903285	3.1789495	3.1789340
16	$\epsilon_{nx,ny}$=1 $\alpha_{x,y}$=0 β_x=0.25m β_y=16m λ1%bandwidth of 555nm aerogel1mm	1.4809827	1.4802813	1.4803128	59.2768380	2.2175278	2.2174474

1+2+5=
1+3+6=
1+4+7=

TABLE : Energy and time resolution at the three focal lines for various conditions e.g. electron beam twiss parameter and normalized emittance (unit of mm mrad), radiator aerogel thickness, and wavelength wavelength-dependent delay (in air and quartz window) for electron momentum pc of 2.4 MeV/c. ref e^- refers to reference electron which hits the radiator at $(x, y) = (0, 0)$.

resolution doesn't depend on x-emittance