Strong coupling of an electron ensemble on the surface of liquid helium to a microwave cavity

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Mission:
• sustainable development of Okinawa
• advancement of science and technology in Japan

People: 56 faculty members / 440 researchers / 134 graduate students

Expect 100 faculty members in 2024 (towards 300 faculty members)

https://www.oist.jp
Quantum Dynamics Unit (QDU)

Nuclear spins ensembles in MnCO₃

Impurity spins in diamond

Current experiments
Avoided crossing: **coupled potential wells**

Particle in two potential wells $U(x)$

$$H = \begin{pmatrix} E_L & 0 \\ 0 & E_R \end{pmatrix}$$

Add tunneling between two wells

$$H = \begin{pmatrix} E_L & g \\ g & E_R \end{pmatrix}$$

Decreasing depth of right well

$$E_{\pm} = \frac{E_L + E_R}{2} \pm \sqrt{\left(\frac{E_L - E_R}{2}\right)^2 + g^2}$$

[Landau and Lifshitz, Quantum Mechanics, Vol 2]
Avoided crossing: cavity QED

\[ H / \hbar = \omega_r a^+ a + \omega_s s_z + g (a s^+ + a^+ s^-) \]

Jaynes-Cummings Hamiltonian

Strong coupling regime: \( g \gg \gamma, \kappa \)

\( \omega_{21} = \omega_c \)

vacuum Rabi splitting

ground state, one photon

excited state, zero photons

[A. Wallraff et al., Nature 431 162 (2004)]
Avoided crossing: coupling to an ensemble of particles

\[ H / \hbar = \omega_r a^+ a + \omega_s \sum_{j=1}^{N} s_j^z + g \sum_{j=1}^{N} (a s_j^+ + a^+ s_j^-) \]

\[ \omega_s S_z \]
\[ g(aS^+ + a^+ S^-) \]

Enhancement of coupling by \( \sqrt{N} \) !!!

\[ |G\rangle = |\uparrow\uparrow\uparrow\ldots\rangle \]
\[ |E\rangle = \frac{1}{\sqrt{N}} \left( |\downarrow\uparrow\uparrow\ldots\rangle + |\uparrow\downarrow\uparrow\ldots\rangle + |\uparrow\uparrow\downarrow\ldots\rangle + \ldots \right) \]

\[ g_N = g \langle G | \hat{a}^+ \sum_{j=1}^{N} \hat{s}_j^- | E \rangle = g \sqrt{N} \]

Dicke model, 1954

[Y. Tabuchi et al., PRL 113, 083603 (2014)]
Applications: hybrid quantum computer

"memory"
spin system:
• difficult to manipulate
• coherence time is long

photons

"processor"
superconducting qubits:
• easy to manipulate
• coherence time is short

Ensemble-photon coupling

SC-qubit-photon coupling

[Y. Tabuchi et al., PRL 113, 083603 (2014)]

Coupling to ensembles: classical or quantum?

Coupled spring-mass oscillators

Coupled quantum oscillators

\[ \kappa \neq 0 \]

\[ \kappa = 0 \]

[\text{L. Novotny, Am. J. Phys. 78, 1199 (2010)}]

Macroscopic spin with

\[ \langle S \rangle = \frac{\hbar N}{2} \gg \hbar \]
Ensemble of electrons on liquid helium

Second Newton law:

\[ m_e \frac{d\vec{v}}{dt} = -e\vec{E}_{MW} - \frac{e}{c} \vec{v} \times \vec{B} - \nu m_e \vec{v} \]

Quantum treatment:

\[ \psi(\vec{r}) = C e^{ik_y y} H_n \left( \frac{x - X}{l_B} \right) \]

\[ j_\pm = \frac{n_s e^2}{m_e} \frac{E_\pm}{\nu - i(\omega \pm \omega_c)} \]

\[ \sigma_\pm = \frac{n_s e^2}{m_e} \frac{1}{M(\omega) - i(\omega \pm \omega_c)} \]

Resonant mode: TEM\(_{00q}\)
Frequency: 35-140 GHz
Quality factor: 1,000-10,000

\[ N \sim 10^8 \]

\[ \lambda / 4 \]
Experimental setup

Semi-confocal FP resonator
TEM\textsubscript{003} mode: 88.5 GHz
Quality factor Q=900

Abdurakhimov et al., PRL117, 056803 (2016)
Avoided crossing: cavity spectrum and electron response

Reflected MW power

Electron’s DC photoresponse

$\omega = \omega_c$

$\omega = \omega_r$

$2\gamma \approx 150 \text{ MHz}$
Quantum or classical??

2DEG in GaAs heterostructures:

\[ g \propto \sqrt{n_e \left( \frac{e^2}{\hbar c} \right)} \]


G. Scalari et al. Science 2012
Our classical model

Coupled equations for two oscillators

Current of 2D electrons: \( j_\pm = \sigma_\pm E_z^{\pm} \)

\[
\sigma_\pm = \frac{n_s e^2}{m_e} \frac{E_\pm}{\nu - i(\omega - \omega_c)}
\]

B.C. for magnetic field: \( H_{z=d_+} - H_{z=d_-} = j_e \)

\[
\begin{pmatrix}
\frac{D}{2c} (\omega - \omega_r + i\gamma) & i\eta_0/2 \\
\sigma_{xx} & 1
\end{pmatrix}
\begin{pmatrix}
E_{z=d} \\
j_s
\end{pmatrix}
= \begin{pmatrix}
E_{in} \\
0
\end{pmatrix}
\]

\[
\text{Det} \begin{pmatrix}
\frac{D}{2c} (\omega - \omega_r + i\gamma) & i\eta_0/2 \\
\sigma_{xx} & 1
\end{pmatrix} = 0
\]

\[
g_N = \sqrt{\frac{n_s e^2}{2m_e \varepsilon_0 D}} = \frac{e}{\hbar} \sqrt{\frac{\hbar}{m_e \omega_c}} \sqrt{\frac{\hbar \omega_c}{2 \varepsilon_0 V}} \sqrt{n_s A} = \frac{eB}{\hbar} E_{\text{rms}} \sqrt{N} \propto \sqrt{n_e \frac{e^2}{\hbar c}}
\]

"quantum" result
Comparison with experiment

Reflected power

$|S_{11}| = \frac{|E_{\text{out}}|}{|E_{\text{in}}|}^2$

DC photoresponse

$\text{Re}(Y) \, (\text{pS})$

AC current $j_e$

$\frac{|l_j|}{(\sigma \xi E_0)}$
**Superradians and Rabi oscillations**

**Hertz dipoles**

\[ Ee^{i(kz - \omega t)} \]

**Mirror**

\[ D \]

Vacuum Rabi oscillations?

\[
\frac{dj_e}{dt} + (i \omega + \nu) j_e = \frac{e^2 n_s}{m_e} E
\]

\[
E = -\eta_0 j_e
\]

\[
\frac{dj_e}{dt} + (i \omega + \Gamma) j_e = 0
\]

\[
\Gamma = \nu + \Gamma_s = \nu + \frac{\eta_0 e^2 n_s}{m_e}
\]

"Superradiant" decay

Qi Zhang et al. PRL, 047601 (2014)

**Frequency of (Rabi-like) oscillations:**

\[
g = \sqrt{\frac{\Gamma_s c}{2D}}
\]

Most recent experiment

Semi-confocal FP resonator
TEM_{003} mode: 35 GHz
Quality factor Q=7,000

\[ E = E^+ + E^- \]
± - right/left circular polarized

\[ j_e^\pm = \frac{n_s e^2 \tau}{m_e} \frac{E^\pm}{1 - i \tau (\omega \pm \omega_c)} \]
Most recent experiment

Looks like resonance induced by $E^+$ mode

$w_{i\rightarrow f} = \frac{2\pi}{\hbar} \langle f | H_{int} | i \rangle + \sum_m \frac{\langle f | H_{int} | m \rangle \langle m | H_{int} | i \rangle}{E_i - E_f} \left[ \delta(E_n - E_i - \hbar\omega) \right]$  

Ripplon-assisted (Raman) transitions

$E^-$

$E^+$
Conclusions

Strong coupling between electron cyclotron motion and a microwave cavity mode

Can be fully described on a completely classical ground

Two coupled oscillators = two coupled linear systems

Cavity QED experiments with electrons on helium? Need a nonlinearity!
Holstein-Primakoff bosons

Consider spin $S = \frac{N}{2}$ which has states $|S, S_z\rangle$

Apply (Holstein-Primakoff) transformation:

$$\hat{S}_z = S - \hat{b}^+ \hat{b}, \quad \hat{S}^+ = \sqrt{2S - \hat{b}^+ \hat{b}} \ \hat{b}, \quad \hat{S}^- = \hat{b}^+ \sqrt{2S - \hat{b}^+ \hat{b}}$$

Bosonic operator $\hat{b}^+(\hat{b})$ creates (annihilates) one spin excitation in N-spin system

In the low-excitation limit ($S - S_z << S$):

$$\hat{S}_z = S - \hat{b}^+ \hat{b}, \quad \hat{S}^+ = \sqrt{2S} \ \hat{b}, \quad \hat{S}^- = \hat{b}^+ \sqrt{2S}$$

Full Hamiltonian becomes:

$$\hat{H} / \hbar = \omega_r \hat{a}^+ \hat{a} - \omega_s \hat{b}^+ \hat{b} + g \sqrt{N} (\hat{a} \hat{b}^+ + \hat{a}^+ \hat{b})$$
Ensemble of two-level systems on helium surface

Use inter-subband resonance rather than CR

Dipole matrix element $\approx a_B \approx 10$ nm

Coupling to single electron:

$$g = e a_B E_{rms} = e a_B \sqrt{\frac{\hbar \omega_{21}}{2\varepsilon_0 V}} \approx 100 \text{ kHz}$$

Nonlinearity: Coulomb shift of energy levels
Cyclotron resonance harmonics

Yamashiro et al., PRL 115, 256802 (2015)

\[
\begin{align*}
\sigma_{xx} (\text{mS}) & \\
\sigma_{x\text{i}} (\text{mS}) & \\
\omega/\omega_c & \\
B (\text{T}) & \\
\end{align*}
\]