A404 Measurement
Report #4 (14/07/2016)
χ^2 test - Cosmic ray cloud chamber
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⋄Use a cloud chamber (Wilson chamber) to record the events of cosmic rays, and use χ^2 test to verify that it is Poisson distributed.

The rate of cosmic ray on earth is low, and its occurrences are (assumed) independent. Therefore, the we expect that its rates of appearances are a Poisson distribution. In this experiment, we measure the number of appearances per 10 seconds and get its distribution. Then, we will use χ^2 test to verify if it's a Poisson distribution.

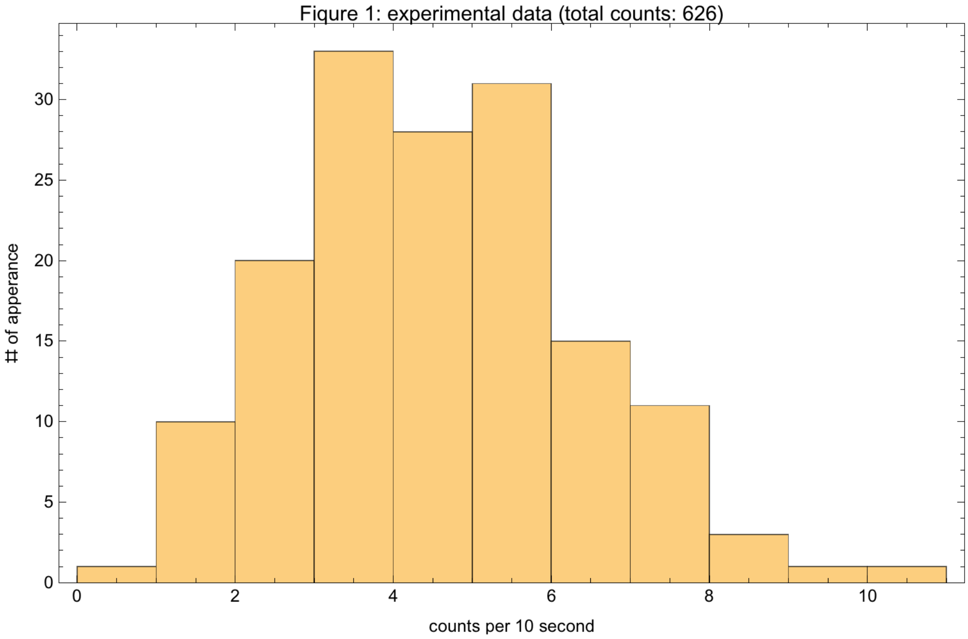
Data sets

```
In[287]:= data = {5, 4, 5, 3, 4, 4, 3, 4, 2, 3, 4, 4, 2, 6, 3, 1, 5, 1, 4, 3, 2, 3, 2, 6, 3,
    3, 5, 6, 5, 6, 1, 4, 4, 4, 4, 4, 3, 5, 6, 1, 3, 3, 2, 0, 1, 3, 2, 5, 3, 7, 2,
    5, 3, 1, 1, 3, 6, 5, 5, 3, 2, 4, 2, 2, 4, 4, 7, 6, 6, 6, 3, 3, 3, 5, 5, 5, 7,
    3, 5, 4, 3, 5, 4, 5, 4, 5, 3, 7, 4, 4, 5, 3, 7, 3, 4, 2, 5, 3, 2, 4, 5, 3, 6,
    2, 2, 5, 3, 6, 4, 9, 7, 6, 3, 6, 5, 5, 2, 5, 5, 1, 7, 7, 4, 3, 5, 8, 3, 8, 5,
    2, 2, 2, 4, 7, 7, 2, 4, 1, 2, 8, 7, 6, 5, 10, 5, 5, 3, 3, 4, 3, 6, 5, 4, 1};
    Ntotal = Total[data]
    mcount = N[Mean[data]]
    SDcount = N[√Mean[data]]
    x1 = mcount - SDcount
    x3 = mcount + SDcount

Out[288]= 626

Out[289]= 4.06494

Out[290]= 2.01617

Out[291]= 2.04877

Out[292]= 6.0811
```

Plot

In[62]:=
```
Histogram[data, Frame → True,
 FrameLabel → {"counts per 10 second", "# of apperance"},
 PlotLabel → "Fiqure 1: experimental data (total counts: 626)", ImageSize → 500]
```

Out[62]=



Discussion

Assume that the distribution is Poisson for cosmic rays. The average counts per 10 seconds here is $v = 4.06$. And for a Poisson distribution, its standard deviation is $\sigma = \sqrt{4.06} = 2.02$. Therefore, the expected Poisson distribution is $P(n,\ v = 4.06) = \frac{e^{-v}\, v^n}{n!}$. The expected number of counts n for $n_L < n < n_H$ is $N \times \sum_{n=n_L}^{n=n_H} P(n)$.

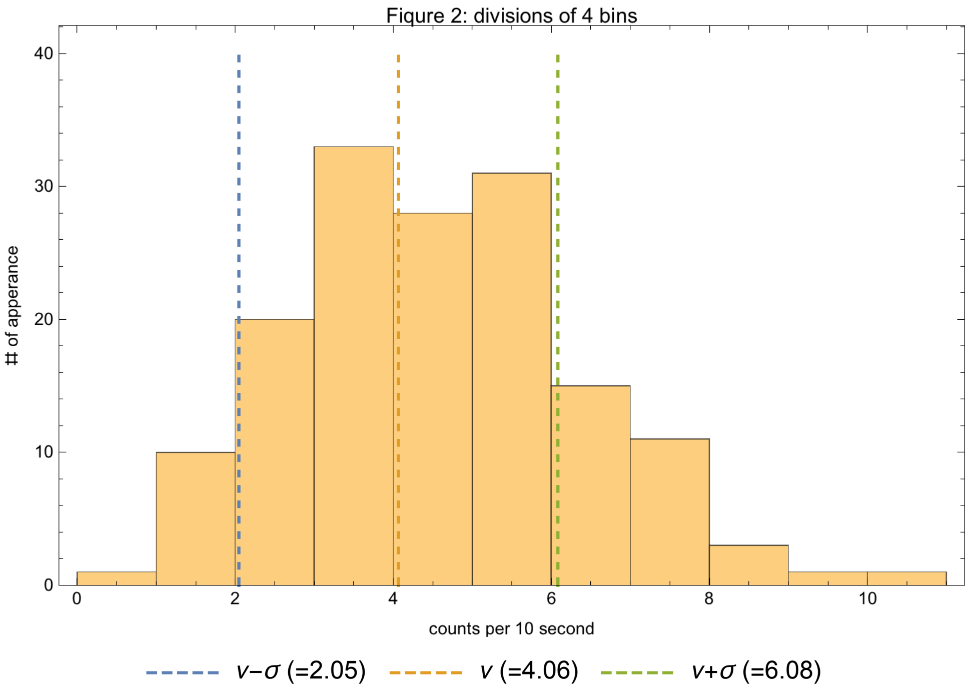
For χ^2-test, if we devided the distribution into 4 bins as $0 \sim (v - \sigma),\ (v - \sigma) \sim v,\ v \sim (v + \sigma),\ (v + \sigma) \sim \infty$. Then, the expected number of counts $E_k \equiv \{E_1, E_2, E_3, E_4\}$ is {35.23, 59.66, 41.01, 18.10}.

In[331]:=
$$Ek = Length[data] * \left\{\sum_{n=0}^{2} \frac{Exp[-mcount]\ mcount^n}{n!}, \sum_{n=3}^{4} \frac{Exp[-mcount]\ mcount^n}{n!}, \sum_{n=5}^{6} \frac{Exp[-mcount]\ mcount^n}{n!}, \sum_{n=7}^{20} \frac{Exp[-mcount]\ mcount^n}{n!}\right\}$$

Out[331]= {35.2263, 59.6612, 41.0099, 18.1026}

In[83]:=
```
p0 = Histogram[data, Frame → True,
   FrameLabel → {"counts per 10 second", "# of apperance"},
   PlotLabel → "Fiqure 2: divisions of 4 bins", ImageSize → 500];

p1 = ListLinePlot[
   {{{mcount - SDcount, 0}, {mcount - SDcount, 5}, {mcount - SDcount, 10},
     {mcount - SDcount, 15}, {mcount - SDcount, 20}, {mcount - SDcount, 25},
     {mcount - SDcount, 30}, {mcount - SDcount, 35}, {mcount - SDcount, 40}},
    {{mcount, 0}, {mcount, 5}, {mcount, 10}, {mcount, 15}, {mcount, 20},
     {mcount, 25}, {mcount, 30}, {mcount, 35}, {mcount, 40}},
    {{mcount + SDcount, 0}, {mcount + SDcount, 5}, {mcount + SDcount, 10},
     {mcount + SDcount, 15}, {mcount + SDcount, 20},
     {mcount + SDcount, 25}, {mcount + SDcount, 30},
     {mcount + SDcount, 35}, {mcount + SDcount, 40}}}, PlotStyle → Dashed,
   PlotLegends → Placed[{"ν-σ  (=2.05)", "ν  (=4.06)", "ν+σ  (=6.08)"}, Bottom]];
Show[
 p0,
 p1]
```

Out[85]=



The observed number of counts $O_k \equiv \{O_1, O_2, O_3, O_4\} = \{31, 61, 46, 16\}$

In[348]:=
```
sortdata = Sort[data];
B1 = sortdata[[1 ;; 31]];
B2 = sortdata[[32 ;; 92]];
B3 = sortdata[[93 ;; 138]];
B4 = sortdata[[139 ;; Length[data]]];
Ok = {Length[B1], Length[B2], Length[B3], Length[B4]}

Total[Table[(Ok[[i]] - Ek[[i]])^2 / Ek[[i]], {i, 1, Length[Ok]}]]
```

Out[353]= {31, 61, 46, 16}

Out[354]= 1.38851

$$\Rightarrow \begin{cases} O_k = \{31, 61, 46, 16\} \\ E_k = \{143.193, 242.519, 166.702, 73.5858\} \end{cases}$$

$\Rightarrow \chi^2 = \sum_{k=1}^{4} \frac{(O_k - E_k)^2}{E_k}$ =1.39

And the degree of freedom here is D=K-Constrains=4-3=1

$\Rightarrow$ the reduced χ^2, $\tilde{\chi}^2 = \frac{\chi^2}{D} = 1.39$

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x ²								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34

accept hypothesis | reject hypothesis | 1.39

According to the table of χ^2 – test, for D=1, the probability of $\tilde{\chi}^2$ = 1.39 is between 0.1~0.25. Assume that the criterion to reject a hypothesis is when the probability ≤0.05, i.e. $\tilde{\chi}^2 \geq 3.84$. Then, for this report, the hypothesis that the comic rays follow the Poisson distribution is acceptable.