I. NOISE CHARACTERISTICS OF AMPLIFIERS AND OTHER TWO-PORT DEVICES

Consider a typical radio-frequency (RF) measurement setup in Fig. 1. Here we excite our device-under-test (DUT), for example the quartz microbalance in your experiment, by an AC signal from generator and measure the device's response using a detector D. Very often, especially when you are dealing with detection of very small signals, you would like to use a low-noise amplifier (LNA) to boost you signal. Besides amplifying your signal by a factor G (which is called the *gain*), you also amplify the noise at the amplifier input. In addition, the amplifier itself add some noise of its own. Therefore, it is very important to know the noise characteristics of your amplifier. In this handout, we discuss the so called *noise figure* and other equivalent characteristics of the two-port devices.

A. Signal-to-noise ratio

As discussed in Handout 3: Noise in Electrical Circuits, any measured signal contains noise, that is random uncorrelated fluctuations of voltage or current in your circuit. Signal-to-noise ratio (SN) is the ratio of the average power S of your signal to the average power N of noise, both dissipated in a matched load. There are several reasons why it is convenient to talk about power rather than amplitude of the signal when dealing with RF measurements. First, remember that usually the RF detectors are *quadratic detectors*, that is their output voltage is proportional to the square of amplitude, that is power, of the input signal. Second,

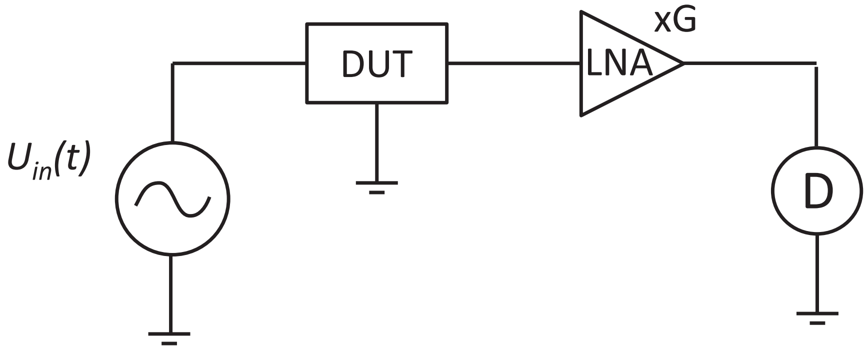

FIG. 1: Typical RF measurement setup which includes a low-noise amplifier (LNA).

as we discussed previously it is convenient to characterize the random noise by its *dispersion*, that is the mean square voltage, which is proportional to the noise average power. Thus, in our definition the noise figure is the power ratio

$$\mathrm{SN} = \frac{S}{N}. \tag{1}$$

B. Noise figure

Now, consider a *two-port network*, that is an electronic device with one input and one output terminals. By definition, the noise figure (NF) of the device is the ratio of the signal-to-noise power ratio at the device input to the signal-to-noise power ratio at its output

$$\mathrm{NF} = \frac{\mathrm{SN}_i}{\mathrm{SN}_o} = \frac{S_i/N_i}{S_o/N_0}. \tag{2}$$

Note that although formally NF is defined for a two-port device, very often it is used (and actually is very useful) for other devices, for example mixers (which is a tree-port device). In this case, the above definition assumes that a constant specified LO power is applied to the LO port of the mixer, and the input and output ports are RF and IF ports.

Now, consider an ideal amplifier having gain G, see Fig. 2(a). Again, for RF amplifiers the gain is usually defined as the ratio of the power of the output signal to the power of the input signal, both dissipated in the matched load. Note that for the most of RF devices, the matched load is 50 Ohm. In the case of an *ideal* amplifier, the output power will be just an input power multiplied by G. Obviously, the noise figure of such an amplifier will be just 1. In the case of a real amplifier, the output will also contain some noise power N_a added by

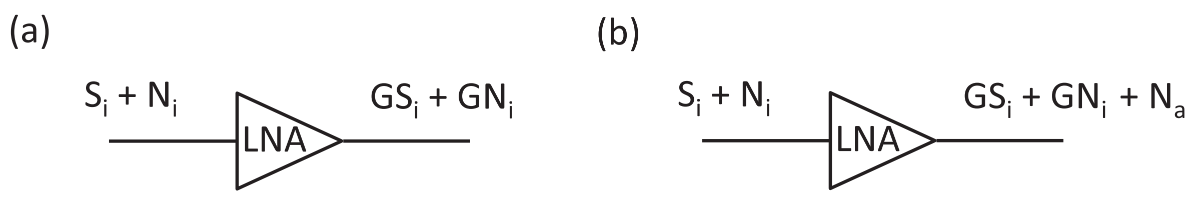

FIG. 2: Input and output of an ideal (a) and real (b) amplifier.

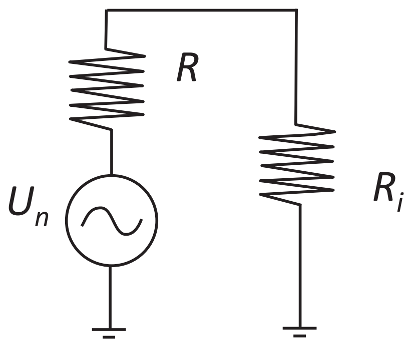

FIG. 3: Thermal noise source loaded onto the input impedance of amplifier R_i.

the amplifier itself, see Fig. 2(b). Using Eq. (2) it is easy to rewrite the noise figure as

$$\mathrm{NF} = \frac{GN_i + N_a}{GN_i}. \tag{3}$$

Thus, the noise figure of any real amplifier is always larger than 1. Very often it is expressed in dB units, in which case we can say that NF of a real amplifier is always larger than 0 dB. Note that the noise figure of amplifier does not depend on the signal power S_i.

C. Thermal noise power

Very often, the input noise to the amplifier is the thermal (Johnston) noise due to the impedance of your DUT. Let's show that the average power of the thermal noise delivered into a matched load is given by $k_B TB$, where T is the temperature of DUT, k_B is the Boltzmann's constant, and B is the bandwidth of your setup. Indeed, consider Figure 3. The source of the thermal noise can be represented as an ideal generator producing RMS voltage $V_n = \sqrt{4Rk_B TB}$ (see the Johnston noise in Handout 3: Noise in electrical circuits) in series with noiseless impedance R. This is connected in series with the input impedance of your amplifier R_i. Suppose that the impedances are perfectly matched, $R_i = R$. The current I in the circuit is

$$I = \frac{V_n}{2R} = \sqrt{\frac{k_B TB}{R}}, \tag{4}$$

thus the noise power dissipated in R_i is

$$N_i = I^2 R_i = k_B T B. \tag{5}$$

Note that the power delivered into the matched load does not depend on the value of R.

Exercise

Derive the thermal noise power delivered into an arbitrary load impedance, that is $R_i \neq R$. Show that the maximum power is delivered when $R_i = R$.

Using the above result, the noise figure is often given in the literature by

$$\mathrm{NF} = \frac{G k_B T B + N_a}{G k_B T B}. \tag{6}$$

By convention, the ambient (room) temperature is taken to be 290 K. It is easy to calculate that the noise average power given by (5) is exactly 4.00×10^{-21} W/Hz, or equivalently -174 dBm/Hz. Thus, if you know the gain G, the bandwidth B, and the noise figure NF of your amplifier, you can estimate the amplifier noise N_a using Eq. (6).

D. Noise temperature

As is clear from Eq. (3) or (6), the noise figure of the amplifier is completely determined by the noise N_a produced by the amplifier itself. Very often, the manufactures choose to provide this number instead of NF. In this case, they use the so called *effective noise temperature* T_n defined by

$$T_n = \frac{N_a}{G k_B B}. \tag{7}$$

In other words, T_n is the temperature of a source impedance that, if connected to the input of an ideal (noiseless) amplifier, would produce the noise power N_a at the amplifier output. From Eq. (6), it is easy to obtain a very simple relation between NF and T_n: $T_n = T(\mathrm{NF}-1)$.

E. NF of amplifier cascades

Very often you want to use several amplifiers connected in series (or as we say in *cascade*), see Fig. 4. In this case, it is straightforward to show that the noise figure of the entire

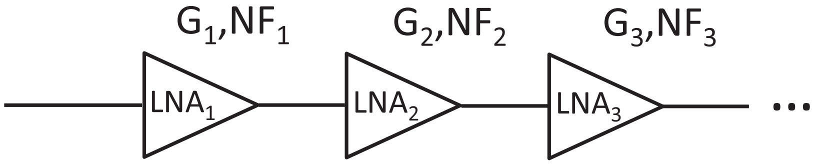

FIG. 4: Amplifier cascade.

cascade is given by

$$\mathrm{NF} = \mathrm{NF}_1 + \frac{\mathrm{NF}_2 - 1}{G_1} + \frac{\mathrm{NF}_3 - 1}{G_1 G_2} + ... \tag{8}$$

This is very interesting and important relation to remember. For example, consider a cascade of several amplifiers. Obviously, the first term on the right side of Eq. (8) will by far dominate providing that the gain of the first amplifier is sufficiently large, and the noise figures of the other amplifiers is not too large. Thus, the noise figure of the entire cascade is dominated by that of the first amplifier! For this reason, you usually would like to use a high-gain low-noise (cryogenic, if possible) amplifier (very often we call it the *preamplifier*) as the first stage of your cascade.

F. How to measure the noise figure

Besides commercially available plug-and-read Noise Figure Analyzers, there are several other methods to measure the noise figure of your amplifier in the lab. One popular way is to use (also commercially available) Noise Sources. This are usually based on semiconductors and, when biased with some DC voltage, can produce a very carefully calibrated noise average powers equivalent to 290-10,000 K thermal noise. By plugging this source to your amplifier input, and measuring the amplifier output P_{out}, you can plot the linear dependence $P_{out} = N_a + G k_B T_s B$ for several (usually just two) different values of the source temperature T_s from the above range, and find N_a from the intersection of the plotted straight line with the vertical axis.

Another way is to try to measure the amplifier noise N_a directly using, e.g. a calibrated quadratic detector. Of cause, the sensitivity of your standard quadratic detector could be by far not enough to detect N_a. In this case, you can use a trick by connecting several amplifiers in cascade, and use the fact that the noise figure will be dominated by the first amplifier in the cascade. This way you can amplify N_a to the level detectable by your detector. We will try this method in the lab.