Spontaneous Symmetry Breaking in Tensor Theories

arXiv:1809.10153 , arXiv:1810.02520, **PD, J.A. Rosabal**

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Fields, Gravity and Strings, IBS

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Three concepts

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- Tensor theories.

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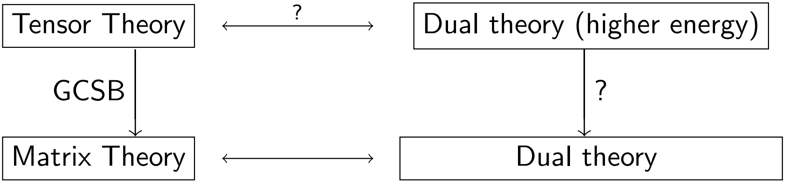

Generalized chiral symmetry breaking

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Generalized chiral symmetry breaking

$$\begin{array}{ccc} U_R(N) \times U_L(N) & \longrightarrow & U_{R+L}(N), \\ (\ g_R\ ,\ g_L^\dagger\ ) & \longrightarrow & (g, g^\dagger) \end{array}$$

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SSB patterns for $d = 2$ (I)

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In usual QCD

$$\begin{array}{c} U_1(N) \\ | \\ U_{\bar{1}}(N) \end{array} \longrightarrow \mathrm{Diag}[U_1(N) \times U_{\bar{1}}(N)]$$

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For $d = 2$ we have three options.

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For $d = 2$ we have three options. First,

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Physics should be independent of the initial color assignment $\rightarrow$ identification of second and third pattern.

SSB patterns for $d = 3$

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Representatives of the equivalence class of SSB patterns for $d = 3$:

1. $\mathrm{Diag}[U_1(N) \times U_{\bar{1}}(N)] \times \mathrm{Diag}[U_2(N) \times U_{\bar{2}}(N)] \times \mathrm{Diag}[U_3(N) \times U_{\bar{3}}(N)]$

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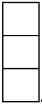
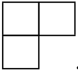
1. $\rightarrow$ , 2. $\rightarrow$ , 3. $\rightarrow$ 

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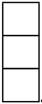
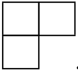
1. $\rightarrow$ , 2. $\rightarrow$ , 3. $\rightarrow$ 

SSB patterns $\longleftrightarrow$ partitions of d

Notation for partitions

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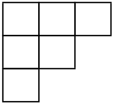
$\mu =$ 

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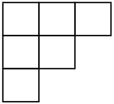
$\mu =$ 

- $|\mu| = 6.$
- $l(\mu) = 3.$
- $\mu_1 = 3,\ \mu_2 = 2,\ \mu_3 = 1.$

SSB patterns for general d

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Generalized chiral symmetry breaking patterns

$$G_{d\bar{d}}(N) = \prod_{k=1}^{d} [U_k(N) \times U_{\bar{k}}(N)] \longrightarrow G_{\mu}(N) = \prod_{\alpha=1}^{l(\mu)} \mathrm{Diag}[H_{\alpha}]$$

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where

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- $H_\alpha \cap H_\beta = \emptyset, \qquad \alpha \neq \beta$.
- $\mathrm{Diag}[H_\alpha]$ is a unitary group $\rightarrow U_\alpha(N)$.
- Number of Goldstone modes $\big(2d - l(\mu)\big) N^2$.

Inducing SSB: ϵ-term method (I)

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Method to induce SSB for scalar field using path integrals **[Matsumoto, Papastamatiou and Umezawa'74]**.

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$$Z_\epsilon\left[J,\overline{J}\right] = \frac{1}{\mathcal{N}}\int D\Phi D\overline{\Phi} \exp\left(\mathrm{i}\int d^4x \Big\{\mathcal{L}\left[\Phi(x)\right] + \overline{J}(x)\cdot\Phi(x) + J(x)\cdot\overline{\Phi}(x) + \mathrm{i}\epsilon|\Phi(x) - v|^2\Big\}\right),$$

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- Under $G_{d\bar{d}}(N)$: $\mathcal{L}[\Phi(x)] = \mathcal{L}[\Phi'(x)]$.
- $v \cdot \Phi(x)$ invariant under $G_\mu(N)$.
- $\epsilon \to 0$ at the end of the calculations.

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- Parametrize the elements of $U_k(N)$ and $U_{\bar{k}}(N)$ as $g_k = e^{\mathrm{i}\theta_a^k T_a}, \quad g_{\bar{k}} = e^{\mathrm{i}\theta_a^{\bar{k}} T_a}, \quad a = 1,\ldots,N^2, \quad k,\bar{k} = 1,\ldots,d.$
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- We derivate these identities with respect to $J(y)$ and $\overline{J}(y)\rightarrow$ identities between different Green functions.

Goldstone modes

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- The first non-trivial identity

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- $m_{B_a^k} = 0 \to B_a^k(x)$ are Goldstone modes.
- The definition of the Goldstone modes is general, valid for different symmetry groups and SSB patterns.

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- **Define** $(B^k)^i_j(x) = \sum_a B^k_a(x)(T_a)^i_j$.

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$$\sum_{\alpha=1}^{l(\mu)} (2\mu_\alpha - 1) = 2d - l(\mu)$$

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$$\mu = \text{ cycle structure of } \sigma'\sigma^{-1}.$$

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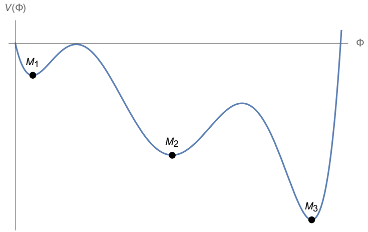

Thanks!