Universality classes in Rank-3 Tensor models

Johannes Lumma

OIST, 2018

In collaboration with
A. Eichhorn, T. Koslowski
and A. D. Pereira

arXiv: 1811.XXXX

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

Emmy
Noether-
Programm

DFG Deutsche Forschungsgemeinschaft

MOTIVATION

- Connections to Quantum Gravity
- Melons are branched polymers [Gurau, Ryan, 2013]
- Different continuum limits might exist (?)

MOTIVATION

- Connections to Quantum Gravity
- Melons are branched polymers (?) [Gurau, Ryan, 2013]
- Different continuum limits might exist

FRG as a tool to discover universality classes

Message: FRG as a tool to discover new universality classes

Outline

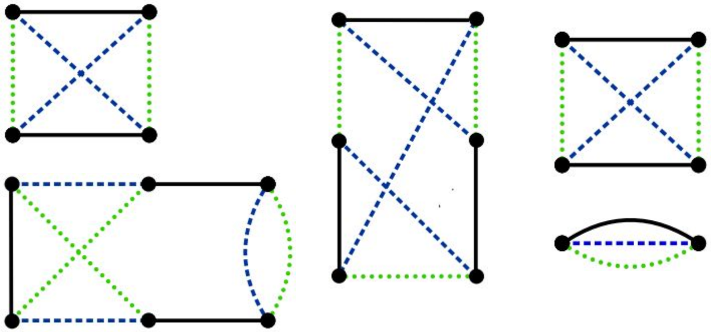
- The stage a.k.a. the model
- Apply the FRG
- Results
- Conclusion

The Model

- Consider a real rank-3 Tensor model s.t.

$$T_{a_1 a_2 a_3} \to O^{(1)}_{a_1 b_1} O^{(2)}_{a_2 b_2} O^{(3)}_{a_3 b_3} T_{b_1 b_2 b_3}$$

- Enlarged Theory space compared to $U(N)^{\otimes 3}$



etc.

What kind of Universality classes do such models have?

Let's find out using the FRG

Applying the FRG

- Coarse-graining in *N* [Eichhorn, Koslowski, 2013]

$$\partial_t \Gamma_N = \frac{1}{2} \mathrm{Tr} \left(\Gamma_N^{(2)} + R_N \right)^{-1} \partial_t R_N$$

⇒ Flow of the Effective Action Γ_N

⇒ Alternative to path-integral formulation

Applying the FRG

- Coarse-graining in *N* [Eichhorn, Koslowski, 2013]

$$\partial_t \Gamma_N = \frac{1}{2} \mathrm{Tr} \left(\boxed{\Gamma_N^{(2)}} + \boxed{R_N} \right)^{-1} \partial_t R_N$$

$$\Gamma_N^{(2)} = \frac{\delta^2 \Gamma_N}{\delta T_{a_1 a_2 a_3} \delta T_{b_1 b_2 b_3}}$$

IR-Regulator

How to cook with the FRG - A recipe

Ingredients:

- A truncation of the effective action
  - *Amount*: Already small truncations can yield viable results
- A regulator R_N

1) $a_1 + a_2 + a_3 < N: \quad R_N > 0$

2) $N < a_1 + a_2 + a_3: \quad R_N = 0$

3) $N \to N' \to \infty: \quad R_N \to \infty$

Instructions:

- Compute $\Gamma_N^{(2)}$
- Specify R_N
- Cook β-fcts (or compute) using FRG
- Fix scaling of couplings

Ingredient I: TRUNCATION

- FRGE cannot be solved exactly

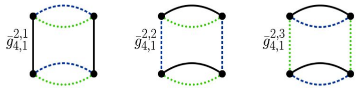
→ Need approximation/ truncation

Ingredient I: TRUNCATION

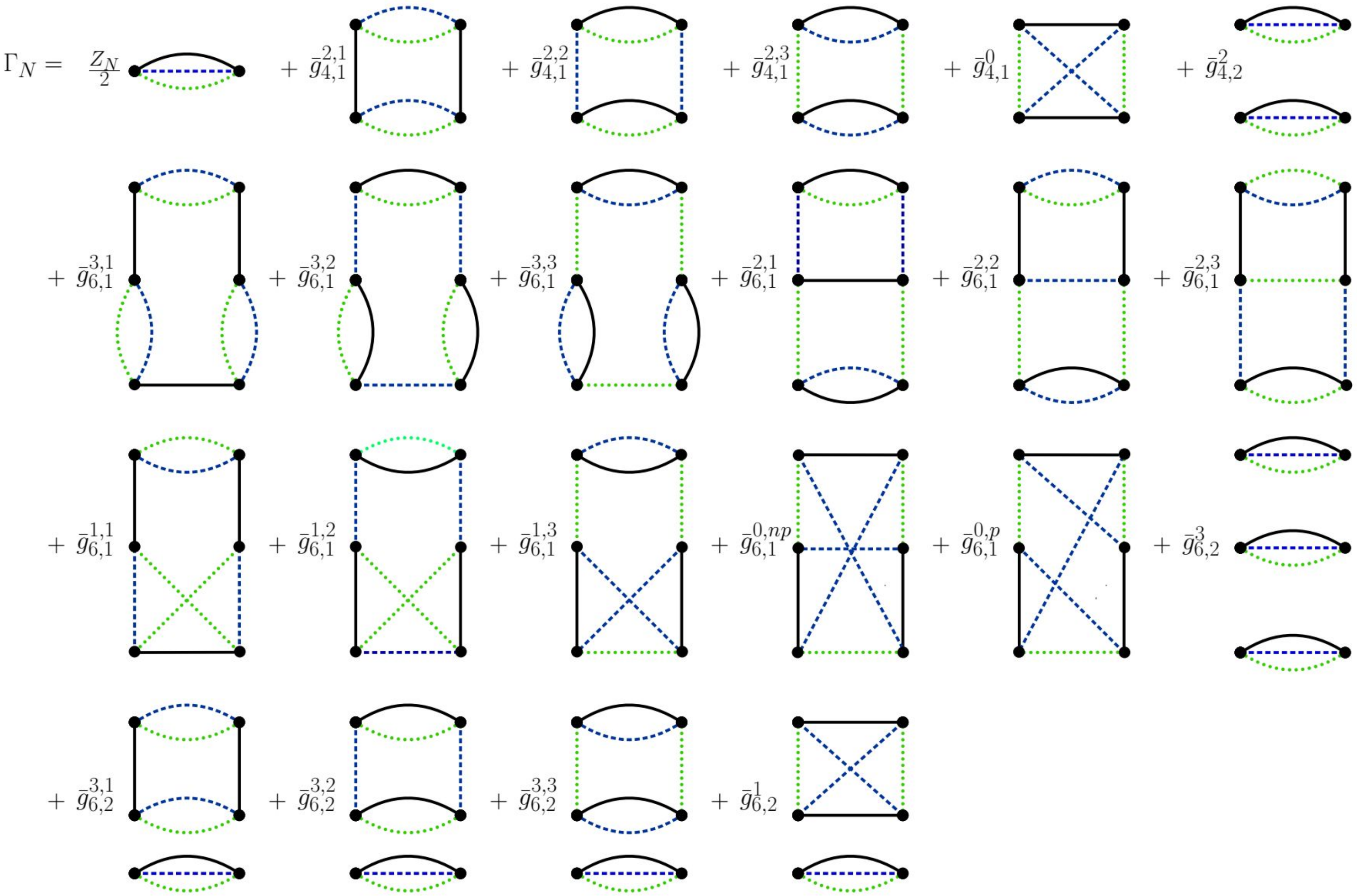
- FRGE cannot be solved exactly

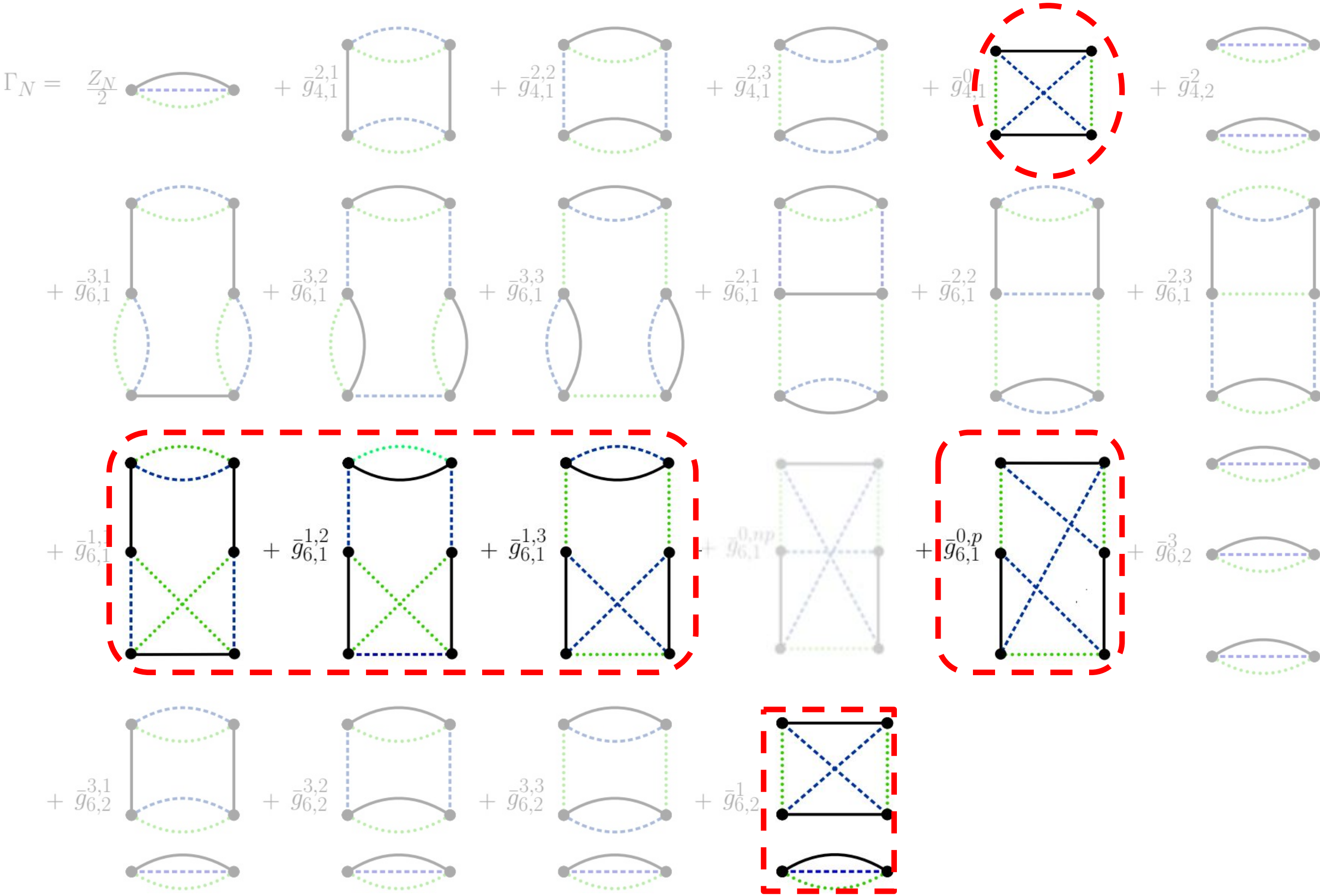
  → Need approximation/ truncation

- Include ALL interactions allowed by symmetry up to 6th order in T



- Dinstinguish interactions by prefered color


$\Gamma_N = \frac{Z_N}{2}$
$+ \bar{g}^{2,1}_{4,1}$
$+ \bar{g}^{2,2}_{4,1}$
$+ \bar{g}^{2,3}_{4,1}$
$+ \bar{g}^{0}_{4,1}$
$+ \bar{g}^{2}_{4,2}$
$+ \bar{g}^{3,1}_{6,1}$
$+ \bar{g}^{3,2}_{6,1}$
$+ \bar{g}^{3,3}_{6,1}$
$+ \bar{g}^{2,1}_{6,1}$
$+ \bar{g}^{2,2}_{6,1}$
$+ \bar{g}^{2,3}_{6,1}$
$+ \bar{g}^{1,1}_{6,1}$
$+ \bar{g}^{1,2}_{6,1}$
$+ \bar{g}^{1,3}_{6,1}$
$+ \bar{g}^{0,np}_{6,1}$
$+ \bar{g}^{0,p}_{6,1}$
$+ \bar{g}^{3}_{6,2}$
$+ \bar{g}^{3,1}_{6,2}$
$+ \bar{g}^{3,2}_{6,2}$
$+ \bar{g}^{3,3}_{6,2}$
$+ \bar{g}^{1}_{6,2}$


$\Gamma_N = \frac{Z_N}{2}$
$+\ \bar{g}^{2,1}_{4,1}$
$+\ \bar{g}^{2,2}_{4,1}$
$+\ \bar{g}^{2,3}_{4,1}$
$+\ \bar{g}^{0}_{4,1}$
$+\ \bar{g}^{2}_{4,2}$
$+\ \bar{g}^{3,1}_{6,1}$
$+\ \bar{g}^{3,2}_{6,1}$
$+\ \bar{g}^{3,3}_{6,1}$
$+\ \bar{g}^{2,1}_{6,1}$
$+\ \bar{g}^{2,2}_{6,1}$
$+\ \bar{g}^{2,3}_{6,1}$
$+\ \bar{g}^{1,1}_{6,1}$
$+\ \bar{g}^{1,2}_{6,1}$
$+\ \bar{g}^{1,3}_{6,1}$
$+\ \bar{g}^{0,np}_{6,1}$
$+\ \bar{g}^{0,p}_{6,1}$
$+\ \bar{g}^{3}_{6,2}$
$+\ \bar{g}^{3,1}_{6,2}$
$+\ \bar{g}^{3,2}_{6,2}$
$+\ \bar{g}^{3,3}_{6,2}$
$+\ \bar{g}^{1}_{6,2}$

Ingredient II: Regulator

$$\boxed{R_N(\{a_i\},\{b_i\}) = Z_N\, \delta_{a_1 b_1}\, \delta_{a_2 b_2}\, \delta_{a_3 b_3} \left(\frac{N^r}{a_1+a_2+a_3} - 1\right) \theta\left(\frac{N^r}{a_1+a_2+a_3} - 1\right)}$$

$$a_1 + a_2 + a_3 < N: \quad R_N > 0$$

$$N < a_1 + a_2 + a_3: \quad R_N = 0$$

$$N \to N' \to \infty: \quad R_N \to \infty$$

No notion of mass dimensionality

$\Longrightarrow$ All choices $r \geq 0$ seem to be allowed

Hundreds of zeros of the beta functions are generated

How to distinguish Physical FP from Unphysical zero?

Trusting the Fixed Point?

- Ensure stability of results by successively increasing truncation
    1. Starting Scheme: Quartic Order with $-\frac{N\partial_N Z_N}{Z_N} \equiv \eta = 0$
    2. Quartic Order with polynomial approximation for η
    3. Quartic Order with full non-polynomial η
    4. Sixth Order, same procedure for η
- Regulator bound $\eta < r$
- Assumption: Canonical guiding principle

Trusting the Fixed Point?

- Ensure stability of results by successively increasing truncation
  1. Starting Scheme: Quartic Order with $-\frac{N\partial_N Z_N}{Z_N} \equiv \eta = 0$
  2. Quartic Order with polynomial approximation for η
  3. Quartic Order with full non-polynomial η
  4. Sixth Order, same procedure for η
- Regulator bound $\eta < r$
- Assumption: Canonical guiding principle

Regulator Bound($\eta < r$) [Meibohm, Pawlowski, Reichert, 2015]

- Rewrite anomalous dimension
  - $\eta = \frac{-N\partial_N Z_N}{Z_N} \implies Z_N \sim N^{-\eta}$
- Consider now our regulator
  - $R_N(\{a_i\},\{b_i\}) = Z_N\, \delta_{a_1 b_1}\delta_{a_2 b_2}\delta_{a_3 b_3}\left(\frac{N^r}{a_1+a_2+a_3} - 1\right)\theta\left(\frac{N^r}{a_1+a_2+a_3} - 1\right)$
- Consider $N \to N' \to \infty$
  - $\lim_{N\to N'\to\infty} R_N \sim Z_N\, N^r \sim N^{r-\eta}$
- Regulator needs to diverge in that limit

N: IR-cutoff

N': UV-cutoff

$\Longrightarrow$ $\boxed{\eta < r}$

Trusting the Fixed Point?

- Ensure stability of results by successively increasing truncation

1. Starting Scheme: Quartic Order with $-\frac{N\partial_N Z_N}{Z_N} \equiv \eta = 0$
2. Quartic Order with polynomial approximation for η
3. Quartic Order with full non-polynomial η
4. Sixth Order, same procedure for η

- Regulator bound $\eta < r$

- Assumption: Canonical guiding principle

Trusting the Fixed Point?

- Ensure stability of results by successively increasing truncation

  1. Starting Scheme: Quartic Order with $-\frac{N\partial_N Z_N}{Z_N} \equiv \eta = 0$
  2. Quartic Order with polynomial approximation for η
  3. Quartic Order with full non-polynomial η
  4. Sixth Order, same procedure for η

- Regulator bound $\eta < r$
- Assumption: Canonical guiding principle

Canonical Guiding Principle

- Idea: Increasing truncation should not induce new relevant directions

$$\left[\bar{g}^{0}_{4,1}\right] = -3/2 \quad \left[\bar{g}^{2,i}_{4,1}\right] = -2 \quad \left[\bar{g}^{2}_{4,2}\right] = -3$$

$$\left[\bar{g}^{1,i}_{6,1}\right] = -7/2 \quad \left[\bar{g}^{2,i}_{6,1}\right] = -4 \quad \left[\bar{g}^{3,i}_{6,1}\right] = -4$$

$$\left[\bar{g}^{0,np}_{6,1}\right] = -3 \quad \left[\bar{g}^{0,p}_{6,1}\right] = -3 \quad \left[\bar{g}^{3,i}_{6,2}\right] = -5$$

$$\left[\bar{g}^{1}_{6,2}\right] = -9/2 \quad \left[\bar{g}^{3}_{6,3}\right] = -6$$

$$\Longrightarrow \max(\theta) - \max(d_{\bar{g}}) \leq 5$$

All couplings with scaling dimension > -5 are included

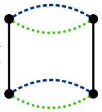
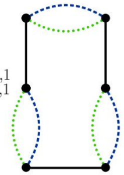
Next coupling with largest scaling dimension $[\bar{g}^{0}_{8,1}] = -5$

$$\xrightarrow{\max(d_{\bar{g}}) = -3/2} \quad \boxed{\max(\theta) \leq 3.5}$$

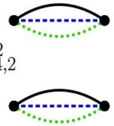
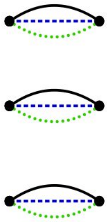
Results (r=1)

Dimensionally-reduced universality classes

- Cyclic Melonic singletrace FP

$\bar{g}_{4,1}^{2,1}$  $\bar{g}_{6,1}^{3,1}$ 

- Multitrace Bubble FP

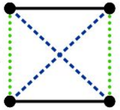
$\bar{g}_{4,2}^{2}$  $\bar{g}_{6,3}^{3}$ 

- Cyclic Melonic Multitrace FP

Cyclic Melons & Multi-Trace $\neq 0$

Candidates for universality classes for 3d Quantum Gravity

- Isocolored FP with tetrahedral interaction

$\bar{g}_{4,1}^{0}$ 

& No preferred color

Results (r=1)

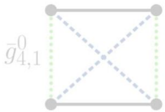
Dimensionally-reduced universality classes

- Cyclic Melonic Singletrace FP
- Multitrace Bubble FP
- Cyclic Melonic Multitrace FP

Cyclic Melonic & Multi-Trace $\neq 0$

Candidates for universality classes for 3d Quantum Gravity

- Isocolored FP with tetrahedral interaction



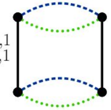
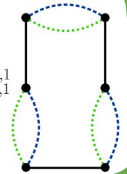
& No preferred color

FRG as a tool to discover Universality classes

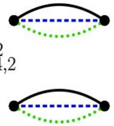
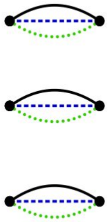
Results (r=1)

Dimensionally-reduced universality classes

- Cyclic Melonic singletrace FP

$\bar{g}^{2,1}_{4,1}$  $\bar{g}^{3,1}_{6,1}$ 

- Multitrace Bubble FP

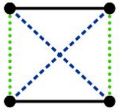
$\bar{g}^{2}_{4,2}$  $\bar{g}^{3}_{6,3}$ 

- Cyclic Melonic Multitrace FP

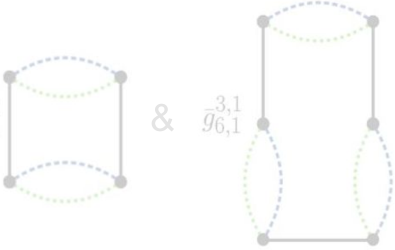
Cyclic Melons & Multi-Trace $\neq 0$

Candidates for universality classes for 3d Quantum Gravity

- Isocolored FP With tetrahedral interaction

$\bar{g}^{0}_{4,1}$ 

& No preferred color



$\bar{g}^{2,1}_{4,1}$ & $\bar{g}^{3,1}_{6,1} \neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0$ —

Stable under extension of truncation

$g^{2,1\,*}_{4,1} = -1$ $\theta_1 = 2$ $g^{2,1\,*}_{4,1} = -0.46$ $\theta_1 =$

$\theta_2 = 1$ $g^{3,1\,*}_{6,1} = -0.50$ $\theta_2 =$

Regulator Bound η

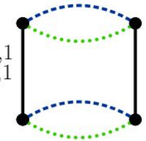
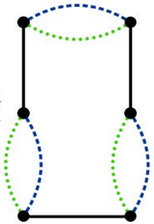
$g^{2,1\,*}_{4,1} = -0.43$ $\theta_1 = 2$ $g^{2,1\,*}_{4,1} = -0.30$ $\theta_1 =$

$g^{3,1\,*}_{6,1} = -0.18$

Canonical Guiding Principle η

$g^{2,1\,*}_{4,1} = -0.38$ $\theta_1 = 2.27$ $g^{2,1\,*}_{4,1} = -0.28$ $\theta_1 = 2.19$

$\theta_2 = -0.16$ $g^{3,1\,*}_{6,1} = -0.15$ $\theta_2 = -0.03$

$\bar{g}^{2,1}_{4,1}$  & $\bar{g}^{3,1}_{6,1}$  $\neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g^{2,1\,*}_{4,1} = -1$ $\quad\quad$ $\theta_1 = 2$

$\theta_2 = 1$

2) Quartic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.43$ $\quad\quad$ $\theta_1 = 2$

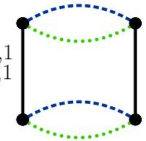
$\theta_2 = -0.14$

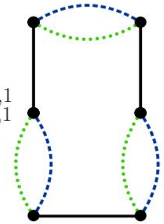
3) Quartic Order with full η

$g^{2,1\,*}_{4,1} = -0.38$ $\quad\quad$ $\theta_1 = 2.27$

$\theta_2 = -0.16$

Cyclic Melonic Singletrace FP



&


$\neq 0$

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g_{4,1}^{2,1\,*} = -1$

$\theta_1 = 2$

$\theta_2 = 1$

2) Quartic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.43$

$\theta_1 = 2$

$\theta_2 = -0.14$

3) Quartic Order with full η

$g_{4,1}^{2,1\,*} = -0.38$

$\theta_1 = 2.27$

$\theta_2 = -0.16$

4) Hexic Order with $\eta = 0$

$g_{4,1}^{2,1\,*} = -0.46$

$g_{6,1}^{3,1\,*} = -0.50$

$\theta_1 = 2$

$\theta_2 = 0.53$

5) Hexic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.30$

$g_{6,1}^{3,1\,*} = -0.18$

$\theta_1 = 2$

$\theta_2 = -0.03$

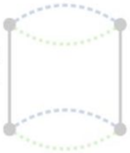
6) Hexic Order with full η

$g_{4,1}^{2,1\,*} = -0.28$

$g_{6,1}^{3,1\,*} = -0.15$

$\theta_1 = 2.19$

$\theta_2 = -0.03$

$\bar{g}^{2,1}_{4,1}$  & $\bar{g}^{3,1}_{6,1}$  $\neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Stable under extension of truncation ✓

$g^{2,1\,*}_{4,1} = -1$ $\theta_1 = 2$ $\theta_2 = 1$

$g^{2,1\,*}_{4,1} = -0.46$ $g^{3,1\,*}_{6,1} = -0.50$ $\theta_1 = 2$ $\theta_2 = 0.53$

2) Quartic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.43$ $\theta_1 = 2$ $\theta_2 = -0.14$

3) Quartic Order with full η

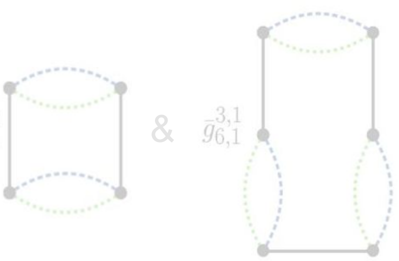
$g^{2,1\,*}_{4,1} = -0.38$ $\theta_1 = 2.27$ $\theta_2 = -0.16$

5) Hexic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.30$ $g^{3,1\,*}_{6,1} = -0.18$ $\theta_1 = 2$ $\theta_2 = -0.03$

6) Hexic Order with full η

$g^{2,1\,*}_{4,1} = -0.28$ $g^{3,1\,*}_{6,1} = -0.15$ $\theta_1 = 2.19$ $\theta_2 = -0.03$



$\bar{g}^{2,1}_{4,1}$ & $\bar{g}^{3,1}_{6,1}$ $\neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Stable under extension of truncation ✓

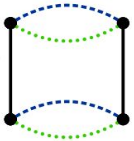
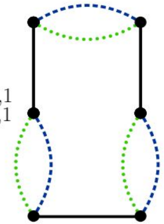
$g^{2,1\,*}_{4,1} = -1$ $\theta_1 = 2$ $\theta_2 = 1$ | $g^{2,1\,*}_{4,1} = -0.46$ $g^{3,1\,*}_{6,1} = -0.50$ $\theta_1 =$ $\theta_2 =$

Regulator Bound

$g^{2,1\,*}_{4,1} = -0.43$ $\theta_1 = 2$ | $g^{2,1\,*}_{4,1} = -0.30$ $= -0.18$ $\theta_1 =$

Canonical Guiding Principle

$g^{2,1\,*}_{4,1} = -0.38$ $\theta_1 = 2.27$ $\theta_2 = -0.16$ | $g^{2,1\,*}_{4,1} = -0.28$ $g^{3,1\,*}_{6,1} = -0.15$ $\theta_1 = 2.19$ $\theta_2 = -0.03$

$\bar{g}^{2,1}_{4,1}$  & $\bar{g}^{3,1}_{6,1}$  $\neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g^{2,1\,*}_{4,1} = -1$ $\quad \theta_1 = 2$, $\theta_2 = 1$ $\quad \eta = 0 < 1$

2) Quartic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.43$ $\quad \theta_1 = 2$, $\theta_2 = -0.14$ $\quad \eta = -0.57 < 1$

3) Quartic Order with full η

$g^{2,1\,*}_{4,1} = -0.38$ $\quad \theta_1 = 2.27$, $\theta_2 = -0.16$ $\quad \eta = -0.58 < 1$

4) Hexic Order with $\eta = 0$

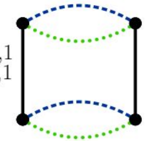
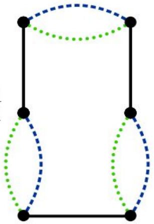
$g^{2,1\,*}_{4,1} = -0.46$, $g^{3,1\,*}_{6,1} = -0.50$ $\quad \theta_1 = 2$, $\theta_2 = 0.53$

5) Hexic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.30$, $g^{3,1\,*}_{6,1} = -0.18$ $\quad \theta_1 = 2$, $\theta_2 = -0.03$

6) Hexic Order with full η

$g^{2,1\,*}_{4,1} = -0.28$, $g^{3,1\,*}_{6,1} = -0.15$ $\quad \theta_1 = 2.19$, $\theta_2 = -0.03$

$\bar{g}^{2,1}_{4,1}$  & $\bar{g}^{3,1}_{6,1}$  $\neq 0$

Cyclic Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g^{2,1\,*}_{4,1} = -1$ $\quad \theta_1 = 2$ $\quad \theta_2 = 1$ $\quad \eta = 0 < 1$

2) Quartic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.43$ $\quad \theta_1 = 2$ $\quad \theta_2 = -0.14$ $\quad \eta = -0.57 < 1$

3) Quartic Order with full η

$g^{2,1\,*}_{4,1} = -0.38$ $\quad \theta_1 = 2.27$ $\quad \theta_2 = -0.16$ $\quad \eta = -0.58 < 1$

4) Hexic Order with $\eta = 0$

$g^{2,1\,*}_{4,1} = -0.46$ $\quad g^{3,1\,*}_{6,1} = -0.50$ $\quad \theta_1 = 2$ $\quad \theta_2 = 0.53$ $\quad \eta = 0 < 1$

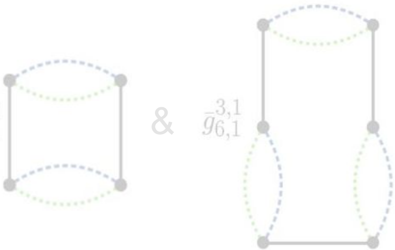
5) Hexic Order with polyn. η

$g^{2,1\,*}_{4,1} = -0.30$ $\quad g^{3,1\,*}_{6,1} = -0.18$ $\quad \theta_1 = 2$ $\quad \theta_2 = -0.03$ $\quad \eta = -0.40$

6) Hexic Order with full η

$g^{2,1\,*}_{4,1} = -0.28$ $\quad g^{3,1\,*}_{6,1} = -0.15$ $\quad \theta_1 = 2.19$ $\quad \theta_2 = -0.03$ $\quad \eta = -0.41$

$\bar{g}^{2,1}_{4,1}$ & $\bar{g}^{3,1}_{6,1}$ $\neq 0$



Cyclic-Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Stable under extension of truncation ✓

$g^{2,1\,*}_{4,1} = -1$	$\theta_1 = 2$	$g^{2,1\,*}_{4,1} = -0.46$	$\theta_1 = 2$
	$\theta_2 = 1$	$g^{3,1\,*}_{6,1} = -0.50$	$\theta_2 = 0.53$

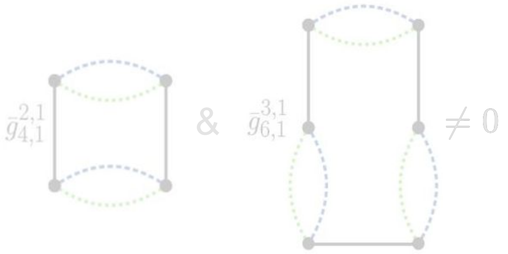
Regulator Bound ✓

2) η η

$g^{2,1\,*}_{4,1} = -0.43$	$\theta_1 = 2$	$g^{2,1\,*}_{4,1} = -0.30$	$\theta_1 = 2$
	$\theta_2 = -0.14$	$g^{3,1\,*}_{6,1} = -0.18$	$\theta_2 = -0.03$

3) Quartic Order with full η 6) Hexic Order with full η

$g^{2,1\,*}_{4,1} = -0.38$	$\theta_1 = 2.27$	$g^{2,1\,*}_{4,1} = -0.28$	$\theta_1 = 2.19$
	$\theta_2 = -0.16$	$g^{3,1\,*}_{6,1} = -0.15$	$\theta_2 = -0.03$



Cyclic-Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Stable under extension of truncation ✓

Regulator Bound ✓

Canonical Guiding Principle

$g^{2,1\,*}_{4,1} = -1$ $\theta_1 = 2$ $\theta_2 = 1$

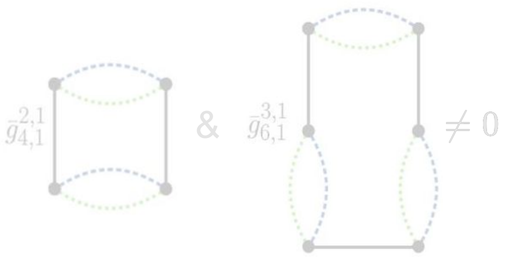
$g^{2,1\,*}_{4,1} = -0.46$ $g^{3,1\,*}_{6,1} = -0.50$ $\theta_1 = 2$ $\theta_2 = 0.53$

$g^{2,1\,*}_{4,1} = -0.43$ $\theta_1 = 2$

$g^{2,1\,*}_{4,1} = -0.30$ $g^{3,1\,*}_{6,1} = -0.18$ $\theta_1 =$

$g^{2,1\,*}_{4,1} = -0.38$ $\theta_1 = 2.27$ $\theta_2 = -0.16$

$g^{2,1\,*}_{4,1} = -0.28$ $g^{3,1\,*}_{6,1} = -0.15$ $\theta_1 = 2.19$ $\theta_2 = -0.03$



Cyclic-Melonic Singletrace FP

Recall: $\max(\theta) \leq 3.5$

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Starting Scheme: Quartic Order with $\eta = 0$

$g_{4,1}^{2,1\,*} = -1$ $\quad\theta_1 = 2$, $\theta_2 = 1$

2) Quartic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.43$ $\quad\theta_1 = 2$, $\theta_2 = -0.14$

3) Quartic Order with full η

$g_{4,1}^{2,1\,*} = -0.38$ $\quad\theta_1 = 2.27$, $\theta_2 = -0.16$

4) Hexic Order with $\eta = 0$

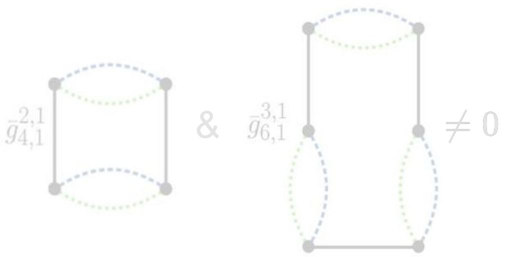
$g_{4,1}^{2,1\,*} = -0.46$, $g_{6,1}^{3,1\,*} = -0.50$ $\quad\theta_1 = 2$, $\theta_2 = 0.53$

5) Hexic Order with polyn. η

$g_{4,1}^{2,1\,*} = -0.30$, $g_{6,1}^{3,1\,*} = -0.18$ $\quad\theta_1 = 2$, $\theta_2 = -0.03$

6) Hexic Order with full η

$g_{4,1}^{2,1\,*} = -0.28$, $g_{6,1}^{3,1\,*} = -0.15$ $\quad\theta_1 = 2.19$, $\theta_2 = -0.03$



Cyclic-Melonic Singletrace FP

- 1 relevant directions @ Quartic and @ Hexic order for all approx. of η

(except @ All Orders, $\eta = 0 \longrightarrow$ 2 rel. dir.)

Stable under extension of truncation

$g^{2,1\,*}_{4,1} = -1$ $\theta_1 = 2$ $\theta_2 = 1$

$g^{2,1\,*}_{4,1} = -0.46$ $g^{3,1\,*}_{6,1} = -0.50$ $\theta_1 = 2$ $\theta_2 = 0.53$

Regulator Bound

η η

$g^{2,1\,*}_{4,1} = -0.43$ $\theta_1 = 2$

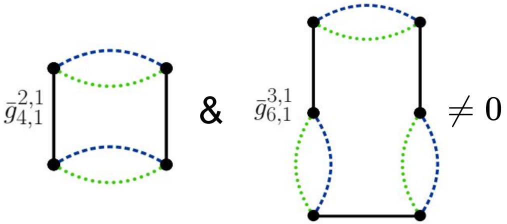
$g^{2,1\,*}_{4,1} = -0.30$ $g^{3,1\,*}_{6,1} = -0.18$ $\theta_1 = 2$

Canonical Guiding Principle

η η

$g^{2,1\,*}_{4,1} = -0.38$ $\theta_1 = 2.27$ $\theta_2 = -0.16$

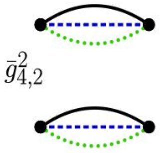
$g^{2,1\,*}_{4,1} = -0.28$ $g^{3,1\,*}_{6,1} = -0.15$ $\theta_1 = 2.19$ $\theta_2 = -0.03$



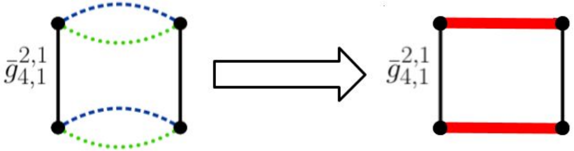
Cyclic Melonic Singletrace FP

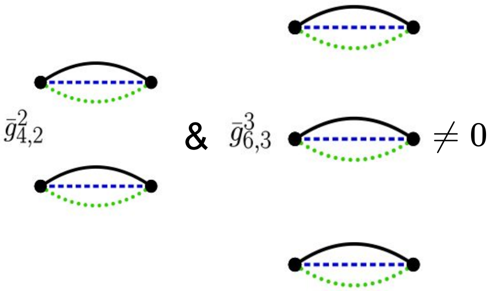
- Also found in complex model

- Distinguishing colours crucial in decoupling



- Dimensional Reduction





Multitrace Bubble FP

Starting Scheme: Quartic Order with $\eta = 0$

$g^{2}_{4,2}{}^{*} = -3.75$

$\theta_1 = 3$

$\theta_2 = -1.5$

2) Quartic Order with polyn. η

$g^{2}_{4,2}{}^{*} = -1.67$

$\theta_1 = 3$

$\theta_2 = 0.17$

$\eta = -0.83$

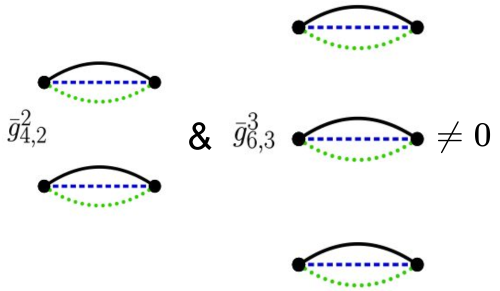
3) Quartic Order with full η

$g^{2}_{4,2}{}^{*} = -1.44$

$\theta_1 = 3.47$

$\theta_2 = 0.18$

$\eta = -0.84$



Multitrace Bubble FP

Starting Scheme: Quartic Order with $\eta = 0$

$${g^2_{4,2}}^* = -3.75$$

$$\theta_1 = 3$$
$$\theta_2 = -1.5$$

2) Quartic Order with polyn. η

$${g^2_{4,2}}^* = -1.67$$

$$\theta_1 = 3$$
$$\theta_2 = 0.17$$
$$\eta = -0.83$$

3) Quartic Order with full η

$${g^2_{4,2}}^* = -1.44$$

$$\theta_1 = 3.47$$
$$\theta_2 = 0.18$$
$$\eta = -0.84$$

4) Hexic Order with $\eta = 0$

$${g^2_{4,2}}^* = -1.73$$
$${g^3_{6,3}}^* = -7.45$$

$$\theta_1 = 3$$
$$\theta_2 = -1.5$$

5) Hexic Order with polyn. η

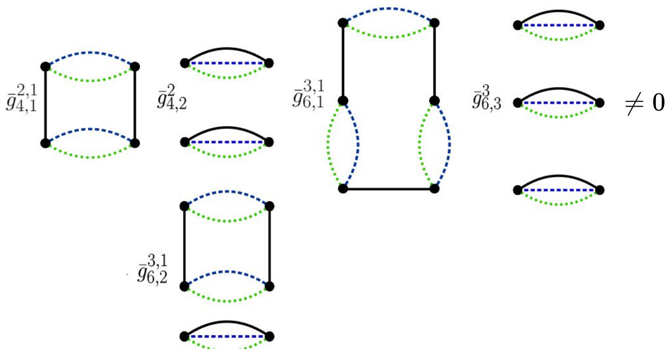
$${g^2_{4,2}}^* = -1.16$$
$${g^3_{6,3}}^* = -2.82$$

$$\theta_1 = 3$$
$$\theta_2 = -0.34$$
$$\eta = -0.58$$

6) Hexic Order with full η

$${g^2_{4,2}}^* = -1.05$$
$${g^3_{6,3}}^* = -2.27$$

$$\theta_1 = 3.32$$
$$\theta_2 = -0.33$$
$$\eta = -0.59$$



Cyclic Melonic Multitrace FP

2 relevant directions @ all orders except quartic order, $\eta = 0$

Sixth order, full η

$$\theta_1 = 2.23$$

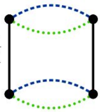
$$\theta_2 = 0.03$$

Results (r=1)

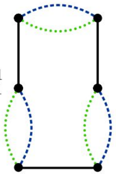
Dimensionally-reduced universality classes

- Cyclic Melonic singletrace FP

$\bar{g}_{4,1}^{2,1}$

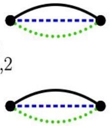


$\bar{g}_{6,1}^{3,1}$

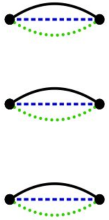


- Multitrace Bubble FP

$\bar{g}_{4,2}^{2}$



$\bar{g}_{6,3}^{3}$



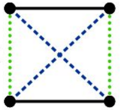
- Cyclic Melonic Multitrace FP

Cyclic Melons & Multi-Trace $\neq 0$

Candidates for universality classes for 3d Quantum Gravity

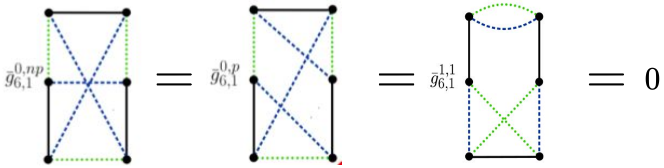
- Isocolored FP with tetrahedral interaction

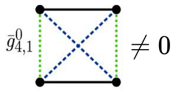
$\bar{g}_{4,1}^{0}$

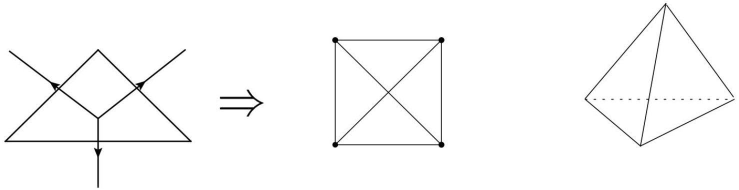


& No preferred color

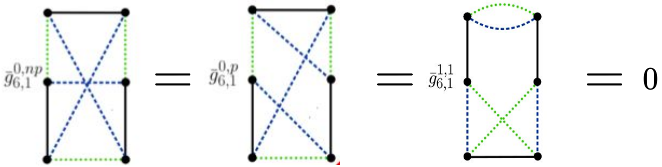
Isocolored FP with tetrahedral interaction

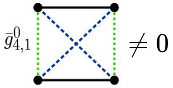




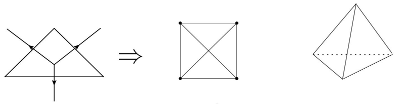


NEW!!! Not featured in the complex model





Isocolored FP with tetrahedral interaction



Starting Scheme: Quartic Order with $\eta = 0$

Not present ($\beta_{g^0_{4,1}} = (\frac{3}{2} + 2\eta) g^0_{4,1}$)

2) Quartic Order with polyn. η

$\theta_1 = 2.98$

$\theta_{2,3} = -0.28 \pm 0.22\, i$

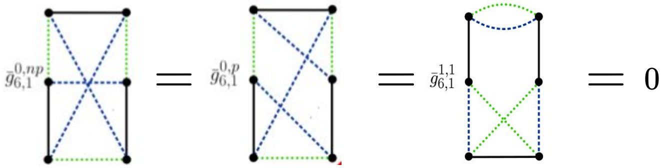
$\eta = -0.75$

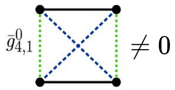
3) Quartic Order with full η

$\theta_1 = 2.60$

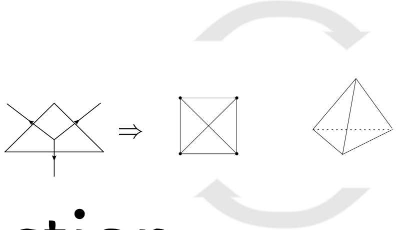
$\theta_{2,3} = -0.27 \pm 0.21\, i$

$\eta = -0.75$





Isocolored FP with tetrahedral interaction



Starting Scheme: Quartic Order with $\eta = 0$

Not present ($\beta_{g^0_{4,1}} = (\frac{3}{2} + 2\eta) g^0_{4,1}$)

2) Quartic Order with polyn. η

$\theta_1 = 2.98$

$\theta_{2,3} = -0.28 \pm 0.22\, i$

$\eta = -0.75$

3) Quartic Order with full η

$\theta_1 = 2.60$

$\theta_{2,3} = -0.27 \pm 0.21\, i$

$\eta = -0.75$

4) Hexic Order with $\eta = 0$

$\theta_{1,2} = 1.95 \pm 0.69\, i$

$\theta_{3,4} = 0.38$

$\theta_{5,6} = -0.03 \pm 5.96\, i$

5) Hexic Order with polyn. η

$\theta_{1,2} = 1.46 \pm 1.39\, i$

$\theta_{3,4} = 0.15$

$\theta_{5,6} = -0.11 \pm 4.83\, i$

$\eta = -0.32$

6) Hexic Order with full η

$\theta_{1,2} = 1.3 \pm 1.56\, i$

$\theta_{3,4} = 0.13$

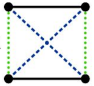
$\theta_{5,6} = -0.02 \pm 5.10\, i$

$\eta = -0.33$

Conclusions

- Indications for two types of universality classes
  - Dimensional Reduction
  - Candidate for 3d Quantum Gravity ?

- New universality class not featured in the complex model

$\bar{g}^0_{4,1}$ 

- FRG is a very useful tool to discover new universality classes

ありがとうございます